Introduction

Within all industries, transportation, order processing and warehousing are the main activities that support their supply chain (SC). In this context, transportation is the main activity that contributes to the emissions of pollutants. Vehicles generate air pollutants such as CO$_2$, NO$_x$, and SO$_2$. On-road vehicles in Europe generate 10% of CO$_2$ world emissions while commercial vehicles in the United Kingdom generate 22% of CO$_2$ emissions in Europe [1, 2].

These emissions are of particular concern due to their contribution to climate change. Recently, the integration of logistics and supply chain management through Green Logistics (GL) and Sustainable Development (SD) has been studied to address the environmental damage within the logistics processes of materials handling, waste management, packaging and transportation [1, 3, 4].
However, good inventory management may lead to increase pollutants due to transportation. High inventory turnover, which under Just-In-Time (JIT) management strategies is associated to minimization of inventory and operational costs, involves an increase of the transportation rate for inventory replenishment [5, 6]. Thus, it is necessary to focus not only on optimizing the levels of inventories but also on decreasing the associated generation of pollutants caused by their transportation.

The relationship between inventory replenishment and CO₂ emissions associated to its transportation needs to be explored to quantitatively assess its economic impact on the SC. Hence, in the present work, a logistic model is developed to analyze the effect of CO₂ emissions on inventory planning to focus on reducing both emissions and inventory costs while defining the replenishment strategy. This leads to the following contributions:

- Integration of CO₂ emissions patterns, costs and transportation distance within the inventory replenishment strategy for uncertain demand, which is frequently observed in practice.
- Normally distributed demand with significant variability is assumed.

The outcomes of this research can be used to explore the assessment of pollution in the economic aspects of SC, particularly on the transportation and inventory replenishment strategies. Also, it can lead to continuing reducing costs in the presence of emission taxes which must be considered in the inventory planning strategy.

**Literature Review**

While there are different pollutants such as N₂O and CH₄, 90% of air pollution is associated to CO₂ emissions which are caused by fuel combustion, representing 33% of greenhouse gas (GHG) emissions. Hence, CO₂ is considered as the main man-caused contributor to climate change [7] and it is associated to contamination and reduction of nutrients in crops [8].

The vehicles used in transportation, which support the global SC, transform 99% of fuel into CO₂ [9]. To reduce these emissions, it is necessary to consider the distance traveled by the vehicles, steep highway roughness, the operational characteristics of the vehicles (i.e., average speed), the behavior of the drivers, and weather and traffic conditions [10].

Because transportation depends on product demand, which is driven by consumer behavior, the CO₂ factor has been studied within inventory replenishment processes. In [11] an economic order quantity (EOQ) strategy, which incorporated a “carbon tax” for CO₂ emissions on holding and ordering costs, was presented.

In [12] the EOQ model was adapted to minimize the CO₂ emission costs in a two-echelon supply chain (retailer and manufacturer). Another work presented an EOQ model that considered CO₂ emission costs associated to the distance traveled between vendor and buyer, vehicle type, age, average speed, and weight [6]. However, these costs had pre-defined fixed/variable values, and the factors such as distance/vehicle type/age/weight were not explicitly analyzed.

In general, emission costs are expressed in terms of constant values and their relationship with the distance traveled has not been explored. Also, there are few studies on the importance of transportation route planning within the inventory replenishment strategy to reduce ordering and emission costs.

Hence, the present work proposes an extended inventory – transportation model to address their effect on CO₂ emissions to reduce operational costs and pollution. In practice, as inventory and transportation depend on variable consumer behavior, the proposed model considers uncertain demand in contrast to deterministic demand which is assumed by standard inventory replenishment strategies.

**Material and Methods**

**Inventory – Transportation Model with CO₂ Emissions**

The first step for the development of the proposed model is to define a CO₂ emission metric based on distance. This is performed to represent the relationship between travel and emission rate. In [13] a normally distributed metric was estimated from real emission data. This metric represented grams of CO₂ per kilometer (gCO₂/km) generated by the most common on-road vehicles and it was defined as:

\[ ECO_2 = 183.89 + Z \times (51.82) \]  \(1\)

...where 183.89 was the mean gCO₂/km, 51.82 was the standard deviation of gCO₂/km, and Z was the number of standard deviations considered to dynamically model the real uncertainty of the emission.

While in [13] Z was randomly defined within the range (-2.326, +2.326) to obtain a general emission value, in the present work a weight – based regression model is considered to estimate the emission for a specific vehicle as defined by its load/cargo. For this work, x is defined as the load’s weight which is measured within the range 1.5 to 7.45 tons, and \( y = f(x) \) is the emission function associated to transporting the load’s weight.

Mathematical modeling of \( y = f(x) \) was performed through polynomial interpolation considering the following steps:

a) Monte-Carlo simulation with the mean and standard deviation parameters defined by (1) was used to generate 120 emission samples;

b) then, these samples were sorted in ascending order and matched with 120 ascending weight values from 1.5 to 7.45;
c) Finally, the matched data were used as training data to estimate the polynomial model.

Fig. 1 presents the training data generated to estimate \( f(x) \) and the polynomial fit considering second and third degree polynomials. As presented, the best fit was obtained with the third degree polynomial which is expressed as:

\[
y = f(x) = 1.5801x^3 - 21.7480x^2 + 119.7086x - 57.7442 \quad \forall \ x \in [1.5, 7.45]
\]  

(2)

It is important to mention that this methodology can be followed to estimate other expressions for \( f(x) \) if more emission data becomes available. Meanwhile, with this expression for \( f(x) \) we can proceed to define the emission associated to a certain distance traveled.

As \( f(x) \) represents \( \text{gCO}_2/\text{km} \), the distance metric must be defined in kilometers. For this purpose, the distance metric based on the arc length \( d_y \) between two geographical locations \((i, j)\) on the spherical Earth was considered. This metric is computed as:

\[
d_y = r \times \arccos(\cos \theta_i \cos \theta_j \cos (\phi_i - \phi_j) + \sin \theta_i \sin \theta_j)
\]  

(3)

...where \( \theta_i \) and \( \theta_j \) are the latitudes and \( \phi_i \) and \( \phi_j \) are the longitudes of two geographical locations \((i, j)\) respectively. Then, \( r \) (6374 km) is the radius of the spherical Earth.

By multiplying (2) and (3) a normally distributed emission factor associated with the distance traveled by a vehicle with a load’s weight given by \( x \) is obtained. Then, by considering a standard emission tax cost \((t\text{CO}_2)\) associated to \( \text{gCO}_2/\text{km} \), the total emission cost of a vehicle transporting a load’s weight \( x \) through a distance between two geographical locations \((i, j)\) is estimated as:

\[
CE_{ij} = f(x) \times d_y \times t\text{CO}_2
\]  

(4)

With (4) it is expected that emission and its cost can be reduced if \( d_y \) is reduced. Another aspect to achieve the reduction of emissions is by controlling the load’s weight which is associated to its cargo whose size can be optimized by the inventory control strategy.

For optimization of the inventory lot size the continuous review model, or \((Q, R)\) model, is considered. In this model, \( R \) is the re-order inventory level which is associated to the order frequency. When the inventory reaches \( R \) an order request of a lot of size \( Q \) is performed [14]. \( R \) is not reached at specific periods because demand is uncertain (non-deterministic), thus, continuous tracking of the inventory level must be performed.

According to [15] the following costs and elements are associated to this model: \( C_s \) is the setup cost per order, \( C_h \) is the holding cost per unit, \( C \) is the purchase cost per unit, \( p \) is the stock-out cost per unit, \( D \) is the cumulative demand through a planning horizon, \( d \) is the average daily, weekly or monthly demand, \( LT \) is the lead time (delivery time of the lot \( Q \)), \( \mu_{LT} \) and \( \sigma_{LT} \) are the mean and standard deviation of the demand during the lead time, \( L(z) \) is the standard loss function, \( z = \Phi^{-1}(1 - (Q/C_h)/(pD)) \) and \( A \) is the expected shortage per cycle. The expected total inventory costs under the

![Stochastic Emissions](image)

**Fig. 1.** Weight-based normally distributed emission data.
(Q, R) strategy through a planning horizon is defined as

\[ EC = TOC + THCCI + THCSS + TSC, \]

where:

- **Total Order Cost (TOC)**
  \[ TOC = \left( \frac{DC_o}{Q} \right) \]
  (5)

- **Total Holding Cost on the Inventory Cycle (THCCI)**
  \[ THCCI = \left( \frac{ChQ}{2} \right) \]
  (6)

- **Total Holding Cost on the Safety Stock (THCSS)**
  \[ THCSS = C_h\left[R - \mu_{LT} + \sigma_{LT} L(z)\right] \]
  (7)

- **Total Shortage Cost (TSC)**
  \[ TSC = \left( \frac{pAD}{Q} \right) \]
  (8)

If each ordered unit has a weight determined by \( w \), then the total weight of the lot \( Q \) is determined by \( wQ \). Because vehicles have limited capacity, a restriction on \( wQ \) must be set. Considering a maximum load of \( 7.5 \) tons as defined by (2) then \( wQ \leq 7.5 \).

With these equations and restrictions, (4) is adapted to estimate the total transportation – emission cost (TTEC) associated to \( Q \) as:

\[ TTEC = \left( \frac{D}{Q} \right) \times \sum_{(i,j) \in B} CE_{ij}X_{ij} \]  
(9)

where \( B \) is the set of all locations between the source location (supplier of \( Q \)) and the destination location (warehouse) and \( X_{ij} \) is a binary arc variable which is equal to 1 if the arc \((i, j)\) belongs to the transportation route / path and it is equal to 0 otherwise. Note that (9) is an adaptation of the shortest path problem \([16]\) which is frequently solved with the Floyd-Warshall algorithm \([17]\).

\( CE_{ij} \) is extended from (4) to consider the outbound and inbound trips that the retailer’s vehicle must perform to collect and receive the lot \( Q \). When the order is requested the vehicle departs to the supplier’s facility (outbound trip) without any cargo (e.g., it departs empty). Hence, only emissions associated to the vehicle’s weight are generated (e.g., \( x = 1 \) ton). Once the vehicle arrives at the supplier’s facility the lot \( Q \) is loaded into the vehicle and the return trip (inbound trip) starts. At this point the vehicle is loaded with \( Q \) units, thus emissions are generated considering the vehicle’s weight plus the cargo’s weight (e.g., \( x = 1 \) ton + \( wQ \)). If distance symmetry is assumed (e.g., \( d_{ij} = d_{ji} \)) the emission costs associated to the outbound and inbound trips can be expressed as:

\[ CE_{ij} = \left[ f(2 \times \text{vehicle’s empty weight} + wQ) \right] \times 2 \times d_{ij} \times tCO_2 \]  
(10)

Finally, the inventory – transportation problem with emissions can be defined with the following mathematical formulation:

\[ \text{Minimize } EC = TOC + THCCI + THCSS + TSC + TTEC \]  
(11)

Subject to:

\[ \sum_{(i,j) \in \sigma^o(i)} X_{ij} - \sum_{(j,i) \in \sigma^e(i)} X_{ji} = \begin{cases} 
1 & \text{if } i = s \\
-1 & \text{if } i = t \\
0 & \text{otherwise} 
\end{cases} \quad \forall i \in V \]  
(12)

\[ \sum_{(i,j) \in \sigma^e(i)} X_{ij} \leq 1 \quad \forall i \in V \]  
(13)

\[ wQ + \text{vehicle’s empty weight} \leq 7.5 \]  
(14)

Fig. 2. Standard outbound/inbound route between the supplier and the retailer for delivery of the product lot \( Q \).
...where $V$ is defined as the set of locations within the transportation network, $s$ is the location of the supplier, and $t$ the location of the retailer. While (11) represents the objective function of the problem, (12) represents the flow conservation constraints between locations, (13) ensures that at most a single location can be reached from another location, and (14) represents the loading capacity restriction.

**Results and Discussion**

The assessment of the model was performed through an instance based on real geographical and inventory data. Fig. 2 presents the geographical data which consists of 1200 location points where locations “s” and “t” represent the supplier and the retailer respectively. Also, the considered standard inbound/outbound transportation route by the retailer, which leads to a total distance of approximately 1640.0568 km, is drawn.

The inventory data, which includes the weight of the product and the capacity of the vehicle, is presented in Table 1.

With this data, the following scenarios were assessed:

a) Standard route with CO$_2$ emissions (STD$\_CO_2$-$Q$): this is the baseline case which is frequently observed in practice, where $Q$ is determined by the maximum capacity of the vehicle and it is ordered on a fixed basis. From Table 1, this leads to consider $Q$ as equal to (7500 kg – 1000 kg)/0.8 kg = 8125 units which are ordered 14616/8125 ≈ 2 times through the planning horizon. The standard delivery route is considered as presented in Figure 1 (total distance = 1640 km). The total cost (EC) for this scenario is estimated by direct substitution of these values on (9) and (11).

b) Standard route with CO$_2$ emissions and $Q$ optimization (STD$\_CO_2$-$Q^*$): this is the partially-optimized case when an inventory supply strategy is considered to determine the best size of $Q$ ($Q^*$) to minimize total costs. The standard delivery route is considered as presented in Fig. 1 (total distance = 1640 km). The total cost (EC) for this scenario is estimated by direct substitution of the standard total distance on (9) and the optimization of $Q$ on (11).

c) Optimal route with CO$_2$ emissions and $Q$ optimization (OPT$\_CO_2$-$Q^*$): this is the complete optimized case (OPT) when the route and inventory supply strategies are considered to determine the optimal size of $Q$ ($Q^*$). Thus, the total cost (EC) for this scenario is estimated by the optimization of $Q$ on (11) and the optimization of the delivery route on (9).

As previously mentioned, optimization of the outbound/inbound delivery route was performed through the Floyd-Warshall algorithm [17]. In contrast, the GRG Nonlinear method was considered for the optimization of $Q$. Microsoft Excel® Solver® was considered as the implementation and solving tool for the proposed model under the STD$\_CO_2$-$Q^*$ and OPT$\_CO_2$-$Q^*$ scenarios.

An important value which has not been discussed is the value for $tCO_2$. Defining a value for $tCO_2$ is a complex task because each region has established it at different rates [18]. Overall, energy taxes depend of the governments’ plans for the development of processes and infrastructure to improve energy generation/consumption and sustainability [19].

This is the reason to perform the assessment with different values for $tCO_2$. Table 2 presents the total costs obtained for all three scenarios for $tCO_2$ within the range from $0.00002$ to $0.10000$ /gCO$_2$.

When compared with the standard scenario STD$\_CO_2$-$Q$, average savings up to 10.22% and 12.77% can be obtained on the total costs with the optimization of the lot size (STD$\_CO_2$-$Q^*$) and with additional optimization of the outbound/inbound delivery route (OPT$\_CO_2$-$Q^*$) respectively. As presented in Fig. 3, as the value of $tCO_2$ increases, the complete optimization...
of EC (11) (OPT_CO2_Q*) leads to more significant savings when compared to the partial optimization of EC (STD_CO2_Q*). In Fig. 4 the optimized route for the OPT_CO2_Q* scenario is drawn. This route leads to a total distance of approximately 1478.7162 km which represents a reduction of 9.8% when compared to the standard route. These savings can be more significant under a multi-route/supplier inventory control strategy.

Additional to this cost reduction, it is important to consider the reduction in CO2 emissions. As presented in Fig. 5, 588 kg/CO2 are estimated to be generated under the standard scenario STD_CO2_Q per outbound/
inbound delivery route. If optimization of the lot size is performed, an average reduction of 36.58% can be achieved (STD_CO2_Q*). Further reductions in CO2 emissions, up to an average of 43%, can be obtained if the outbound/inbound delivery route is optimized (OPT_CO2_Q*). As observed, tCO2 is important to determine the lot size Q which is directly associated to the vehicle’s weight and thus, to the CO2 emissions per kilometer.

**Conclusions**

From the business perspective, it is important to reduce operational costs associated with its SC. However, in practice, this may lead to negative impacts on the environment. High inventory turnover, which is a desirable performance metric within the SC of all industries, involves increasing the transportation rate for inventory replenishment. As consequence, this can
increase the GHG emissions generated by on-road transportation.

While the standard policy to add an emission tax to transportation aims to provide governments with economic resources for the development of green technologies and sustainable infrastructure, it does not necessarily lead to reduce GHG emissions. Within this context, the present work addressed the impact of route planning and inventory control on the emissions of CO₂ and operational costs. For this purpose, an integrated inventory – transportation mathematical model was developed.

When considering the standard scenario, where route planning and inventory levels are not optimized, there are significant operational costs and CO₂ emissions. Just on the economic aspect, operational costs can be reduced by 10.22% and 12.77% if inventory levels and the transportation route are optimized. However, the outcomes are more significant on the environmental aspect. By just optimizing the inventory levels an average reduction of 36.58% can be achieved on CO₂ emissions. Additionally, if route planning is performed, a reduction up to 43.02% can be achieved.

Hence, the results of the present work provide insights regarding the importance of the lot size and route planning to reduce the emissions per delivery routes and thus, to reduce operational costs in accordance to emission reduction. Application of the mathematical model described in (11) - (14) can support companies to visualize how their operations contribute to transportation aims to provide governments with economic resources for the development of green technologies and sustainable infrastructure, it does not necessarily lead to reduce GHG emissions. Within this context, the present work addressed the impact of route planning and inventory control on the emissions of CO₂ and operational costs. For this purpose, an integrated inventory – transportation mathematical model was developed.

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Hence, the results of the present work provide insights regarding the importance of the lot size and route planning to reduce the emissions per delivery routes and thus, to reduce operational costs in accordance to emission reduction. Application of the mathematical model described in (11) - (14) can support companies to visualize how their operations contribute to CO₂ emissions and develop internal strategies to reduce them in accordance to their operational costs. This can lead to incentives for internal improvement and long-term compliance with environmental policies.

Future work can be performed on the following key aspects:
- Determination of emission tax value for specific industries and businesses.
- Development of transportation infrastructure to reduce distances and use alternative vehicles with less emissions for long routes.
- Adaptation of alternative supply strategies for inventory control.

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Conflict of Interest

The authors declare no conflict of interest.

References


