Risk Assessment of Water Inrush in Karst Tunnels Based on the Ideal Point Method

Yingchao Wang1*, C. Guney Olgun2, Linbing Wang3, Bo Meng4

1State Key Laboratory for Geomechanics and Deep Underground Engineering, China University of Mining and Technology, Xuzhou, Jiangsu, China
2Civil, Architectural, and Environmental Engineering Department, Missouri University of Science and Technology, Rolla, MO, USA
3Department of Civil and Environmental Engineering, Virginia Tech, Blacksburg, VA, USA
4School of Mechanics and Civil Engineering, China University of Mining and Technology, Xuzhou, Jiangsu, China

Received: 24 October 2017
Accepted: 10 February 2018

Abstract

Water inrush is one of the typical geological hazards in the construction of high-risk tunnels, and has caused severe losses. To predict water inrush accurately, a novel model was put forward for karst tunnels in the present study. The ideal point method coupled with the analytic hierarchy process method (AHP) was applied for risk assessment of water inrush. First, the ideal point method was introduced as a brand-new way to predict the risk level of water inrush. Second, the water inrush risk in karst tunnels was discussed in terms of influencing factors. With the consideration of karst hydrological and engineering geological conditions, seven key factors were selected as evaluation indices, including formation lithology, unfavorable geological conditions, groundwater level, landform and physiognomy, modified strata inclination, contact zones of dissolvable and insoluble rock, and layer and interlayer fissures. Then the ideal point method was used to deal with the multiple evaluation indices to determine the ideal point and the anti-ideal point. Meanwhile, the analytic hierarchy process method (AHP) was applied to determine the weight coefficient of each evaluation index. Thus, the minkowski distances respectively for the ideal point and the anti-ideal point were calculated. Based on the discriminant analysis theory, the closeness degrees to the ideal points were brought out to specify the risk level of water inrush. Finally, the proposed model was applied to a typical deep-buried karst tunnel: Jigiongling Tunnel in China. The obtained results were compared with the results of the relevant methods and the practical findings, and reasonable agreements could validate the presented approach. The obtained results not only provide guidance for the construction of high-risk tunnels, but also bring out an alternative way for risk assessment of water inrush.

Keywords: water inrush, risk assessment, ideal point method, close degree

*e-mail: wych12345678@126.com
Water inrush is one of the serious geological disasters of deep-buried underground engineering. As an accidental disaster, water inrush has some typical characteristics, including the complexity of the water source, the diversity of geological structures, and multiple disaster-causing mechanisms. Water inrush frequently occurs in the construction of high-risk tunnels, like karst tunnels, which has caused great economic loss and heavy casualties. It is reported that 97 water inrush disasters have occurred in karst tunnels in China from 2001 to 2010 [1]. Meanwhile, about 100 people have lost their lives during these water inrush disasters. The cost induced by water inrush increases year by year, particularly with the rapid development of the economy. Therefore, research on risk assessment of water inrush is indeed necessary.

A great amount of research has been devoted to the catastrophic evolution process of water inrush [2-4] and the risk assessment of water inrush, while the main focus has been on the risk of floor water inrush in coal mines [5-6]. Many new methods have been put forward to predict water inrush, such as the support vector machine [7], BP neural network and DS theory [8], geographic information system [9], a secondary fuzzy comprehensive evaluation system [10], and four-zone theory [11], as well as fault tree analysis [12]. Meanwhile, some probability coefficients have been presented to forecast floor water inrush, including a conventional water inrush coefficient [13], vulnerability index [14-15], the analytical approach about water-inrush-factor [7, 16], and the coefficient of water bursting from coal seam floor [17]. The above achievements have important theoretical significance on the risk assessment of floor water inrush in coal mines. Different methods still have their special characteristics and limitations within certain scopes. Due to the complicated karst hydrologic and engineering geological conditions in underground engineering, it is still difficult to predict water inrush accurately in coal mines. When deep-buried tunnels are considered, these methods for coal mines could provide useful guidance, but have certain limitations for tunnel applications [7].

The mechanism and influencing factors of water inrush are different between karst tunnels and coal mines [18]. With the consideration of water inrush in karst tunnels, many scholars have conducted significant progress toward understanding the mechanical mechanism of water inrush. Meanwhile, analytical solutions, numerical research, and experimental studies have been performed to predict water inrush. However, due to the multi-scale complexity of the system, some numerical researches are always performing with certain given assumptions, which might lead to obvious deviation from practical situations. It is also difficult to determine the geological and hydrogeological parameters for the numerical calculation. The measurement techniques for local experiments are very limited. Thus, analytical solutions are popular on risk assessment of water inrush and other disasters [19-23].

Recently, analytical solutions on risk assessment of tunnel engineering have become a hot topic. A large number of researchers have proposed various methods derived from other fields and are used to evaluate the risk of water inrush in tunnels. Li et al. [18] put forward an attribute synthetic evaluation system for the risk evaluation of water inrush in karst tunnels. The geographic information system (GIS) technology was presented to predict dynamically the water inrush risk and to develop appropriate protective measures [24]. A software system was proposed for risk assessment of water inrush [1]. In the software system, fuzzy mathematics and analytical hierarchy process (AHP) were used to quantitatively describe the risk levels for each factor. Meanwhile, a new risk evaluation model based on the fuzzy topsis method was presented with the consideration of both the uncertainties and the new factors [25]. In addition, set pair analysis [26], the efficacy coefficient method [27], and the cloud model [28] were also successfully applied in risk assessment of water inrush in karst tunnels.

The above methods and models have achieved reasonable results that provide reference values for risk assessment of water inrush in karst tunnels. Due to the complexity of karst tunnels, it is still difficult to reveal the quantified relationship between water inrush and its influencing factors. Therefore, it is necessary to propose a novel and reliable method for assessing the risk of water inrush in karst tunnels.

In the present study, the ideal point method was coupled with AHP to predict water inrush in karst tunnels. First, a hierarchy model of the influence factors was established for water inrush. Then, the analytic hierarchy process method was presented to determine the weight coefficient of each evaluation index. Moreover, the ideal point and the anti-ideal point were determined and the Minkowski distance was calculated. Furthermore, based on the discriminant analysis theory, the closeness degree for the ideal point was brought out to specify the risk level of water inrush. Finally, the established model was applied to Jigongling Tunnel on Fanba Expressway in China. The results of the proposed model are compared with the results of the relevant methods and the practical situation, and reasonable agreements are shown.

**Introduction**

The mechanism and influencing factors of water inrush are different between karst tunnels and coal mines [18]. With the consideration of water inrush in karst tunnels, many scholars have conducted significant progress toward understanding the mechanical mechanism of water inrush. Meanwhile, analytical solutions, numerical research, and experimental studies have been performed to predict water inrush. However, due to the multi-scale complexity of the system, some numerical researches are always performing with certain given assumptions, which might lead to obvious deviation from practical situations. It is also difficult to determine the geological and hydrogeological parameters for the numerical calculation. The measurement techniques for local experiments are very limited. Thus, analytical solutions are popular on risk assessment of water inrush and other disasters [19-23].

Recently, analytical solutions on risk assessment of tunnel engineering have become a hot topic. A large number of researchers have proposed various methods derived from other fields and are used to evaluate the risk of water inrush in tunnels. Li et al. [18] put forward an attribute synthetic evaluation system for the risk evaluation of water inrush in karst tunnels. The geographic information system (GIS) technology was presented to predict dynamically the water inrush risk and to develop appropriate protective measures [24]. A software system was proposed for risk assessment of water inrush [1]. In the software system, fuzzy mathematics and analytical hierarchy process (AHP) were used to quantitatively describe the risk levels for each factor. Meanwhile, a new risk evaluation model based on the fuzzy topsis method was presented with the consideration of both the uncertainties and the new factors [25]. In addition, set pair analysis [26], the efficacy coefficient method [27], and the cloud model [28] were also successfully applied in risk assessment of water inrush in karst tunnels.

The above methods and models have achieved reasonable results that provide reference values for risk assessment of water inrush in karst tunnels. Due to the complexity of karst tunnels, it is still difficult to reveal the quantified relationship between water inrush and its influencing factors. Therefore, it is necessary to propose a novel and reliable method for assessing the risk of water inrush in karst tunnels.

In the present study, the ideal point method was coupled with AHP to predict water inrush in karst tunnels. First, a hierarchy model of the influence factors was established for water inrush. Then, the analytic hierarchy process method was presented to determine the weight coefficient of each evaluation index. Moreover, the ideal point and the anti-ideal point were determined and the Minkowski distance was calculated. Furthermore, based on the discriminant analysis theory, the closeness degree for the ideal point was brought out to specify the risk level of water inrush. Finally, the established model was applied to Jigongling Tunnel on Fanba Expressway in China. The results of the proposed model are compared with the results of the relevant methods and the practical situation, and reasonable agreements are shown.

**Principle of the Ideal Point Method**

**Definition**

The ideal point method is a generalization of discriminant analysis theory. As a comprehensive evaluation method, the ideal point method can simultaneously deal with the comprehensive evaluation of multiple factors and multiple objects [29-31]. The ideal point method has been applied in many fields, such as forest harvest regulation [32], stochastic multiple attribute decision [33], power restoration strategy for a distribution
network [34], environmental quality evaluation [35], and an evaluation of land use planning [36].

The kernel of the ideal point method is to find a point. The point can approach the ideal point as possible based on a defined model. The distance between the point and the ideal point is minimum, while the distance between the point and the anti-ideal point is maximum.

The Evaluation Process of the Ideal Point Method [19]

(1) Evaluation indices matrix
For an evaluation object $R$, it can be assumed that there are $n$ evaluation indices $(x_1, x_2, \ldots, x_n).$ Each index has its own objective function, such as $f_1(x)$, $f_2(x)$, ..., and $f_n(x).$ A vector function can be defined as $F(x) = [f_1(x), f_2(x), \ldots, f_n(x)].$ The weights of evaluation indices are described as $\omega_1$, $\omega_2$, ..., and $\omega_n$, respectively. The evaluation indices matrix, $X$, is presented as:

$$X = \{x_1, x_2, \ldots, x_n\} \omega = \{x_1, x_2, \ldots, x_n\} \omega_1 \omega_2 \cdots \omega_n$$

(1)

(2) The ideal point and the anti-ideal point
The evaluation indices can be divided into two types of indices [35]: the positive indices and the inverse indices. For the positive indices, the bigger the positive indices are, the better the evaluation object will be. For the inverse indices, the smaller the relative indices are, the better the evaluation object will be. When the values of evaluation indices change with linear law, the ideal point and the anti-ideal point can be described as follows:

For the positive indices:

$$\begin{align*}
  f_i^+(+) &= \max f_i(x) & i = 1, 2, \cdots, n \\
  f_i^+(-) &= \min f_i(x) & i = 1, 2, \cdots, n
\end{align*}$$

(2)

where $f_i^+(+)$ and $f_i^+(-)$ are respectively the ideal point vector and the anti-ideal point vector of $i$-th evaluation index. $f_i(x)$ is the value of $i$-th evaluation index.

For the inverse indices:

$$\begin{align*}
  f_i'(+)& = \min f_i(x) & i = 1, 2, \cdots, n \\
  f_i'(-) &= \max f_i(x) & i = 1, 2, \cdots, n
\end{align*}$$

(3)

(3) The evaluation function of the ideal point
The optimized solution of the evaluation index is the point, $P$, which can furthest approach the ideal point and is far from the anti-ideal point. Thus, the distance from $P$ to the ideal point is minimum ($\|f(x) - f^+(\cdot)\| \rightarrow \min$). At the same time, the distance from $P$ to the anti-ideal point is maximum ($\|f(x) - f^-(\cdot)\| \rightarrow \max$). In the present study, Minkowski distance is used to determine the ideal point and the anti-ideal point, which are given as follows.

The distance from $P$ to the ideal point, $d_1$, is calculated as

$$d_1 = \|f(x) - f^+(\cdot)\|^\frac{1}{p} = \left\{\sum_{i=1}^{n} \omega_i |f_i(x) - f_i^+(\cdot)|^p\right\}^{\frac{1}{p}}$$

(4)

The distance from $P$ to the anti-ideal point, $d_2$, is calculated as

$$d_2 = \|f(x) - f^-(\cdot)\|^\frac{1}{p} = \left\{\sum_{i=1}^{n} \omega_i |f_i(x) - f_i^-(\cdot)|^p\right\}^{\frac{1}{p}}$$

(5)

…where $p$ is a parameter that can be changed according to the practical evaluation problems. When $p$ is equal to 1, $d_1$ and $d_2$ are defined as the hamming distance and the absolute distance, respectively. When $p$ is equal to 2, $d_1$ and $d_2$ are defined as Euclid distance. When $p$ approaches infinity as a limit ($p \approx \infty$), $d_1$ and $d_2$ are defined as Chebyshov distance.

(4) Calculating the closeness degree for the evaluation object to the ideal point
The closeness degree for the evaluation object to the ideal point, $T$, is calculated as

$$T = \frac{d_2}{d_1 + d_2}$$

(6)

…where $0 \leq T \leq 1$. The bigger $T$ is, the better the evaluation object will be. Meanwhile, the bigger $T$ indicates that the smaller the distance for the evaluation object to the ideal point and the bigger distance for the evaluation object to the anti-ideal point.

Principle of the Analytic Hierarchy Process Method

The analytic hierarchy process (AHP) [27, 28] was presented to determine the weights of each evaluation index. AHP is mainly performed with mathematics and psychology. AHP has been validated as an effective way to analyze the complicated decisions, and it has been employed in many fields.

For an investigated object, the complicated decision is treated as a series of pairwise comparisons. Then the pairwise comparisons is analyzed and synthesized, which are used to capture both the subjective aspects and the objective aspects of the decision. Additionally, AHP is used to check the consistency of evaluations from the decision maker, which can reduce the prejudice on decision making.
Wang Y., et al.

(1) A hierarchy for the objective problem
First, the objective problem is treated as a hierarchy based on the above processes. Then the objectives and the relative problems are analyzed, and multiple influence factors are selected as the evaluation indices. The evaluation indices are divided into several levels, which range from the highest to the lowest. In addition, each element in each level is independent from each other.

(2) Structure judgment matrices
Based on the hierarchy construction, the various elements are compared with each other at a time, and evaluated with consideration of their impact on an element above them in the hierarchy. The relative importance between two criteria is assigned values that range from 1 to 9, as shown in Table 1. Thus, the structure of judgment matrices is constructed, as shown in Table 2.

(3) Calculating weight vectors
Each pairwise in the comparison matrix is analyzed. The maximum eigenvalues are calculated as well as the corresponding eigenvectors, which are implemented by the summation method as follows:

\[ \omega' = \frac{b_j}{\sum_{i=1}^{n} b_j} \]  

(7)

\[ \omega' = \frac{\sum_{j=1}^{n} \omega'_{ij}}{\sum_{j=1}^{n} \omega_{ij}} \]  

(8)

\[ \omega_i = \sum_{j=1}^{n} \omega'_i \]  

(9)

\[ \lambda_{max} = \frac{1}{n} \sum_{i=1}^{n} (B \omega_i) \]  

(10)

(4) Consistency test of judgment matrix
The judgment matrix is generally established based on the subjective judgment of the actual situation, which may cause some inaccuracies in the numerical matrix. Therefore, a criterion is presented to test the consistency of the judgment matrix. For the consistency test, the formula can be described as:

\[ CR = CI / RI \]  

(11)

\[ CI = (\lambda_{max} - n)/(n-1) \]  

(12)

...where \( \omega'_{ij} \) is the column vector of the judgment matrix, \( b_{ij} \) is an element of the judgment matrix \( B \), and each element \( b_{ij} \) represents the importance of the \( i \)th criterion relevant to the \( j \)th criterion. If \( b_{ij} > 1 \), the \( i \)th criterion is more important than the \( j \)th criterion, while if \( b_{ij} < 1 \), the \( i \)th criterion is less important than the \( j \)th criterion. If the two criteria have the same importance, the entry \( b_{ij} \) is equal to 1. The value of \( b_{ij} \) is measured according to a numerical scale from 1 to 9, as shown in Table 1.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Value of \( b_{ij} \) & Interpretation \\
\hline
1 & \( i \) and \( j \) are equally important \\
3 & \( i \) is slightly more important than \( j \) \\
5 & \( i \) is more important than \( j \) \\
7 & \( i \) is strongly more important than \( j \) \\
9 & \( i \) is absolutely more important than \( j \) \\
2, 4, 6, 8 & The middle of two adjacent judgments \\
Reciprocal & When \( i \) and \( j \) are compared, the scalar is the reciprocal of \( i \) and \( j \) scalar \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Order of judgment matrix & 1 & 2 & 3 \\
\hline
RI & 0 & 0.58 & 0.90 \\
\hline
\end{tabular}
\end{table}
Risk Assessment of Water Inrush...

The Coupling Model of the Ideal Point Method with AHP for Risk Analysis of Water Inrush

Analysis of Evaluation Indices and Evaluation System

The occurrence mechanism of water inrush is complicated in deep-buried karst tunnels. Many factors affect water inrush, such as disaster-causing construction, a hazard-inducing environment, construction methods, and so on. With the comprehensive consideration of the multiple factors, seven key factors were selected as the evaluation indices according to the latest relative research [18, 37]: formation lithology, unfavorable geological conditions, groundwater level, landform and physiognomy, modified strata inclination, contact zones of dissolvable and insoluble rock, and layer and interlayer fissures. Meanwhile, the unfavorable geological conditions were divided into three groups of secondary indices: water-bearing structure, karst water system, and fracture zone. Then, the evaluation index system of water inrush was established, as shown in Fig. 1. The relationships between the risk levels and each evaluation index were summarized in Table 4. The influence of the seven evaluation indices to water inrush [28] is described in detail as follows.

1. Formation lithology ($l_1$)

Formation lithology includes rock characteristics, rock composition, and structural and mechanical properties of rock. Soluble rock and non-soluble rock are formed in different geological ages and geological environments. They have different structures and mineral compositions and different solubility and permeability. Karst caves easily form in the soluble rock. The bigger the thickness

<table>
<thead>
<tr>
<th>Evaluation index</th>
<th>Formation lithology</th>
<th>Unfavorable geological conditions</th>
<th>Groundwater level (m)</th>
<th>Landform and physiognomy (proportion of negative landform area), %</th>
<th>Modified strata inclination, °</th>
<th>Contact zones of dissolvable and insoluble rock</th>
<th>Layer and interlayer fissures</th>
</tr>
</thead>
<tbody>
<tr>
<td>I(Very high)</td>
<td>0.508~0.254</td>
<td>0~60</td>
<td>0~60</td>
<td>120~60</td>
<td>100~60</td>
<td>25~45</td>
<td>0~60</td>
</tr>
<tr>
<td>II(High)</td>
<td>0.104~0.254</td>
<td>60~70</td>
<td>60~70</td>
<td>30~60</td>
<td>40~60</td>
<td>10~25</td>
<td>60~70</td>
</tr>
<tr>
<td>III(Medium)</td>
<td>0.042~0.104</td>
<td>70~85</td>
<td>70~85</td>
<td>10~30</td>
<td>20~40</td>
<td>5~10</td>
<td>70~85</td>
</tr>
<tr>
<td>IV(Low)</td>
<td>0~0.042</td>
<td>85~100</td>
<td>85~100</td>
<td>0~10</td>
<td>0~20</td>
<td>0~5</td>
<td>85~100</td>
</tr>
</tbody>
</table>

Table 4. Evaluation indices and risk grade standards of water inrush.

![Fig. 1. Hierarchy model of the influence factors for water inrush in karst tunnels.](image)
Wang Y., et al.

of karst strata, the better the karst development. The contribution of formation lithology to water inrush can be described as rock solubility $t$ [37] or by using the experts’ grading method, as shown in Table 4.

(2) Unfavourable geological conditions ($I_2$)

Unfavourable geological conditions, such as water conductive faults and plentiful water caves. Water abundance, water conductivity, and spatial position of the unfavourable geological body greatly influence the risk level of water inrush in tunnelling engineering. It is difficult to quantify the influence of unfavourable geological conditions and establish risk-grade standards of water inrush. In the present study, the contributions of unfavourable geological conditions to water inrush are calculated by using the experts’ grading method.

(3) Groundwater level ($I_3$)

Groundwater level is one of the critical factors of water inrush. In the groundwater concentration zone, the strength of the rock mass is low and water inrush easily occurs. In this work, the height deviation between groundwater level and tunnel floor, $h$, is selected as the evaluation index. According to the statistical research of water inrush cases, the ground level is divided into four levels, as shown in Table 4.

(4) Landform and physiognomy ($I_4$)

In different landforms and physiognomies, the possibility of water inrush and the water irruption quantity are different. For a cross-section of landforms, the water irruption quantity of tunnels in mountain v alley is maximum. For the longitudinal profile of landforms, the water irruption quantity of tunnels in the basin type landform is maximum. In this work, the proportion of negative landform area [38] is selected as the evaluation index. The influence of landform and physiognomy is divided quantitatively into four risk grades: 60-100%, 40-60%, 20-40%, and 0-20% based on the proportion of negative landform area.

(5) Modified strata inclination ($I_5$)

In karst tunnels, structural fractures of strata and fold morphology greatly influence karst development. Water inrush mainly occurs in the strong tectonic seismic activity zone and at the interface of thin limestone and thick limestone. According to relative studies [39], wings of syncline or anticline with a strata inclination of 25-45° are most favourable for karst development. In this work, modified strata inclination is selected as the evaluation index (Table 4).

(6) Contact zones of dissolvable and insoluble rock ($I_6$)

The influence of corrosive water in the contact zones of soluble and insoluble rock probably induces water inrush in the construction of tunnels. When soluble rock is on insoluble rock, ground water permeates the soluble rock and many karst caves form in the bottom of the soluble rock. In this work, the contributions of contact zones of dissolvable and insoluble rock are calculated by using the experts’ grading method.

(7) Layer and interlayer fissures ($I_7$)

The development of layer and interlayer fissures greatly influences water inrush in tunnelling engineering.

Table 5. Judgment matrix for weights analysis of each index $I_i$.

<table>
<thead>
<tr>
<th>Evaluation indices</th>
<th>$I_1$</th>
<th>$I_2$</th>
<th>$I_3$</th>
<th>$I_4$</th>
<th>$I_5$</th>
<th>$I_6$</th>
<th>$I_7$</th>
<th>$\omega_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_1$</td>
<td>1</td>
<td>1/3</td>
<td>1/2</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>0.15</td>
</tr>
<tr>
<td>$I_2$</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>3</td>
<td>6</td>
<td>0.35</td>
</tr>
<tr>
<td>$I_3$</td>
<td>2</td>
<td>1/2</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td>0.22</td>
</tr>
<tr>
<td>$I_4$</td>
<td>1/2</td>
<td>1/4</td>
<td>1/3</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>0.09</td>
</tr>
<tr>
<td>$I_5$</td>
<td>1/5</td>
<td>1/8</td>
<td>1/6</td>
<td>1/3</td>
<td>1</td>
<td>1/4</td>
<td>1/2</td>
<td>0.03</td>
</tr>
<tr>
<td>$I_6$</td>
<td>1/2</td>
<td>1/3</td>
<td>1/2</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>0.11</td>
</tr>
<tr>
<td>$I_7$</td>
<td>1/3</td>
<td>1/6</td>
<td>1/4</td>
<td>1/2</td>
<td>2</td>
<td>1/3</td>
<td>1</td>
<td>0.05</td>
</tr>
</tbody>
</table>

The condition with $\lambda_{max} = 7.265$, $CI = 0.044$, $RI = 1.32$ and $CR = 0.033 < 0.1$ can satisfy the consistency check requirement.

Table 6. Judgment matrix for weights analysis of $I_{2i}$.

<table>
<thead>
<tr>
<th>Evaluation indices</th>
<th>$I_{21}$</th>
<th>$I_{22}$</th>
<th>$I_{23}$</th>
<th>$\omega_{2i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{21}$</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>0.65</td>
</tr>
<tr>
<td>$I_{22}$</td>
<td>1/3</td>
<td>1</td>
<td>2</td>
<td>0.23</td>
</tr>
<tr>
<td>$I_{23}$</td>
<td>1/5</td>
<td>1/2</td>
<td>1</td>
<td>0.12</td>
</tr>
</tbody>
</table>

The condition with $\lambda_{max} = 3.004$, $CI = 0.002$, $RI = 0.58$ and $CR = 0.0038 < 0.1$ can satisfy the consistency check requirement.

Fig. 2. Weights of evaluation indices for water inrush.
Groundwater seepage and karst development are closely related to the development of layer and interlayer fissures. The influence degree of layer and interlayer fissures on karst development is divided into four levels. To meet the needs of quantitative evaluation, the contributions of layer and interlayer fissures are calculated using the experts’ grading method.

**Determination Weights with AHP**

According to the calculation processes of AHP, the weights of the first-grade evaluation indices $I_i$ can be derived, including formation lithology, unfavorable geological conditions, groundwater level, landform and physiognomy, modified strata inclination, contact zones of dissolvable and insoluble rock, and layer and interlayer fissures. Judgment matrix for weights analysis of each index $I_i$ is shown in Table 5. Meanwhile, the weights of the evaluation indices $I_{ij}$ can also be derived, including water-bearing structure, catchment area of karst water system, and the width of fault fracture zone. The conducted judgment matrix is presented in Table 6.

The weights are obtained for the selected evaluation indices of water inrush, as shown in Fig. 2. There are obvious differences between the weights of different evaluation indices, which follows the decreasing order: $I_2 > I_3 > I_1 > I_6 > I_4 > I_7 > I_5$. Among the seven evaluation indices of water inrush, the weight of the unfavorable geological condition ($I_2$) is equal to 0.35. It is concluded that unfavorable geological condition ($I_2$) is the main influence factor of water inrush.

**Determining the Ideal Point Matrix and the Anti-Ideal Point Matrix**

In the evaluation indices system of water inrush, some indices were regarded as the positive indices, including formation lithology, groundwater level, landform and physiognomy (proportion of negative landform area), and modified strata inclination. The bigger these positive indices are, the higher the risk level of water inrush. Meanwhile, the other indices were regarded as inverse indices, such as unfavorable geological conditions, contact zones of dissolvable and insoluble rock, and layer and interlayer fissures. The smaller these inverse indices are, the higher the risk level of water inrush. According to Eq. 2, Eq.3 and risk level standards of water inrush in Table 4, the ideal point matrix ($F'(+)\)) and the anti-ideal
The point matrix \( F'(-) \) of water inrush can be obtained as:

\[
F'(-) = \begin{bmatrix}
0.508 & 0 & 120 & 100 & 45 & 0 & 0 \\
0.254 & 60 & 60 & 60 & 25 & 60 & 60 \\
0.104 & 70 & 30 & 40 & 10 & 70 & 70 \\
0.042 & 85 & 10 & 20 & 5 & 85 & 85 \\
0 & 100 & 0 & 0 & 0 & 100 & 100
\end{bmatrix}
\]  \tag{13}

\[
F'(+)^{} = \begin{bmatrix}
0.254 & 60 & 60 & 60 & 25 & 60 & 60 \\
0.104 & 70 & 30 & 40 & 10 & 70 & 70 \\
0.042 & 85 & 10 & 20 & 5 & 85 & 85 \\
0 & 100 & 0 & 0 & 0 & 100 & 100
\end{bmatrix}
\]  \tag{14}

**Computational Procedure**

The presented model was developed based on Visual Basic. First, the value of each evaluation index was obtained from field survey and site monitoring. Second, the weights of each evaluation index were calculated based on AHP. Then, the distances to the ideal point and the anti-ideal point were calculated. Finally, the closeness degree for the ideal point was determined. According to the obtainable results, the risk level of water inrush for the evaluation sample could be provided. The computational procedure and its flow chart are presented in Fig. 3.

**Model Test**

In order to test the validity of the proposed model, some measured samples were selected from a typical karst tunnel in China, as shown in Table 7. The measured samples were evaluated on risk assessment of water inrush in the karst tunnel. For sample 1, the closeness degree to the ideal point of each level are listed as follows: (1) 0.2342 under level I (very high), (2) 0.5185 under level II (high), (3) 0.5641 under level III (medium), and 0.5587 under level IV (low). The greatest closeness degree is 0.5641 under level III (medium). Thus the risk level of water inrush for sample 1 belongs to grade III (medium). Moreover, the obtainable results were compared with the field-observed results of the karst tunnel, and good agreement could be gained, which could provide useful consult for risk assessment of water inrush in the karst tunnel.

The obtained results were compared with the results of the relevant methods (Table 8). It was found from Table 8 that the results from the proposed model agreed with results from set pair analysis [26], the efficacy coefficient method [27], and the cloud model [28]. Therefore, the proposed method used to assess the risk level of water inrush in karst tunnels is feasible and effective, and convenient to operate. And it has a specific and significant advantage as the ideal point method is a generalization of discriminant analysis theory. As a comprehensive evaluation method, the ideal point method can simultaneously deal with the comprehensive evaluation of multiple factors and multiple objects. It also can make up for the shortcoming of traditional evaluation methods, providing reliable data from evaluation results. The presented model could provide scientific evidence for risk assessment of water inrush in karst tunnels.

**Engineering Application**

**Engineering Background**

To further validate the proposed model, a practical project, Jigongling Tunnel, was selected as the investigated object for its risk assessment of water inrush. Jigongling [26-28] is located in the karst mountain areas of Hubei Province in China, which possesses the common characteristics of karst tunnels. In detail, Jigongling is a typical deep-buried tunnel, and the maximum overburden thickness is 338 m.
The project is 4.5 km long, which goes through multiple geological environments. Particularly, when the tunnel goes from K19 + 240 to K20 + 180, the undergoing environments are very complicated with shale, marl, and dolomitic limestone. Moreover, groundwater has direct influence on the tunnel, which should be considered. When the tunnel goes from K19 + 450 to K19 + 760, the tunnel passes a weak karst aquifer and a strong karst aquifer [18].

Risk Evaluation of Water Inrush

Based on the present risk evaluation model, the risk level of water inrush was investigated for Jigongling from K19+509 to K19+539. The closeness degree of the investigated object to the ideal point were obtained and listed as follows: $T(I) = 0.1613$, $T(II) = 0.6756$, $T(III) = 0.6142$, and $T(IV) = 0.5668$. The greatest closeness degree was 0.6756 under level II (high). As a result, the risk level of water inrush was regarded as Level II, which belonged to high risk.

The present result was compared with the results from the attribute mathematics theory [18], set pair analysis [26], the efficacy coefficient method [27], and the cloud model [28] in Table 9. It was found from Table 9 that reasonable agreements validate the present approach. The present result was also compared with the field-observed results from the tunnel (Fig. 4), and good agreement could be gained. Therefore, the present model with the ideal point method and AHP is feasible for risk assessment of water inrush in karst tunnels. The model also provides a novel way for risk assessment of water inrush and other disasters.

**Conclusions**

Risk assessment of water inrush is complicated with many uncertainties. Water inrush is also affected by complex factors. To deal with the multiple factors and the complicated objects, the ideal point method was put forward in the present study. Risk levels of water inrush
in karst tunnels were predicted with the generalization of discriminant analysis theory. First, the karst hydrology and the engineering geological conditions were analyzed to select the evaluation indices. Several influence factors were considered as evaluation indices, including formation lithology, unfavorable geology and groundwater level, landform and physiognomy, modified strata inclination, contact zones of dissolvable and insoluble rock, and layer and interlayer fissures. Then each evaluation index was treated with the analytic hierarchy process (AHP), which can avoid the individual influence on subjective methods. Thus, weight coefficients were determined for each evaluation index, which were then used for the confirmation of risk level of water inrush. Based on the established model, some measured samples from a typical karst tunnel were investigated with good agreement when compared with the field-observed results. Furthermore, risk assessment was implemented in a typical karst tunnel, Jigongling. The evaluation results were compared with the results form the attribute mathematics theory, set pair analysis, the efficacy coefficient method, the cloud model, and field-observed findings. Reasonable agreements validate the model again. Therefore, the coupling model with the ideal point method and AHP is feasible for the risk assessment of water inrush in karst tunnels. Meanwhile, the present method could also provide results with relatively high accuracy. The model is also simple and feasible for the construction of karst tunnels. In addition, the present model can be extended for other risk assessments.

Acknowledgements

Financial support from the Fundamental Research Funds for the Central Universities (2017XKQY074) are sincerely acknowledged.

Conflict of Interest

The authors declare no conflict of interest.

References

35. WANG J. Comprehensive assessment underground water quality assessment in Huaibei city area based on multiple-goal decision-making and ideal point method. Environmental science and management [J], 25, 2005.