Introduction

With the rapid development of industrialization, people’s living standards have been greatly improved. However, the consumption of a mass of fossil fuels has also led to a sharp increase in CO₂ emissions [1]. Intergovernmental Panel on Climate Change (IPCC) report: greenhouse gas emissions from human activities are the main cause of global warming and the frequency of extreme weather events [2]. Many countries and regions have promulgated various policies to limit carbon emissions, among which the most widely used policy is carbon emissions permit trading. Carbon emission trading refers to enterprises in the same industry, if the actual carbon emissions of the enterprise are lower than the government quota by means of increasing technological innovation, the enterprise can sell the extra quota in the carbon trading market to gain profits, if the carbon emission of an enterprise exceeds the quota, it needs to purchase the remaining quota.

Original Research


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Abstract

In the context of green development, cooperation between government and enterprises in emission reduction is an effective way to improve the efficiency of enterprise emission reduction. In carbon trading policies, this paper assumes that the consumer demand influenced by product green level and enterprise goodwill, considering the continuous production in a multiple cycle under the condition of dynamic change characteristics, constructs the government and enterprise cooperation to reduce emissions of three kinds of differential game model, to explore the effective way of government and enterprise cooperation. The results show that cost-sharing contracts can achieve Pareto improvement of revenue for the government, enterprise under a certain condition; the revenue of the government, enterprises, and systems are optimal under collaborative cooperation. Finally, we prove the validity of the conclusion through an example analysis, and the sensitivity of related parameters is analyzed.

Keywords: carbon emissions permit trading, low-carbon emission reduction efforts, low carbon preference, cost-sharing, differential game

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of other enterprises in the market to maintain production [3]. Carbon emission trading can effectively control the production emissions of enterprises, but it is at the cost of reducing the production scale of enterprises and slowing down economic development [4]. Therefore, only by promoting enterprises to actively carry out green transformation and upgrading, can we achieve both economic growth and environmental protection.

In recent years, many scholars have studied enterprise emission reduction under carbon trading. In terms of production and operation management of enterprises, Dobos used the dynamic Arrow-Karlin model to analyze the influence of the carbon emission trading mechanism on enterprise production and inventory strategy [5]. Du et al. studied the optimal production model of an enterprise with multiple emission access channels under the carbon emission trading machine [6]. Zhang and Xu proposed a linear solution with high complexity to analyze the optimal production strategy under the constraints of carbon cap and trade [7]. Xu et al. considered the influence of carbon cap-and-trade and deduced the optimal total emissions and optimal output of the product [8]. The above literature makes an important contribution to the application of carbon emission trading at the micro-level. However, they fail to consider the relationship between government and enterprises.

In terms of cooperative emission reduction, some scholars have studied the cooperative emission reduction of supply chain under the carbon trading mechanism. Such as, Yang et al. [9] studied supply chain pricing and carbon emission reduction decisions under vertical and horizontal cooperation considering the influence of carbon trading mechanism. Hou et al. [10] studied the investment decisions of manufacturers and retailers in emission reduction technologies under the mechanism of carbon trading and emission reduction cost sharing. Yin et al. [11] studied the carbon emission reduction and coordination strategy of the closed-loop supply chain of new energy vehicles.

Wang et al. [12] studied the two-way cost sharing and emission reduction decisions of manufacturers and retailers based on the carbon cap-and-trade system.

Research on the cooperation between government and enterprises on emission reduction mainly focuses on carbon subsidies. Li et al. studied the role of government subsidy policies in reducing carbon emissions [13]. Hafezalkotob et al. found that appropriate government subsidies can encourage manufacturers to carry out low-carbon production and reduce the environmental impact of the supply chain [14]. Zhang et al. studied the influence of government subsidy policies on manufacturers’ emission reduction decisions under the carbon cap-and-trade system [15]. Cao et al. [16] studied the optimal production and carbon emission reduction level of manufacturers under carbon cap-and-trade and low-carbon subsidy policies. Different from the above studies, this paper studies the cooperation between government and enterprises from the aspects of green goodwill and cost sharing.

In the process of pushing enterprises to reduce emissions, consumers’ low-carbon preference plays a key role [17]. Some scholars have studied consumers’ low-carbon preferences, Wang et al. assumed that consumers had a preference for low-carbon products, studied the emission reduction investment behaviors and strategies of upstream and downstream enterprises [18]. Li et al. found that more and more consumers pay attention to enterprises’ environmental records when making purchase decisions [19]. Fei et al. found the understanding degree of carbon label, the acceptability degree of carbon label, the credibility of carbon label, the reference group, and the price of low-carbon products are all important factors influencing consumers’ willingness to buy low-carbon products [20]. Under the low carbon preference of consumers, the market demand mainly depends on the green goodwill and the green level of products of enterprises [21, 22]. However, the formation of green goodwill of enterprises requires not only needs the enterprises’ own low-carbon emission reduction efforts but also needs the government’s low-carbon efforts (low-carbon publicity).

In the dual role of government and enterprises, the green goodwill of enterprises is constantly changing.

In reality, cooperation between government and enterprises to reduce emissions is a long-term and dynamic process. As an important dynamic game model, differential game can solve dynamic equilibrium results in continuous time [23]. The early application of differential game theory to the pursuit model in the military field, recently, more and more literature has applied this method to the field of management science, such as technological innovation management [24], environmental management [25], and production management [26]. Some researchers have developed game models in Low carbon emissions, mainly focused on the emission reduction of supply chain enterprises [27-30], but they do not involve the research on the government incentive for enterprise emission reduction. Therefore, assuming that consumer demand is affected by the green level of products and green goodwill of enterprises, and considering the carbon trading mechanism, this paper use the differential game model to analyzes the cooperative emission reduction between government and enterprises from the dynamic perspective.

The main contributions of this paper are as follows. (1) We link the demand of consumers with the green level of products and the green goodwill of enterprise, and fundamentally analyze the motivation of enterprise emission reduction. (2) The influence of carbon emission trading is considered in the process of enterprise emission reduction. (3) The differential game model is used to study the emission reduction efforts of governments and enterprises from a dynamic perspective.
Material and Methods

Problem Description

This paper takes the emission reduction system composed of the government and individual enterprises under the background of low carbon as the research object, studying the joint emission reduction problem between the government and enterprises. In the joint emission reduction process, the government shares part of the cost of low-carbon emission reduction for enterprises and publicizes their emission reduction behaviors (advertising effect), thus encouraging enterprises to actively invest in low-carbon emission reduction.

Considering that the government plays an important role in the process of low-carbon emission reduction, we take the government as the leading party of the Stackelberg differential game, and the enterprise is the follower. The decision-making process of governments and enterprises is as follows: First, the government decides the level of publicity effort and the proportion of emission reduction costs shared by enterprises. Then, enterprises determine low-carbon emission reduction efforts according to the government’s decision.

In the whole process, the revenue of the enterprise is mainly brought by the increase of consumer demand caused by the improvement of green goodwill and the green level of the product. The revenue of the government mainly includes two parts. First, the social benefits of environmental improvement brought about by enterprise low-carbon emission reduction. Second, an increase in tax revenue resulting from the revenue increase of the enterprise. The decision-making process is shown in Fig. 1.

Conditional Hypothesis

Hypothesis 1 – The green goodwill of enterprises is related to the level of publicity efforts of the government and the level of low-carbon emission reduction efforts of enterprises, and it is a dynamic process. Drawing on Nerlove-Arrow’s classic advertising goodwill model [31]. The change process of green goodwill can be expressed by the following differential equation:

$$\dot{R}(t) = \lambda_1 S_1(t) + \lambda_2 S_2(t) - \delta R(t)$$

Where $R(t)$ represents the green goodwill of the enterprise at moment $t; S_1(t)$ represents the level of propaganda efforts of the government at moment $t; S_2(t)$ represent the low carbon emission reduction efforts of the government and enterprises at moment $t; \lambda_1$ and $\lambda_2$ respectively represent the influence coefficients of the emission reduction efforts of the government and enterprises on the green goodwill of the enterprise, $\delta$ represents the natural attenuation coefficient of enterprise green reputation; $R(t)$ represents the change rate of the enterprise’s green goodwill with time $t$.

Hypothesis 2 – Under the carbon emission trading mechanism, the flow of carbon trading funds of enterprises is expressed as follows:

$$T = p_r [eq_b - S_2(t) - U]$$

Where $T$ represents the capital flow of carbon emissions permit trading, $p_r$ represents the price of carbon emissions permit trading, $e$ represents the carbon emission per unit product produced, $q_b$ represents the output of the product, $U$ represents the carbon quota obtained by the enterprise.

Hypothesis 3 – The green level of the product is consistent with the level of the enterprise's low-carbon emission reduction efforts. Consumers have green preference psychology, and their purchasing behavior is jointly influenced by the green goodwill of the enterprise and the green level of the product. The demand function of the enterprise's product can be expressed as follows:

Hypothesis 3 – Consumer demand is affected by the green goodwill of the enterprise and the green level of the product. We assumed that the green level

Fig. 1. Schematic diagram of government and enterprise cooperation in emission reduction.
of the product is consistent with the enterprise’s emission reduction efforts. The consumer demand is expressed as:

\[ Q(t) = \mu S_2(t) + \theta R(t) \]

Where, \( \mu \) represents the influence coefficient of the green level of product on consumer demand, \( \theta \) represents the influence coefficient of green goodwill on consumer demand.

**Hypothesis 4** – Considering the convexity of effort costs, based on the effort cost in previous literature [32], we assume that the cost of government publicity efforts and the low carbon emission reduction costs of enterprises at moment \( t \) are as follows:

\[ C_1(t) = \frac{k_1}{2} S_1^2(t), \quad C_2(t) = \frac{k_2}{2} S_2^2(t) \]

Where \( C_1 \) and \( C_2 \) respectively represent the costs of the government and enterprises at the moment \( t \); \( k_1 \) and \( k_2 \) represent the cost coefficients of the government and enterprises respectively.

**Hypothesis 5** – The target revenue function of government and enterprise are expressed as follow:

\[
\begin{align*}
\max_{S_1(t), \pi_1} J_1 &= \int_0^\infty e^{-\rho t} [\pi_1 S_1(t) + \pi_2 Q(t) - C_1(t) - L(t)C_2(t)] dt \\
\max_{S_2(t), \pi_2} J_2 &= \int_0^\infty e^{-\rho t} [bQ(t) - p_e(eq_b - S_2(t) - U) - (1 - L(t))C_2(t)] dt
\end{align*}
\]

Where \( \pi_1 \) represents the influence coefficient of enterprise’s emission reduction efforts on government revenue; \( \pi_2 \) and \( b \) represent the influence coefficient of consumer demand on government and enterprise revenue respectively; \( \rho \) represent discount rate. \( L(t) \) represents the cost sharing ratio.

In the case of dynamic parameters in the model, it will be very difficult to solve, so this paper assumes that the parameters in the model are time-independent according to the processing method. In addition, for the convenience of writing, the unit of time \( t \) will be omitted in the following writing.

**Non-Cooperative Contract**

Under the non-cooperative contract, the government and the enterprise make decisions independently to maximize their respective revenue. Learning from the literature [33], the objective function of the government and the enterprise can be expressed as:

\[
\begin{align*}
\max_{S_1(t), \pi_1} J_1 &= \int_0^\infty e^{-\rho t} [\pi_1 S_1(t) + \pi_2 Q(t) - C_1(t) ] dt \\
\max_{S_2(t), \pi_2} J_2 &= \int_0^\infty e^{-\rho t} [bQ(t) - p_e(eq_b - S_2(t) - U) - C_2(t)] dt
\end{align*}
\]

**Proposition 1** – Under the non-cooperative contract, the optimal publicity effort level of government and the optimal emission reduction effort level of enterprises are:

\[
(S_1^*, S_2^*) = \left( \frac{\lambda \pi_1 \theta}{k_1 (\rho + \delta)}, \frac{(\mu b + p_e)(\rho + \delta) + \lambda \theta}{k_2 (\rho + \delta)} \right)
\]

**Results and Discussion**

**Model Analysis**

Based on the description and hypothesis of the problem in the previous section, this section further analyzes whether the cost-sharing contract can make the optimal decision of the government and enterprises to reach the optimal level of cooperation. If it cannot reach the optimal level of cooperation, the paper analyzes whether the cost-sharing contract can make the optimal revenue of the government and enterprises realize pareto improvement and the degree of improvement, and to provide the decision-making basis for the future government-enterprise cooperation emission reduction.
According to the structure forms of (10) and (11), we can infer that the optimal linear function $V, V_2$ conforms to the solution conditions of the HJB equation [35]. Therefore, we assume that $V_1 = c_1 R + c_3, V_2 = d_1 R + d_3$, where $c_1, c_3, d_1, d_3$ is a constant. Substituting $V_1, V_2$ into (10) and (11), we have:

$$\rho_{V_1}^N = \left[ \frac{\pi \mu b + \pi \alpha \mu b V_1^N + \pi \alpha \mu b + \pi \gamma b V_1^N + \mu p - \theta R}{k_1} \right] + \frac{1}{2} \left( \frac{(\alpha \mu b V_1^N)^2}{k_1} + \frac{(\alpha \mu b + \alpha \gamma b V_1^N)^2}{k_1} + \frac{(\alpha \mu b + \alpha \gamma b V_1^N)^2}{k_1} - \delta R \right)$$

(10)

$$\rho_{V_2}^N = \left[ \frac{b(\mu b + \alpha \mu b + \mu p + \theta R) - p_e \lambda_1 (\mu b + \alpha \mu b + \mu p - U)}{k_3} \right] + \frac{1}{2} \left( \frac{(\mu b + \alpha \mu b + \mu p)^2}{k_3} + \frac{(\mu b + \alpha \mu b + \mu p)^2}{k_3} + \frac{(\mu b + \alpha \mu b + \mu p)^2}{k_3} - \delta R \right)$$

(11)

By sorting out Equations (12) and (13), the coefficient of the optimal linear function is:

$$c_1 = \frac{\pi \mu b + \pi \alpha \mu b V_1^N + \pi \alpha \mu b + \pi \gamma b V_1^N + \mu p - \theta R}{k_1} \left[ \frac{1}{2} \left( \frac{(\alpha \mu b V_1^N)^2}{k_1} + \frac{(\alpha \mu b + \alpha \gamma b V_1^N)^2}{k_1} + \frac{(\alpha \mu b + \alpha \gamma b V_1^N)^2}{k_1} - \delta R \right) \right]$$

(12)

$$c_2 = \frac{b \mu b + b \alpha \mu b + b \mu p + b \theta R - p_e \lambda_1 (\mu b + \alpha \mu b + \mu p - U)}{k_3} \left[ \frac{1}{2} \left( \frac{(\mu b + \alpha \mu b + \mu p)^2}{k_3} + \frac{(\mu b + \alpha \mu b + \mu p)^2}{k_3} + \frac{(\mu b + \alpha \mu b + \mu p)^2}{k_3} - \delta R \right) \right]$$

(13)

Finally, substituting equations (16) and (17) into equations (8) and (9), we can get equation (5).

**Corollary 1** – Under the non-cooperative contract, the optimal publicity effort level of government $S_1$ is negatively correlated with the government’s emission reduction cost coefficient $k_1$, discount rate $\rho$, and goodwill attenuation coefficient $\delta$; and is positively correlated with the influence coefficient $\gamma_1$ of green goodwill of enterprises on demand, influence coefficient $\pi_1$ of consumer demand on government revenue.

**Corollary 2** – The optimal emission reduction effort $S_2$ of the enterprise is negatively correlated with the cost coefficient $k_2$, the discount rate $\rho$ and the goodwill attenuation coefficient $\delta$; and is positively correlated with the influence coefficient $\mu$ of the green level of the products on consumer demand, the influence coefficient $b$ of consumer demand on enterprise revenue, the carbon trading price $\rho_1$, the influence coefficient $\pi_2$ of consumer demand on government revenue. The carbon trading price $\rho_1$ is positively correlated with their own revenue. This shows that both sides make decisions from the perspective of maximizing their own interests without considering the revenue of the system. When both sides formulate their own strategies based on the maximization of the revenue of the system, both the government and the enterprise will improve their revenue situation. We can see from corollary 2 that the enterprise’s emission reduction effort level is negatively affected by its emission reduction cost coefficient. The emission reduction level of enterprises can be improved by reducing their emission reduction costs. Therefore,
the government can encourage enterprises to invest in low-carbon emission reduction by sharing their emission reduction costs.

**Cost Sharing Contract**

Under the cost-sharing contract, the government chooses to share a certain proportion of emission reduction costs for enterprises to encourage enterprises to carry out low-carbon emission reduction. From the perspective of long-term dynamics, the decision of low-carbon emission reduction efforts between the government and enterprises constitutes a government-led Stackelberg game. The government first makes decisions on the level of optimal publicity effort and the proportion of cost sharing, and then the enterprises determine their own low-carbon emission reduction efforts according to the decision of the government. In this case, the revenue function of the government and enterprises can be expressed as:

\[
\max_{S_1} J_1^0 = \int_0^\infty e^{-\rho t} [\pi S_1 + \pi Q - C_1 - LC_2] dt
\]

(18)

\[
\max_{S_1} J_1^1 = \int_0^\infty e^{-\rho t} [bQ - p_t \{eq_s - S_2 - U\} - (1 - L)C_2] dt
\]

(19)

**Proposition 2** – Under the cost-sharing contract, the optimal equilibrium strategy between the government and enterprises is:

\[
S_2^{\rho*} = \frac{\lambda \pi \theta}{k (\rho + \delta)}
\]

(20)

\[
S_2^{\rho*} = \begin{cases} 
\frac{2(\mu + \pi \mu + b \mu + p_t) + \lambda \theta (2\pi + b)}{2k_1 (\rho + \delta)}, & 2B > A \\
\frac{(\mu b + p_t k_1) \rho + \lambda \theta b}{k_1 (\rho + \delta)}, & 2B \leq A 
\end{cases}
\]

(21)

\[
L^* = \begin{cases} 
\frac{2(\pi + \pi \mu + \frac{\lambda \pi \theta}{\rho + \delta}) - (b \mu + p_t + \frac{\lambda \theta b}{\rho + \delta})}{2(\mu + \pi \mu + \frac{\lambda \pi \theta}{\rho + \delta}) + (b \mu + p_t + \frac{\lambda \theta b}{\rho + \delta})}, & 2B > A \\
0, & 2B \leq A 
\end{cases}
\]

(22)

where \(A = b\mu + p_t + \frac{\lambda \theta b}{\rho + \delta}, \ B = \pi + \pi \mu + \frac{\lambda \pi \theta}{\rho + \delta}\).

**Proof** We use the optimal control method to solve the equation, the HJB equation of the enterprise can be expressed as:

\[
\rho V_1^{\rho} = \max_{S_1} \left\{ b(\mu S_1 + \pi Q + \pi R - p_t (eq_s - S_2 - U) - (1 - L) \frac{k_1 S_2^2}{2} + \frac{\lambda S_1 + \lambda S_2 - \delta R}{2} \right\}
\]

(23)

According to (22), by solving the first-order condition of \(S_2\), we can obtain:

\[
S_2^{\rho} = \frac{\mu + p_t + \lambda V_2^{\rho}}{(1 - L)k_1}
\]

(24)

The government determine its level of publicity effort and cost sharing ratio according to the rational response of enterprises. At this time, the government’s HJB equation can be expressed as:

\[
\rho V_1^{\rho} = \max_{S_1} \left\{ \pi (2B + A)B + \pi \mu (2B + A)B + \pi \theta R - \frac{(\lambda V_2^{\rho})^2}{2k_1} - \frac{(2B - A)B^2}{8k_1 A^2} + \frac{\lambda V_2^{\rho}}{k_1} - \frac{\lambda (2B + A)B}{2Ak_2} - \delta R \right\}
\]

(25)

Substituting Equation (24) into Equation (25), and solve the first derivative of publicity effort level \(S_1\) and cost sharing ratio \(L\), we can get:

\[
S_1^{\rho} = \frac{\lambda V_2^{\rho}}{k_1}
\]

(26)

\[
\rho V_1^{\rho} = \max_{S_1} \left\{ \frac{(2B + A)B + \pi \mu (2B + A)B + \pi \theta R - \frac{(\lambda V_2^{\rho})^2}{2k_1}}{2Ak_2} - \frac{(2B - A)B^2}{8k_1 A^2} + \frac{\lambda V_2^{\rho}}{k_1} - \frac{\lambda (2B + A)B}{2Ak_2} - \delta R \right\}
\]

(28)

Similarly, let \(V_1^{\rho} = p_t R + p_t, V_2^{\rho} = q_t R + q_t\), where \(p_t, p_t, q_t, q_t\) are constants. Substituting \(V_1^{\rho}, V_2^{\rho}\) into (28) and (29) respectively, we get:

\[
\rho (p_t R + p_t) = \max_{S_1, S_2} \left\{ \frac{(2B + A)B + \pi \mu (2B + A)B + \pi \theta R - \frac{(\lambda V_2^{\rho})^2}{2k_1}}{2Ak_2} - \frac{(2B - A)B^2}{8k_1 A^2} + \frac{p_t (\lambda V_2^{\rho})}{k_1} - \frac{\lambda (2B + A)B}{2Ak_2} - \delta R \right\}
\]

(30)
The coefficients of the optimal linear function can be obtained as follows:

\[
\rho(q,R + q) = \frac{b\mu(2B + A)B}{2Ak_2} + b\theta R - p_e(eq_s - \frac{(2B + A)B}{2Ak_2} - U) - q(\frac{\lambda_1}{k_1} - \frac{\lambda_2}{2Ak_2} - \delta R)
\]

(31)

The government's publicity effort cost coefficient is negatively correlated with the discount rate correlated with proportion of government contribution of emission reduction for enterprises. The optimal only when 2B > A, the government will share the cost of emission reduction for enterprises, which reduces the pressure of emission reduction and improves their optimal emission reduction efforts, which is consistent with the expectation above.

### Cooperation Contract

In this section, we discuss the collaborative efforts of government and industry to reduce emissions. We assume that both government and business make their own decisions with the goal of maximizing the revenue of the system.

**Proposition 3** – Under the cooperation contract, the optimal publicity effort level of the government and the optimal emission reduction effort level of the enterprise are:

\[
S_{\gamma}^* = \frac{\lambda_2(\pi_2 + b)\theta}{k_2(\rho + \delta)}
\]

(34)

\[
S_{\gamma}^* = \frac{(\rho + \delta)[(\pi_1 + b)\mu + p_e] + \lambda_2(\pi_2 + b)\theta}{k_1(\rho + \delta)}
\]

(35)

**Proof** The objective function of system revenue can be expressed as:

\[
\max_{S_1, S_2} J^s = \int_0^\infty e^{-\rho t}[\pi_1 S_1 + (\pi_2 + b)Q - p_e(eq_s - S_2 - U) - C_1 - C_2]dt
\]

(36)

At this point, the system optimal revenue function \(V^s\) satisfies the HJB equation, we have:

\[
\rho V^s = \left\{ \begin{array}{ll}
\max_b [\pi_1 S_1 + (\pi_2 + b)Q - p_e(eq_s - S_2 - U) - C_1 - C_2] + \\
V_{\gamma}^s (\lambda_2 S_2 - \lambda_1 S_1 - \delta R)
\end{array} \right.
\]

(37)

Solving the first-order conditions of \(S_1, S_2\), we can get:
where \( V_c^i = \frac{\partial V_c^i}{\partial R} \), substituting (35) and (36) into (34), we can get:

\[
\rho V_c^i = \left\{ \begin{array}{l}
\pi_1 + (\pi_2 + b)\mu + p_c + \lambda V_c^i \\
\pi_1 + (\pi_2 + b)\mu + p_c + \lambda V_c^i - U \end{array} \right\} - \left( \frac{\partial V_c^i}{\partial k_i} \right)\frac{\partial k_i}{\partial R}
\]

We assume that the linear optimal function \( V_c^i \) satisfies the solution of the HJB equation, let

\[
V_c^i (R) = m_i R + m_2
\]

where \( m_1, m_2, \) is a constant.

Similarly, by substituting (41) into (40), the coefficient of the optimal revenue function can be obtained:

\[
\begin{align*}
\rho_i &= \frac{\pi_i + (\pi_2 + b)\mu + p_c + \lambda_i}{\pi_i + (\pi_2 + b)\mu + p_c + \lambda_i - U} \\
\rho_i &= \frac{\pi_i + (\pi_2 + b)\mu + p_c + \lambda_i}{\pi_i + (\pi_2 + b)\mu + p_c + \lambda_i - U} - \left( \frac{\partial V_c^i}{\partial k_i} \right) \frac{\partial k_i}{\partial R}
\end{align*}
\]

where \( B_i = \pi_i + (\pi_2 + b)\mu + p_c + \lambda_i m_i \).

Substituting \( m_1, m_2 \) into Equation (41), we can obtain the optimal revenue function of the system as follows:

\[
V_c^i = \left( \pi_i + (\pi_2 + b)\mu + p_c + \lambda_i \right) R - \frac{\pi_i B_i}{\pi_i + (\pi_2 + b)\mu + p_c + \lambda_i} - \left( \frac{\partial V_c^i}{\partial k_i} \right) \frac{\partial k_i}{\partial R}
\]

Taking the derivative of Equation (42) with respect to \( R \) and substituting theirs in (38) and (39), we can get (34) and (35).

**Corollary 6** – Under the government-enterprise cooperation emission reduction contract, the government’s optimal level of propaganda efforts \( S_1^{ce} \) is positively correlated with the influence coefficient \( \lambda_i \) of the government’s propaganda efforts on enterprise green goodwill, the influence coefficient \( \pi_2 \) of consumer demand on government revenue, the influence coefficient \( b \) of consumer demand on enterprise revenue and the influence coefficient \( \theta \) of green goodwill on consumer demand; is negatively correlated with discount rate \( \rho \), the coefficient \( k_i \) of the government’s propaganda efforts cost and the coefficient \( \delta \) of enterprise green goodwill attenuation.

**Corollary 7** – The optimal emission reduction effort level \( S_1^{ce} \) is positively correlated with the influence coefficient \( b \) of consumer demand on enterprise revenue, the influence coefficient \( \pi_2 \) of consumer demand on government revenue, the influence coefficient \( \pi_1 \) of enterprise emission reduction on social benefit, carbon trading price \( p_c \) and the influence coefficient \( \lambda_i \) of enterprise emission reduction effort level on enterprise green goodwill; is negatively correlated with discount rate \( \rho \), the cost coefficient \( k_i \) of Enterprise emission reduction efforts and the attenuation coefficient \( \delta \) of enterprise green goodwill.

We can see from corollary 6 and 7, the cost coefficient of the government and enterprises has an important impact on their propaganda efforts and emission reduction efforts, which indicates that the improvement of the overall revenue of the system lies in the improvement of its own emission reduction efficiency. When the emission reduction cost is low, the system revenue can be better improved.

**Comparison of Equilibrium Results**

By comparing the equilibrium strategies and optimal returns of non-cooperative contracts, cost-sharing contracts and cooperative contracts, we can draw the following conclusions.

**Proposition 4** – Under the cooperative contract, the propaganda efforts of government and emission reduction efforts of enterprises are reach the highest level.

**Prove** first, we compare the level of government propaganda efforts, according to equations (5), (20) and (34), we can get:

\[
S_{1'}^{nc} = S_{1'}^{nc} = \frac{\lambda_1 \pi_2 \theta}{k_1 (\rho + \delta)}, \quad S_{1'}^{nc} = \frac{\lambda_1 (\pi_2 + b) \theta}{k_1 (\rho + \delta)}
\]

Therefore \( S_{1'}^{nc} = S_{1'}^{nc} < S_{1'}^{nc} \).

Then, we compare the low carbon emission reduction efforts of enterprises, according to equations (5), (21) and (35), when \( 2B > A \), we can get:

\[
S_{2'}^{nc} - S_{2'}^{nc} = \frac{2(\pi_1 + \pi_2 \mu + \lambda_2 b \theta \rho + \delta) - (\lambda_2 b \theta \rho + \delta) + \mu b + p_c)}{2k_2} > 0
\]

\[
S_{2'}^{nc} - S_{2'}^{nc} = \frac{(\rho + \delta)(\mu b + p_c) + \lambda_2 b \theta \rho + \delta)}{2k_2} > 0
\]

So, when \( 2B > A \), we have \( S_{2'}^{nc} > S_{2'}^{nc} > S_{2'}^{nc} \).

where \( A = b \mu + p_c + \lambda_2 b \theta \rho + \delta, \quad B = \pi_1 + \pi_2 \mu + \lambda_2 \pi_1 \theta \). According to proposition 4, under non-cooperation contract and cost-sharing contract,
the level of propaganda efforts of the government remains unchanged, while the enterprise’s emission reduction effort level increase, this is consistent with the conclusion of study [36]. Under the cooperation contract, both the government’s publicity effort level and the enterprise’s low-carbon emission reduction effort level are the highest.

**Proposition 5** – Compared with non-cooperative contracts, the revenue of both government and enterprise are pareto improved under cost-sharing contracts.

**Proof** First, we compare the revenue of government, according to (16) and (32), we can get:

\[
V_1^{D^*} - V_1^{N^*} = \frac{[(2\pi + 2\mu b + p_x + \pi, \mu)(\rho + \delta) + \lambda, \theta(2\pi_x - b)]^2}{8k_x, \rho(\rho + \delta)} > 0
\]

So, we have \(V_1^{D^*} - V_1^{N^*} > 0\), that is \(V_1^{D^*} > V_1^{N^*}\).

Then, we compare corporate earnings, from (17) and (33), we obtained:

\[
V_2^{D^*} - V_2^{N^*} = \frac{2(\pi, \mu \rho + \lambda, \theta(\rho + \delta) - \lambda, \theta + \mu b + p_x)^2}{4k_x, \rho} > 0
\]

So, when \(2B > A\), we have \(V_2^{D^*} - V_2^{N^*} > 0\), that is \(V_2^{D^*} > V_2^{N^*}\).

According to proposition 5, under the cost-sharing contract, the revenue of both government and the enterprise are greater than those of the non-cooperative contract. This shows that by sharing the cost of emission reduction for enterprises, the revenue of both sides have achieved pareto improvement. Cost-sharing contracts can coordinate supply chains significantly by changing enterprises’ carbon reduction decisions and benefits (Wang et al., 2019).

**Proposition 6** – Under the cooperative emission reduction contract, the revenue of the system is greater than that of the other two contracts.

**Proof** From proposition 5, we know that:

\[
V_1^{D^*} + V_2^{D^*} > V_1^{N^*} + V_2^{N^*}
\]

According to equations (42), (32) and (33), we can get:

\[
V_1^{C^*} - (V_1^{D^*} + V_2^{D^*}) = \frac{(b\theta \lambda, \theta)^2}{2k_x, \rho(\rho + \delta)} + \frac{[(b\mu + p_x)(\rho + \delta) + \lambda, \theta \rho]^2}{8k_x, \rho(\rho + \delta)} > 0
\]

So, we have \(V_1^{C^*} > V_1^{D^*} + V_2^{D^*} > V_1^{N^*} + V_2^{N^*}\).

To sum up, when \(2B > A\), cost-sharing contracts achieve pareto improvements in the revenue of all parties compared to non-cooperative contracts, this is consistent with the conclusion of study [37]. The optimal revenue of the system under the cost sharing contract are greater than those under the non-cooperation contract, while the optimal revenue of the system under the cooperation contract are greater than those under the cost sharing contract. The revenue distribution between the government and enterprises under the cooperation contract mainly depends on the bargaining power of both parties. If a reasonable revenue distribution mechanism can be established, it will be beneficial to both parties as well as the whole society.

**Numerical Results**

In this section, the effectiveness of the three contracts and the influence of key parameters are further analyzed through numerical simulation. We use real data for analysis to make the numerical experiment more convincing. We selected a chemical enterprise in Shanghai, which chose to invest in carbon capture and storage technology (CCS) to reduce carbon emission levels. The following data is obtained from the actual operation status of the enterprise, China Energy Statistical Yearbook, China Environmental Statistical Yearbook and some research on CCS technology investment. The specific data are as follows:

The price per unit of product is 1.5 yuan, the carbon trading price in the carbon trading market is 20-50 yuan/ton, which is mainly determined according to the actual trading price in the current carbon market; the carbon emission of each product produced by the enterprise is 0.02 kg; the production quantity is 2 million units, the carbon quota allocated by the government is 5 ton; Carbon emissions without CCS is 0.02 kg/unit; the cost coefficients of government and enterprise emission reduction efforts are 0.3 and 0.7 respectively; the influence coefficient of enterprise emission reduction on government revenue is 0.6, which is mainly determined through expert consultation; the influence coefficients of product green level and green goodwill on demand are 0.3 and 0.7 respectively; the influence coefficients of government and enterprise emission reduction efforts on enterprise green goodwill are 0.6 and 0.4 respectively; the influence coefficients of consumer demand on government and enterprise revenue are 0.4 and 0.7 respectively; we suppose that when the emission reduction efforts of both the government and the enterprise are zero, the green goodwill decay rate of the enterprise is 0.3; government and business have the same discount rate 0.6.

Fig. 2 and Fig. 3 compared the level of advocacy efforts of governments and the level of mitigation efforts of enterprises under the three contracts. Fig. 4 and Fig. 5 compared the revenue of government and enterprises under non-cooperative contracts and cost-sharing contracts. Fig. 6 compared the revenue of the system under the three contracts. Fig. 7 described the impact of relevant parameters on the cost-sharing ratio.

According to Fig. 2, we can see that the level of government propaganda efforts increases as the influence coefficient \(\theta\) increases and decreases as...
the cost coefficient $k_1$ increase under the three contracts. The level of government advocacy efforts was the same in both the non-cooperation contract and cost-sharing contract, that is $S_1^{N^*} = S_1^{D^*}$. The level of government propaganda efforts under the contract of cooperation is the highest of the three contracts.

According to Fig. 3, we can see that the effort level of enterprise emission reduction increases with the increase of carbon trading price $p_e$ and decreases with the increase of cost coefficient $k_2$ under three contracts. At the same time, we found that when the carbon trading price is low, the enterprise's emission reduction level under the cost-sharing contract is higher than that without the cost-sharing contract, that is $S_2^{P^0} > S_2^{N^*}$. However, when the carbon trading price exceeds a certain value ($p_e$ satisfies $2B > A$), the enterprise's emission reduction effort level under the non-cooperative contract is higher than that under the cost-sharing contract, that is $S_2^{P^0} < S_2^{N^*}$. This shows that under certain conditions, government incentive measures will make enterprises realize expected earnings in advance, thus enterprises will scale back the level of their low-carbon emission reduction efforts.

The enterprise's low-carbon emission reduction efforts reach the optimal level under the cooperation contract, namely, $S_2^C > S_2^{P^0} > S_2^{N^*}$. This shows that the cooperation contract can effectively improve the level of enterprise emission reduction efforts.

According to Fig. 4, we found that when $p_e$ and $\pi_1$ satisfy $2B > A$, the value of $L$ is positive, this indicates that only when $p_e$ and $\pi_1$ satisfy $2B > A$, the government will choose to share the cost of emission reduction for enterprises. At the same time, we found that $L$ increases gradually with the increase of $\pi_1$, this means that when the government gains more social revenue from enterprises' low-carbon emission reduction, the government will increase the share proportion of emission reduction costs to encourage enterprises to improve emission reduction level. In addition, we can also see that $L$ decreases gradually with the increase of carbon trading price, this is mainly because when the trading price of carbon emission permits is high, enterprises will take the initiative to increase the carbon emission reduction level to save the cost of purchasing carbon emission permits, when the government realizes this, it will gradually reduce the cost sharing.

According to Fig. 5, we found that the government's revenue shows an increasing trend and gradually tends to be stable with the passage of time. This shows that as the government continues to invest in reducing emissions, the government's revenue has improved. At the same time, we found the government's revenue decreases gradually with the increase of attenuation coefficient. We can also see that cost-sharing contracts
achieve pareto improvements in government revenue compared with non-cooperative contracts. This is mainly because the government shares the cost of emission reduction of enterprises, so enterprises will increase emission reduction level, and this will lead to increased consumer demand, which will lead to pareto improvements in government revenues.

According to Fig. 6, we found that the enterprise’s revenue shows an increasing trend and gradually tends to be stable with the passage of time. This shows that with the improvement of enterprise emission reduction level, enterprise revenue has been improved. We can also see that under the cost sharing contract, the enterprise’s revenue has achieved pareto improvement. This is mainly because the government shares the cost of emission reduction of enterprises, and enterprises will increase their emission reduction efforts to improve their green goodwill and green level of product, this will lead to increased consumer demand, which will lead to pareto improvement in enterprise’s revenue.

Similarly, we can see from Fig. 7, the revenue of the system also presents an increasing trend and eventually tends to be stable with the passage of time, however, it gradually decreases with the increase of attenuation coefficient. We can also see that the revenue of the emission reduction system is largest under the cooperative contract, while the revenue of the system is the smallest under the non-cooperative contract. This shows that cost sharing contract can realize pareto improvement of system revenue. The revenue of the system is higher than the other two contracts under the cooperation contract, it shows that stronger cooperation between government and enterprises can create more revenue.

The previous literature on government and corporate and cooperative emission reduction mainly focus on government subsidies under static conditions (13-16), without considering the dynamic relationship between consumer preferences and enterprises’ emission reduction. The research results of this paper can effectively improve the efficiency of government-enterprise cooperation in emission reduction.

Conclusions and Suggestions

Conclusions

In recent years, the emission of greenhouse gases has led to the occurrence of climate warming and extreme weather. As the main body of carbon emission, enterprises should bear the main responsibility of climate change. In order to control the carbon
emissions of enterprises, the government has made many policies, such as carbon trading, carbon subsidies. However, government incentives alone are not enough to drive companies to actively reduce emissions. Consumer demand is also needed. In view of this, from the perspective of government cost sharing and consumer low-carbon preference, this paper constructs three differential game models of government and enterprise joint emission reduction and consumer demand-driven. Through solving and analyzing the model, the following conclusions can be obtained.

(1) Under certain conditions (2B>A), the government share part of the cost of emission reduction for enterprises, which can improve the revenue of both the government and enterprises. This also verifies the rationality of China’s carbon emission subsidies for enterprises.

(2) In the absence of cost-sharing and cost-sharing contracts, governments and enterprises have different emission reduction efforts. The enterprise’s emission reduction effort level will increase because the government shares a certain proportion of emission reduction costs, and the government’s emission reduction effort level will remain unchanged.

(3) When the government and enterprises make joint decisions, the sum of the revenue of the government and enterprises is the largest, which also provides a theoretical basis for the coordination of government and enterprise emission reduction.

Suggestions

Based on the above analysis, we found that the cooperation between government and enterprises in emission reduction can promote the improvement of emission reduction efficiency. Therefore, we can draw the practical management enlightenment in this paper as follows:

(1) Strengthen institutional design and build a long-term mechanism for cooperation on emission reduction. The government and enterprises sign clear cooperation agreements and put specific cooperation mechanisms into practice. To ensure positive revenue for the government and enterprises and enhance the stability of cooperation.

(2) Formulate preferential policies to reduce the cost of low-carbon emission reduction for enterprises. For example, developing green finance business and providing financial support to green transformation enterprises; establishing a flexible and diversified carbon tax system; and giving tax incentives and exemptions to enterprises that are good at low-carbon emission.

(3) Cultivate public awareness of environmental protection and encourage consumers to buy low-carbon products. Strengthen the publicity of green and low-carbon; organize green public welfare activities; provide subsidies to consumers who buy low-carbon products.

This paper considers that the cooperation contract can achieve the optimization revenue of the system, but does not consider the distribution and coordination of government and enterprise revenue under the cooperation contract, which needs further research in the future.

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Conflict of Interest

The authors declare no conflict of interest.

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