Introduction

A long-term estimation of water balance is very important to: 1) understand the hydrological cycles over large temporal scales, 2) describe the dynamics of hydrological systems, and 3) predict the rapidly changing trends of hydro-climatic variables [1]. Water balance is a most basic and important concept for catchment hydrology. Moreover, it is a common concept for studying hydrological behaviors [2-5]. One of the core questions for estimating long-term water balance is the partition of mean annual precipitation into mean annual evapotranspiration and mean annual streamflow [6]. Until now, none of the methods has been developed for directly measuring evapotranspiration on a large spatial scale [2].

In recent times, climate change and human activities have led to a large change in hydrological processes and water availability in many regions of the world [7-10]. The link between climate change and hydrological response is one of the main questions to be researched in the past and even today in hydrology. Many attempts have been made to formulate the mean annual water-energy balance [11-14]. Budyko (1974) assumed that the actual evapotranspiration is a function of the aridity index and precipitation, and it can be expressed as \( E/P = f(E_0/P) \) [15].

The energy-based theoretical equations, which describe the climate and the water balance, have been developed and applied in a “top-down” fashion [16-18]. The developments of Budyko (1974) and Fu’ (1981) [19]...
Renner et al. (2012) developed a similar equation [14], which is very simple for the purposes of estimation. The aforesaid equation can be used to predict the effects of climate change on streamflow.

In this article, a theoretical pattern of the dependence for the aridity index ($\phi = E_0/P$) and the catchment characteristics on the basis of Renner’s equation are analyzed. The aim of the research is to discuss the effects of climate on evapotranspiration by evaluating Renner’s water-energy balance framework, and to show the variances of different regions and of different climatic conditions.

Table 1. Some information of the 10 regions.

<table>
<thead>
<tr>
<th>ID</th>
<th>Regions</th>
<th>Area (km²)</th>
<th>P (mm)</th>
<th>Q (mm)</th>
<th>E (mm)</th>
<th>$E_0$ (mm)</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Songhuajiang basin</td>
<td>93.5</td>
<td>461</td>
<td>134</td>
<td>327</td>
<td>730</td>
<td>1.58</td>
</tr>
<tr>
<td>2</td>
<td>Liaohe basin</td>
<td>31.4</td>
<td>514</td>
<td>138</td>
<td>376</td>
<td>870</td>
<td>1.69</td>
</tr>
<tr>
<td>3</td>
<td>Haihe basin</td>
<td>32</td>
<td>490</td>
<td>81</td>
<td>409</td>
<td>970</td>
<td>1.97</td>
</tr>
<tr>
<td>4</td>
<td>Yellow River basin</td>
<td>79.5</td>
<td>443</td>
<td>79</td>
<td>364</td>
<td>950</td>
<td>2.14</td>
</tr>
<tr>
<td>5</td>
<td>Huaihe basin</td>
<td>33</td>
<td>869</td>
<td>314</td>
<td>555</td>
<td>990</td>
<td>1.13</td>
</tr>
<tr>
<td>6</td>
<td>Yangtze River basin</td>
<td>180</td>
<td>1,055</td>
<td>532</td>
<td>523</td>
<td>830</td>
<td>0.78</td>
</tr>
<tr>
<td>7</td>
<td>Southeastern rivers basin</td>
<td>24.5</td>
<td>1,663</td>
<td>944</td>
<td>719</td>
<td>950</td>
<td>0.57</td>
</tr>
<tr>
<td>8</td>
<td>Pearl River basin</td>
<td>57.8</td>
<td>1,536</td>
<td>811</td>
<td>725</td>
<td>1,000</td>
<td>0.65</td>
</tr>
<tr>
<td>9</td>
<td>Southwestern rivers basin</td>
<td>84.4</td>
<td>1,071</td>
<td>667</td>
<td>404</td>
<td>1,050</td>
<td>0.98</td>
</tr>
<tr>
<td>10</td>
<td>Northwestern rivers basin</td>
<td>336.2</td>
<td>172</td>
<td>42</td>
<td>130</td>
<td>1,100</td>
<td>6.39</td>
</tr>
</tbody>
</table>
mountainous and plain areas account for about 60% and 40%, respectively. The topography is characterized by low mountains in the west and plains in the east. This region has a warm-temperate monsoon climate, the mean annual temperatures are 7.4ºC in the mountainous area and 12.7ºC in the plain area, and mean annual precipitation is 563 mm (with about 80% distributed between July and September). Its mean annual pan evaporation is 1,500-2,000 mm.

Data

The annual precipitation and streamflow data of the 10 basins were obtained from the “water resources bulletin” (1997-2015) of each basin, respectively. The aridity index data of China was derived from the National Science and Technology Infrastructure of China, the National Earth System Science Data Sharing Infrastructure (www.geodata.cn). The annual runoff data (1960-2010) of Baiyangdian catchment were obtained from the Hydrological Yearbook of the People’s Republic of China. And the annual precipitation and potential evapotranspiration data (1960-2010) of the catchment were obtained from the national weather station of Baoding via the China Meteorological Data Sharing Service System (cma.gov.cn).

Methods

Water-Energy Balance with CCUW Hypothesis

Tomer and Schilling (2009) employed the two non-dimensional variables, i.e., relative excess water (W) and relative excess energy (U) [20], to establish a framework to reflect the hydro-climatic state of a given catchment. These two variables were mathematically derived and are shown as follows:

\[
W = 1 - \frac{E}{P}, \quad U = 1 - \frac{E}{E_0},
\]

...where \(E_0\) is mean annual potential evapotranspiration and \(W\) and \(U\) denote the proportions of the available water and energy that are unused, i.e., in excess of the requirements.

Tomer and Schilling (2009) have introduced a conceptual model based on the hypothesis that the direction of a temporal change in a relationship between \(U\) and \(W\) can be used to distinguish the effects of a change in the patterns of land use and climate on a water budget in a given basin [20]. The conceptual model states that the changes in the climate and basin characteristics qualitatively lead to a different change in partitioning the water \((W)\) and the energy \((U)\) at the surface. A change in the climatic conditions of a long-term average, i.e., \(P\) and \(E_0\), would lead to the changes of the \(U\) and the \(W\) in an opposite direction. By taking this assumption, Renner and Bernhofer (2012) [14] have concluded that:

\[
\Delta U / \Delta W = -1
\]

The aforementioned hypothesis can be abbreviated as “CCUW.”

On the basis of the definition of the \(W\) and the \(U\), their change in the two different periods can mathematically be expressed by virtue of equation (3), and it can be shown as follows:

\[
\Delta W = \frac{E_1}{P_1} - \frac{E_2}{P_2}, \quad \Delta U = \frac{E_1}{E_{0,1}} - \frac{E_2}{E_{0,2}}
\]

...and, in accordance with equation (4), it may be changed as follows:

\[
\Delta U + \Delta W = 0
\]

Thereafter, we can get:

\[
\frac{E_1}{P_1} + \frac{E_1}{E_{0,1}} = \frac{E_2}{P_2} + \frac{E_2}{E_{0,2}}
\]

...i.e.:

\[
\frac{E}{P} + \frac{E}{E_0} = C_E = \text{const}
\]

It is important to note that the sum of \((E/P)\) and \((E/E_0)\) for a given basin is constant. And that constant is called catchment efficiency \((C_E)\).

Re-arranging equation (8), we get:

\[
\frac{E}{P} = C_E \cdot \frac{E_0}{P + E_0}
\]

Then Renner et al. (2012) [14] developed this new water-energy balance that could be used on long-term time and catchment scales, and could be used to predict the mean annual evapotranspiration of a catchment.

Sensitivity Coefficient of \(E\) to \(P\) and \(E_0\)

The sensitivity concept, as described by Schaake and Liu (1989) [22], shows that the relative changes in a streamflow are proportional to the inverse of the runoff ratio (i.e., \(P/Q\)). As has been shown in an analytical equation, Equation (9) can be differentiated with respect to \(P\) or \(E_0\). The partial derivatives describe the effect of a hydrological cycle as a result of rapidly changing \(P\) and/or \(E_0\), which can be written as follows:

\[
\frac{\Delta U}{\Delta W} = -1
\]
\[
\frac{\partial E}{\partial P} = C_E \left(\frac{E_0}{P+E_0}\right)^2 = C_E \left(\frac{E_0}{P+1+E_0}\right)^2
\]

\[
\frac{\partial E}{\partial E_0} = C_E \left(\frac{P}{P+E_0}\right)^2 = C_E \left(\frac{1}{P+1+E_0}\right)^2
\]

(8)

...where \(\partial E/\partial P\) represents the change in actual evapotranspiration (\(E\)) divided by the change in precipitation (\(P\)). Similarly, \(\partial E/\partial E_0\) represents the change in the actual evapotranspiration (\(E\)) divided by the change in potential evapotranspiration (\(E_0\)).

And, we also used the elasticity coefficient (\(\epsilon\)) to represent a proportional change in the evapotranspiration (\(E\)) divided by the change in a climatic variable (\(X\)), such as precipitation (\(P\)) or potential evapotranspiration (\(E_0\)). It can be written as follows:

\[
\epsilon = \frac{\partial E}{\partial X} \cdot \frac{X}{E} = \frac{\partial E}{\partial X} \cdot \frac{X}{E}
\]

(9)

In accordance with equations (19) and (20), the elasticity coefficient of the evapotranspiration to the precipitation (\(\epsilon_P\)) and the potential evapotranspiration (\(\epsilon_{E0}\)) can be obtained as follows:

\[
\epsilon_P = \frac{\partial E}{\partial P} \cdot \frac{P}{E} = \frac{E_0}{P+E_0} \cdot \frac{P}{P+E_0} = \frac{E_0}{P+E_0} = \phi
\]

\[
\epsilon_{E0} = \frac{\partial E}{\partial E_0} \cdot \frac{E_0}{E} = C_E \left(\frac{P}{P+E_0}\right)^2 \cdot \frac{P}{P+E_0} = \frac{P}{1+\phi}
\]

(10)

State Space of the Water-Energy Balance Equation

Fig. 4 shows the curves of the CCUW hypothesis for different values of \(CE\). These curves are strongly determined by virtue of a \(CE\), which are similar to determine the effect of different values for catchment parameter \(w\) in a parameterized Fu’s theoretical equation \([14, 19, 23]\).

Under extreme wet conditions, \(E\) approaches the \(E_0\), and, thereafter, it would not increase with precipitation (\(P\)), because of the condition that the \(E\) is limited by potential evapotranspiration (\(E_0\)). Whereas under extreme dry conditions, \(E\) approaches precipitation (\(P\)) and, thereafter, it would not increase with potential evapotranspiration (\(E_0\)). As has been shown in Fig. 1, the two asymptotes join at point A, at which \(E, P, E_0\) are equal; OA is the wet edge at which \(E = E_0\) and AB is the dry edge at which \(E = P\).

In accordance with the Budyko space of \((E/P)\) and \((E_0/P)\), there is an energy limit (i.e., \(y = x\)) and a water limit (i.e., \(y = 1\)), which are based on the fact that \(E<P\) and \(E<E_0\) \([24-26]\). In many conditions, as shown in Fig. 4, the curves of the CCUW-based hypothesis overstepped the aforementioned two limiting boundary lines \([27-28]\), which can mathematically be expressed as:

\[
\frac{E}{P} = \frac{C_E \cdot E_0}{P+E_0} < 1
\]

\[
\frac{E}{P} = \frac{C_E \cdot E_0}{P+E_0} < \frac{E_0}{P}
\]

\[
C_E = \frac{E}{P} + \frac{E}{E_0} < 2
\]

(11)

Then the limits of \(C_E\) can be written as:

\[
C_E < 1 + \frac{E_0}{P} \left(\text{when } \frac{E_0}{P} \leq 1\right)
\]

\[
C_E < 1 + \frac{P}{E_0} \left(\text{when } \frac{E_0}{P} \geq 1\right)
\]

(12)

Results and Discussion

Application and Verification of CCUW Hypothesis in Baiyangdian Catchment

Just a single parameter of the mean annual water-energy balance equation represents the integrated effects of the catchment and of the vegetation characteristics, which have a significant effect on evapotranspiration. This parameter mainly includes the plant-available water, the average slope, land use, vegetation cover, etc. Renner et al. (2012) has called that parameter the \(C_E\) (i.e., catchment efficiency) \([14]\), which was calibrated using
the data of annual precipitation, actual evapotranspiration, and potential evapotranspiration in the catchment. Fig. 2 showed the annual $C_E$ of Baiyangdian from 1960 to 2010, and suggested a mean value of 1.20. The Mann-Kendall change point test (not shown here) of the $C_E$ parameter indicates an abrupt change in 1980 at the 0.05 significance level. Analysis of the $C_E$ in 1960-80 and 1981-2010 suggests that it both had obviously constant trends in these two periods, respectively, and this verified the theory of CCUW hypothesis and equation (6), so it could be applied in the following analysis.

$C_E$ was named “catchment efficiency,” and its change reflects the change of catchment characteristics. The Mann-Kendall test result of $C_E$ showed an abrupt change in 1980, and that means the characteristics of Baiyangdian catchment had some changes that year. A comparison of these two periods showed that the $C_E$ in 1981-2010 (the mean value was 1.08) was slightly bigger than in 1960-80 (the mean value was 1.27). In 1980, China’s Three-North Shelterbelt Program came into operation, and the forest coverage improved greatly in the study area. The increase of forest area made actual evapotranspiration have an increase, and this might be the reason for the increase of $C_E$.

The Sensitivity Analysis of $E$ to $P$ and $E_0$ and its Regional Differences

The sensitivity and elasticity coefficients of actual evapotranspiration to precipitation and potential evapotranspiration were determined by equations (8) and (10). The values of $\partial E/\partial P$ and $\partial E/\partial E_0$ of Baiyangdian catchment were 0.51 and 0.14, respectively, and indicated that a 1 mm increase in $P$ would result in 0.51 mm in actual evapotranspiration, while a 1 mm increase in $E_0$ would result in 0.14 mm in actual evapotranspiration. And the values of $\varepsilon_P$ and $\varepsilon_{E_0}$ of Baiyangdian catchment were 0.66 and 0.34, respectively, and indicated that a 10% increase in $P$ would result in 6.6% in actual evapotranspiration.
while a 10% increase in $E_o$ would result in 3.4% in actual evapotranspiration.

Regional Differences of the Sensitivities in China

According to equations (8) and (10), the sensitivity and elasticity coefficients of the 10 regions in China were calculated, and the results are shown in Table 2. The values of $\frac{\partial E}{\partial P}$ and $\frac{\partial E}{\partial E_0}$ in China were 0.16–0.65 and 0.02–0.48, and the values of $\epsilon_E$ and $\epsilon_{E_0}$ were 0.36–0.86 and 0.14–0.64.

The northwestern rivers basin is an extremely arid region in China, with an aridity index of 6.39 and much larger than other regions, the results of $\frac{\partial E}{\partial P}$ showed that it is also as large as 0.65, meaning that if precipitation decreased by 1 mm, actual evapotranspiration would decrease by 0.65 mm. The southeastern rivers basin is the extreme humid region in China, its aridity index is only 0.57, and the results of $\frac{\partial E}{\partial P}$ showed that it also smallest at 0.19. For $\frac{\partial E}{\partial E_0}$, inversely, the northwestern rivers basin had the smallest value of 0.02, meaning that if potential evapotranspiration decreased by 1 mm, actual evapotranspiration would decrease by only 0.02 mm. And the southeastern rivers basin had the largest value of 0.48. Like $\frac{\partial E}{\partial P}$ and $\frac{\partial E}{\partial E_0}$, the values of $\epsilon_E$ and $\epsilon_{E_0}$ showed the same trend among these 10 regions.

The Sensitivity of $P$ and $E_o$ on $E$ with Aridity Index

As seen in Table 2, the values of $\frac{\partial E}{\partial P}$ and $\epsilon_E$ in arid regions ($\phi<1$) were larger than in humid regions ($\phi>1$). This means that precipitation changes would lead to more changes in evapotranspiration in arid regions. While the values of $\frac{\partial E}{\partial E_0}$ and $\epsilon_{E_0}$ in humid regions ($\phi<1$) were larger than in arid regions ($\phi>1$). This means that potential evapotranspiration changes would lead to more changes in evapotranspiration in humid regions.

Table 3 and Fig. 3 show the correlation analysis and relationships between the coefficients ($\frac{\partial E}{\partial P}$, $\frac{\partial E}{\partial E_0}$, $\epsilon_E$, and $\epsilon_{E_0}$) and aridity index ($\phi$), respectively. The results suggest that the coefficients have significant correlations with $\phi$ at the 0.01 level, implying that the sensitivity of climate on $E$ was significantly influenced by aridity. The relationships between the coefficients of $P$ ($\frac{\partial E}{\partial P}$ and $\epsilon_E$) and $\phi$ are significantly nonlinear and positive ($y = aln(x+b)+c$), and the relationships between the coefficients of $E_0$ ($\frac{\partial E}{\partial E_0}$ and $\epsilon_{E_0}$) and $\phi$ are significantly nonlinear and negative ($y = -aln(x+b)+c$). This means that actual evapotranspiration is more sensitive to precipitation in a drier region and more sensitive to potential evapotranspiration in a wetter region.

Effects of Climate on Evapotranspiration with Aridity Index

Fig. 5 shows a sketch of the relationship between the sensitivity coefficients of $E$ to climate ($P$ and $E_0$) and aridity index ($\phi$). As seen in this sketch, the sensitivity of $E$ to $P$ is increased with the increase of aridity index, and the sensitivity of $E$ to $E_0$ is decreased with the increase of aridity index. Before and after the point of $\phi = 1$, the comparisons between the sensitivity to $P$ and to $E_0$ have opposite results.

According to equation (8), we compared $\frac{\partial E}{\partial P}$ with $\frac{\partial E}{\partial E_0}$ and got ($\frac{\partial E}{\partial P}$)$\times$($\frac{\partial E}{\partial E_0}$) = ($E_o/P$)$^2$ = $\phi$. In accordance with this equation, $[\frac{\partial E}{\partial P}$($\frac{\partial E}{\partial E_0}$)] can be in direct proportion to the square of an aridity index ($\phi$), which means that in a humid region (i.e., $\phi<1$), the $\frac{\partial E}{\partial P}$ is smaller than that of $\frac{\partial E}{\partial E_0}$. Therefore, evapotranspiration is more sensitive to potential evapotranspiration; in an arid region (i.e., $\phi>1$), $\frac{\partial E}{\partial P}$ is larger than $\frac{\partial E}{\partial E_0}$, meaning that evapotranspiration is more sensitive to precipitation.

Equation (10) shows that $\epsilon_E$ and $\epsilon_{E_0}$ had a significant correlation with the aridity index ($\phi$). So $\epsilon_E$ is increased with the increase of aridity index and $\epsilon_{E_0}$ is decreased with the increase of an aridity index. Compared with $\epsilon_E$ and $\epsilon_{E_0}$, we could get $\epsilon_E / \epsilon_{E_0} = \phi$. The $\epsilon_E$ and $\epsilon_{E_0}$ had the same value of 0.5 when $\phi = 1$; whereas $\epsilon_E$ is smaller than that of $\epsilon_{E_0}$ in a humid region ($\phi<1$). This also means that evapotranspiration is much more sensitive to potential evapotranspiration; and the $\epsilon_E$ is larger than that of $\epsilon_{E_0}$ in an arid region (i.e., $\phi>1$), which means evapotranspiration is much more sensitive to precipitation.

Conclusions

For this study we used a more simple water and energy balance equation to compare the effects of precipitation and potential evapotranspiration on actual evapotranspiration, mathematically and theoretically. And the conclusions can be summarized as follows:

1. In Baiyangdian catchment, a 1 mm or 10% increase in precipitation would lead to 0.51 mm or 6.6% in actual evapotranspiration, and a 1 mm or 10% increase in potential evapotranspiration would lead to 0.14 mm or 3.4% in actual evapotranspiration.
2. The regional differences in the 10 regions of China showed that the effects of climate on actual evapotranspiration had significant spatial heterogeneity and were significantly influenced by the aridity index (φ).

3. In humid regions (φ<1), evapotranspiration is more sensitive to potential evapotranspiration, and in arid regions (φ>1) evapotranspiration is more sensitive to precipitation.

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