Original Research

Vibration Characteristics in Subway Operation and Environmental Responses of Ancient Buildings

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Abstract

Seismic waves travel faster in rock stratum and have greater environmental responses. This paper analyzed the transmission regulation, vibration frequency, and amplitude attenuation characteristics of waves in layered elastic medium. Also, impacts of the elastic modulus of homogeneous medium, subway tunnel depth, and other factors on the environmental response in the runtime were investigated. Based on typical lots of Qingdao Subway, environmental response of train vibration was analyzed. The horizontal vibration response of Sensitive Point 1 indicates that the horizontal velocity reaches its peak (0.4 mm/s) at 25 m away from the vibration source, and it minimizes with the increase of horizontal distance. Vertical vibration of Sensitive Point 2 shows that the maximum of Z vibration level occurred right above the tunnel, and numerical value of Z vibration level decreased with the increase of horizontal distance. The results provide data for the vibration damping and isolation during subway tunnel construction in rock stratum, and meaningful references to the control of environmental response and vibration problems during the operation of a subway train.

Keywords: rock strata, subway train operation, vibration characteristics, ancient buildings, environmental response

Introduction

The vibration and noise caused by subway trains have raised public concern since the first subway was constructed. A subway train generates random excitation of a certain amount, which spreads to the ground and above-ground structures via the track, ballast, tunnel, rock, and soil media. The excitation then induces a vibration, known as secondary vibration, that triggers shaking in doors, windows, and other facilities, which is harmful to the environment. In general, subway lines are bored beneath densely populated areas or industrial estates where a number of residential buildings, workshops, ancient buildings, and other facilities are located, which are extremely sensitive to vibration. Buildings and residents suffer a lot from ground vibration and noise caused by subway trains. The vibration and noise will become public hazards after the metro lines have constituted a metro network system.

Complaints about damages to buildings and disturbance to civilian's lives, which were caused by subway trains, have occured in many countries. According to statistics, besides of the factory noise and building construction, the environmental vibration caused by transportation systems is the most strongly reflected pollution, accounting for 14.2% of the total complaints. Paris Metro Lines No. 7 and No. 13 travel beneath the New Opera House in Bastille, and Paris Metro has had to apply certain vibration isolation measures to prevent vibration hazards [1, 2].

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Many researchers have started their research of vibration characteristics and environmental response during subway train operation long before. J. Melke and his colleagues presented a measurement that was based on pulse stimulation and test analysis to predict the ground vibration level around subway lines. He also gathered transfer functions of different test points and analyzed the regular vibration wave transmission pattern. British Rail Authority once conducted a survey of ground vibration triggered by trains, which mainly focused on resonance and the relationships among driving velocity, excitation frequency, and track parameters. In Switzerland, A. Zach and G. Rutishauser studied the vibration frequency and acceleration characteristics of subway trains and tunnel structures. They presented methods to reduce underground and above-ground structure vibration triggered by subway trains [3-5].

The research noted above mainly focuses on the vibration characteristics and environmental responses in soil stratum. But as for rock stratum, in which the seismic wave transfers faster and the environmental response is more intense, related research is much less. So research on vibration characteristics and related environmental problems during subway operation in rock stratum may provide information for vibrational damping and isolation, which is meaningful to subway operation.

Transmission Characteristics of Seismic Wave in Rock Stratum

Propagation, Frequency, and Amplitude Attenuation with Time and Space of Elastic Wave

Under the effect of vertical harmonic vibration on elastic half-space surface, the amplitude of near-field body wave matches that of the Rayleigh wave. The superposition of the three kinds of waves produces a characteristic interference wave, whose wavelength is determined by frequency and Poisson's ratio. Generally, the vertical displacement component of Rayleigh wave decays faster than the horizontal component, the horizontal component of body wave attenuates more slowly than the vertical one.

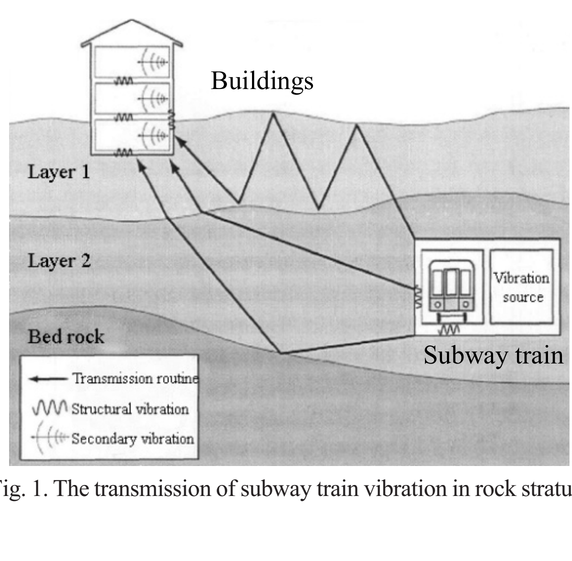

Fig. 1. The transmission of subway train vibration in rock stratum.

Power base, transportation, construction, and other major artificial wave sources transfer the combination of P wave, S wave, and Rayleigh wave via rock medium. Miller approximately calculated the distribution of the energy of these waves on elastic half-space surfaces under the round vertical resonant excitation using Poisson's ratio v = 0.25.

The energy distribution of the three waves is: Rayleigh wave 67%, S wave 26%, P wave 7%. Woods drew the famous long-distance wave field map according to this result [6-13].

With the increase of distance, ground vibration wave attenuates, wavefront's acre increases, and wave energy density and displacement amplitude of the wave decrease. In the media, the attenuation relationship between amplitude of a body wave's displacement and distance is 1/r, while on the half-space surface the attenuation relationship is l/r^2. In the media, the attenuation relationship between amplitude of Rayleigh wave's displacement and the distance is $l/r^{0.5}$, while on the half-space surface they are exponentially related. The fact that 2/3 of the total energy amount at the far field is passed along by Rayleigh wave from the surface vibration source and that Rayleigh wave attenuates much slower than body wave in horizontal direction indicates that under vertical dynamic loads, the far-field ground vibration is mainly Rayleigh wave.

Kelvin-Voigt viscoelastic body physics equation:

$$\sigma = E\varepsilon + \eta\dot{\varepsilon} \tag{1}$$

Longitudinal wave equation:

$$\rho\frac{\partial^2 u}{\partial t^2} = E\frac{\partial^2 u}{\partial x^2} + \eta\frac{\partial^3 u}{\partial x^2 \partial t} \tag{2}$$

Kelvin-Voigt spatial attenuation regulation of body stress wave's amplitude

$$u = u_0 e^{-\alpha x} e^{i(\omega t - k x)} \tag{3}$$

...where u_0 is the amplitude, ω is the vibration frequency, α is the attenuation coefficient of vibration peak with the space, and k is the number of spatial response waves of viscoelastic body.

Taking the viscosity of rock stratum into account, the spatial attenuation coefficient of vibration amplitude at one point is:

$$\alpha^2 = \frac{\rho E\omega^2}{2(E^2 + \eta_V^2\omega^2)}\left[\sqrt{1 + \frac{\eta_V^2\omega^2}{E^2}} - 1\right] \tag{4}$$

The attenuation coefficient of vibration amplitude with time at one point is as follows:

$$a_t = \frac{\eta_V E_V \omega_q^2}{4(E_V^2 + \eta_V^2\omega_q^2)}\left(\sqrt{1 + \frac{\eta_V^2\omega_q^2}{E_V^2}} + 1\right) \tag{5}$$

The relationship between wave frequency and vibration frequency is:

$$\omega_w^2 = \frac{E_V^2\omega_q^2\left(\sqrt{1 + \frac{\eta_V^2\omega_q^2}{E_V^2}} + 1\right)}{2(E_V^2 - \eta_V^2\omega_q^2)}\left[1 - \frac{\eta_V^2\omega_q^2}{8(E_V^2 - \eta_V^2\omega_q^2)}\left(\sqrt{1 + \frac{\eta_V^2\omega_q^2}{E_V^2}} + 1\right)\right] \tag{6}$$

Based on the hypothesis that the rock is viscoelastic, the ratio of wave frequency and vibration frequency, namely r, is always less than 1, which indicates that the frequency of stress wave attenuates during its propagation in viscoelastic medium. R decreases with the increase of viscosity coefficient; the higher the frequency, the faster r decreases. R increases with the increase of elastic modulus; the higher the frequency, the faster r increases. Whether spatial attenuation or temporal attenuation, high-frequency harmonic waves attenuate faster than low-frequency harmonic waves. The increase of viscosity coefficient will trigger the increase of attenuation coefficient of a stress-amplitude wave's amplitude.

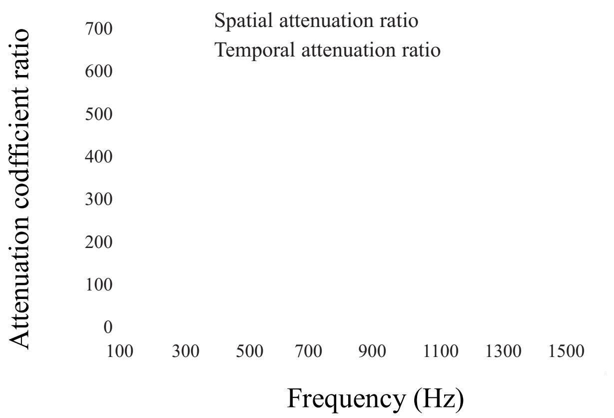

Fig. 2. The relationship between attenuation coefficient and frequency.

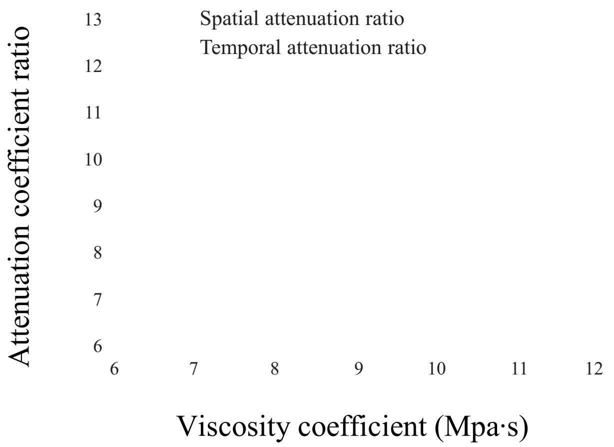

Fig. 3. The relationship between attenuation coefficient and viscosity coefficient.

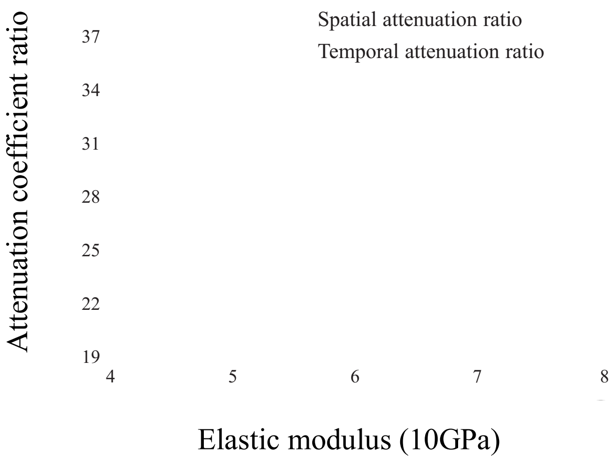

Fig. 4. The relationship between attenuation coefficient and elastic modulus.

Increase in the elastic modulus of the rock will lead to a decrease in spatial attenuation coefficient of a stress wave's amplitude. The relationship between elastic modulus and temporal attenuation coefficient can be approximated as a linear relationship, while the relationship between elastic modulus and spatial attenuation coefficient can be approximated as an exponential relationship. But spatial attenuation is more sensitive to elasticity modulus than temporal attenuation. The greater the elastic modulus is, the slower spatial attenuation will be.

Wave Properties in Layered Elastic Medium

Reflection, transmission, and refraction will occur when shock wave reaches the interface of two different kinds of rock media.

Incident wave f1 travels form media 1 to media 2, the two media's propagation constants are assumed to be p_1 and p_2, respectively. Transmission wave f_2 and reflected wave g_1 were generated at the interface, as shown:

Displacement reflection coefficient: $R = \dfrac{1-\beta}{1+\beta}$

Displacement transmission coefficient: $T = \dfrac{2}{1+\beta}$

In the above formula, β is the impedance ratio of media 2 to media 1.

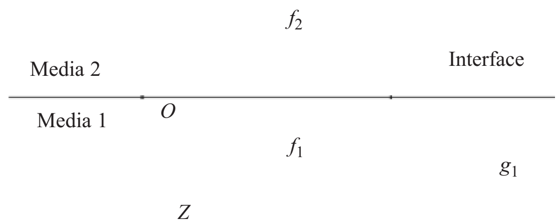

Fig. 5. Incident wave, transmission wave, and reflected wave.

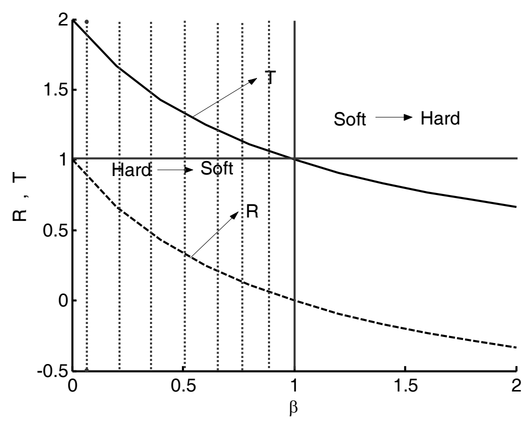

Fig. 6. The transmission coefficient and reflection coefficient of displacement wave.

Table 1. Layer parameters.

	Silty clay	Strongly weathered granite	Moderately weathered granite	Slightly weathered granite
Elastic Modulus (Mpa)	289	60	5,000	22,000
Poisson's ratio	0.31	0.28	0.25	0.22
Density (kg/m³)	3	2,300	2,450	2,620

Influencing Factor of Environmental Response in Subway Operation

The Effect of Homogeneous Medium's Elastic Modulus

Four kinds of layers were selected in the research, respectively silty clay, strongly weathered granite, moderately weathered granite, and slightly weathered granite. The regulation of vibration propagation in single stratum was studied first. The parameters were shown in Table 1.

As can be seen from Fig. 8, the frequency of horizontal vibration velocity and vertical vibration acceleration are mainly concentrated in the range of 10 Hz or less. With the decrease of a layer's elastic modulus, the frequency corresponding to the maximum vibration amplitude gradually decreases and the mid-frequency range amplitude gradually disappears.

Figs. 9 (a-d) and 10 (a-d) show that the greater the elastic modulus is, the smaller the maximum of horizontal vibration velocity will be. When the elastic modulus is relatively small, there will be a partial amplification area beyond the peak point of horizontal velocity. The bigger the elastic modulus is, the lower the peak of vertical Z vibration level and the smaller the range of influence will be.

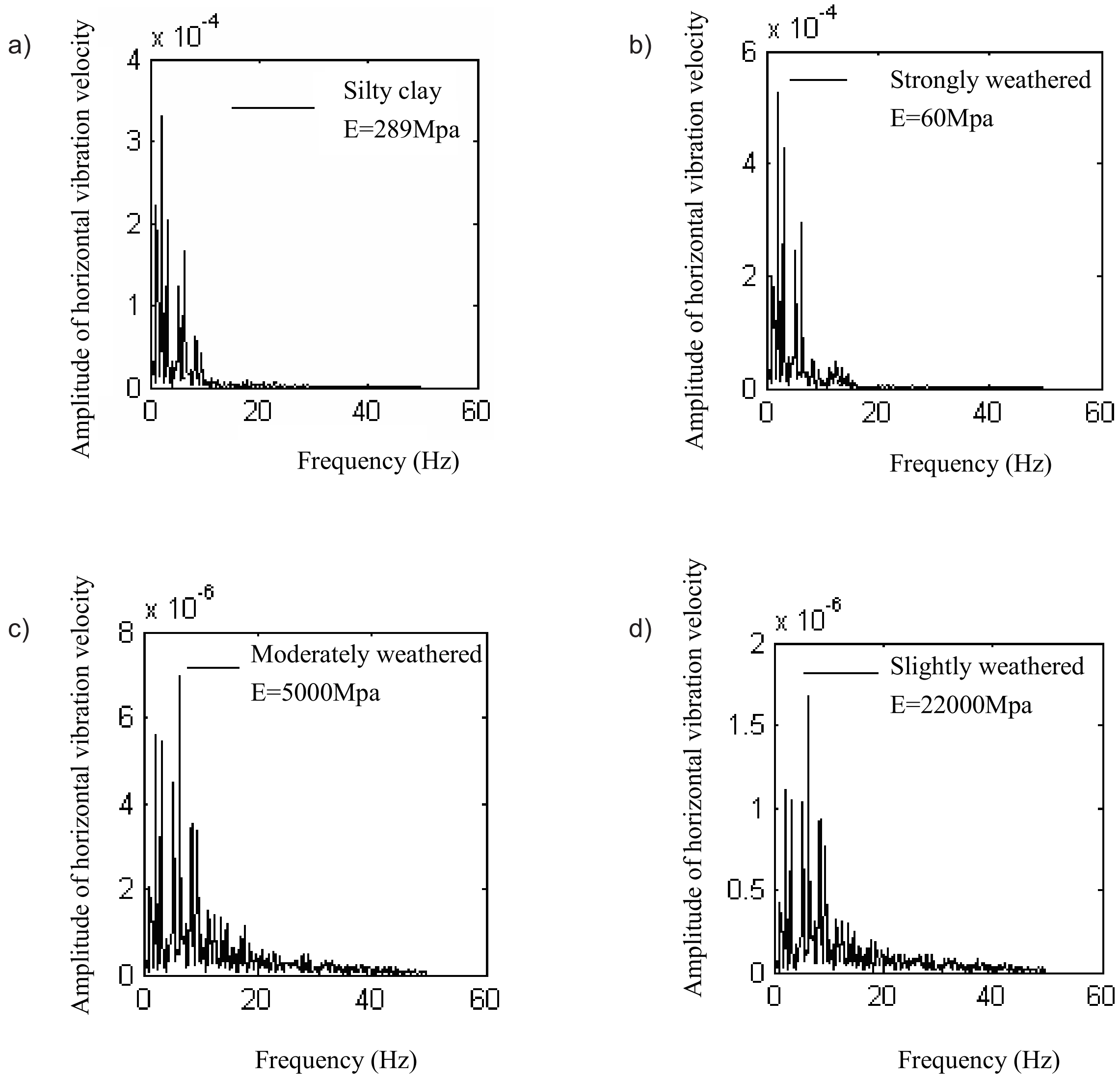

Fig. 7. Amplitude of horizontal vibration velocity 10 m from the center of the tunnel.

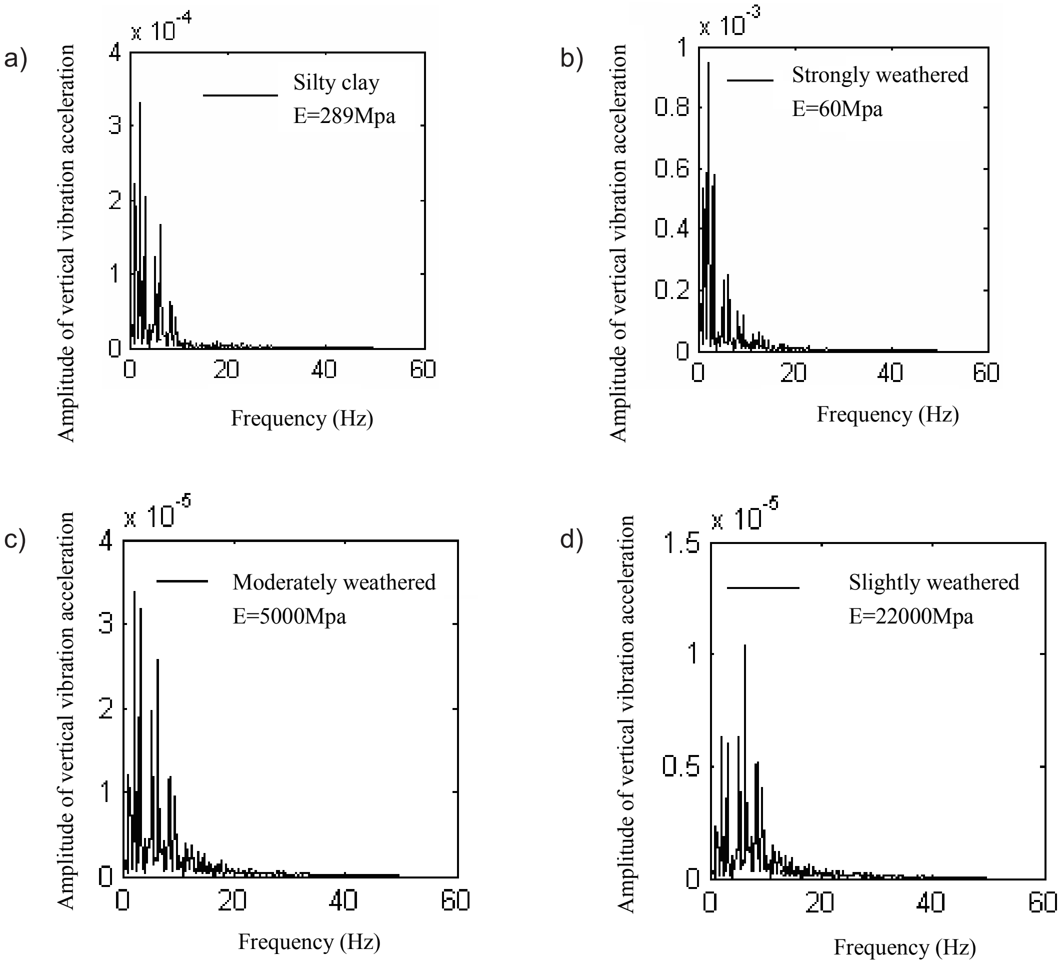

Fig. 8. Amplitude of vertical vibration acceleration at the center of the tunnel.

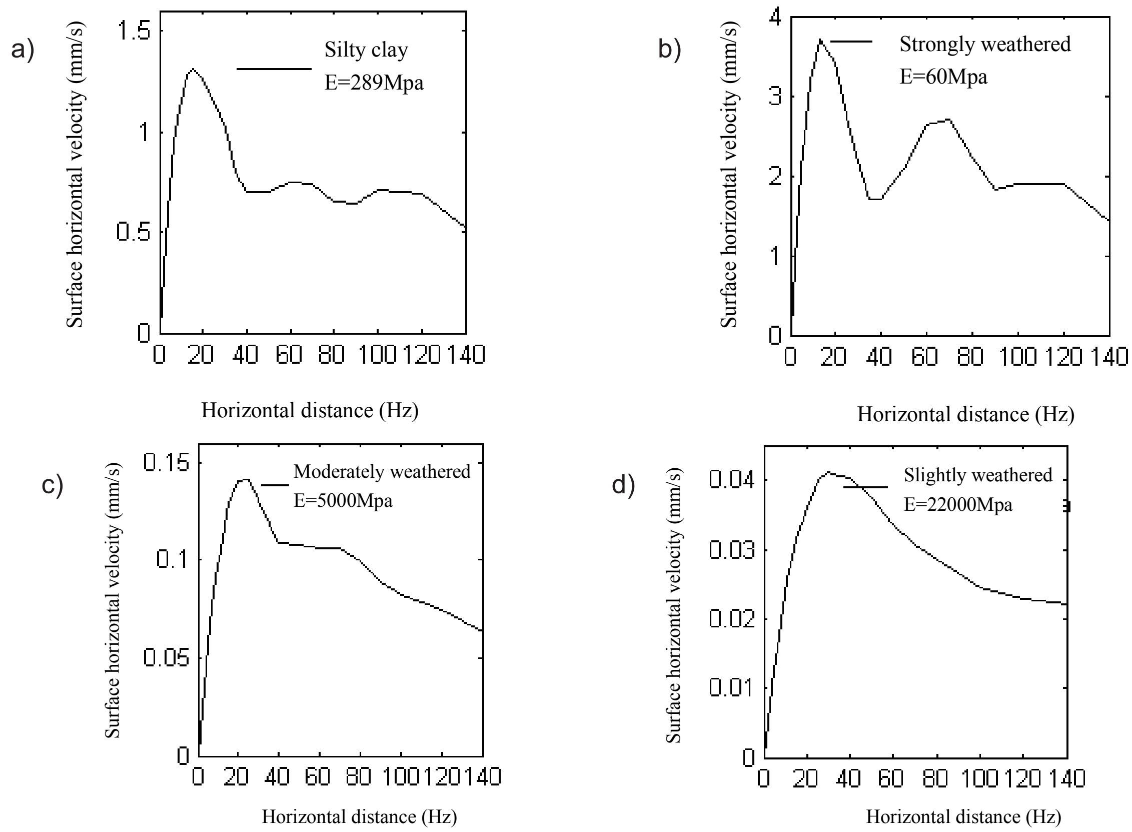

Fig. 9. The variation of surface horizontal velocity with horizontal distance.

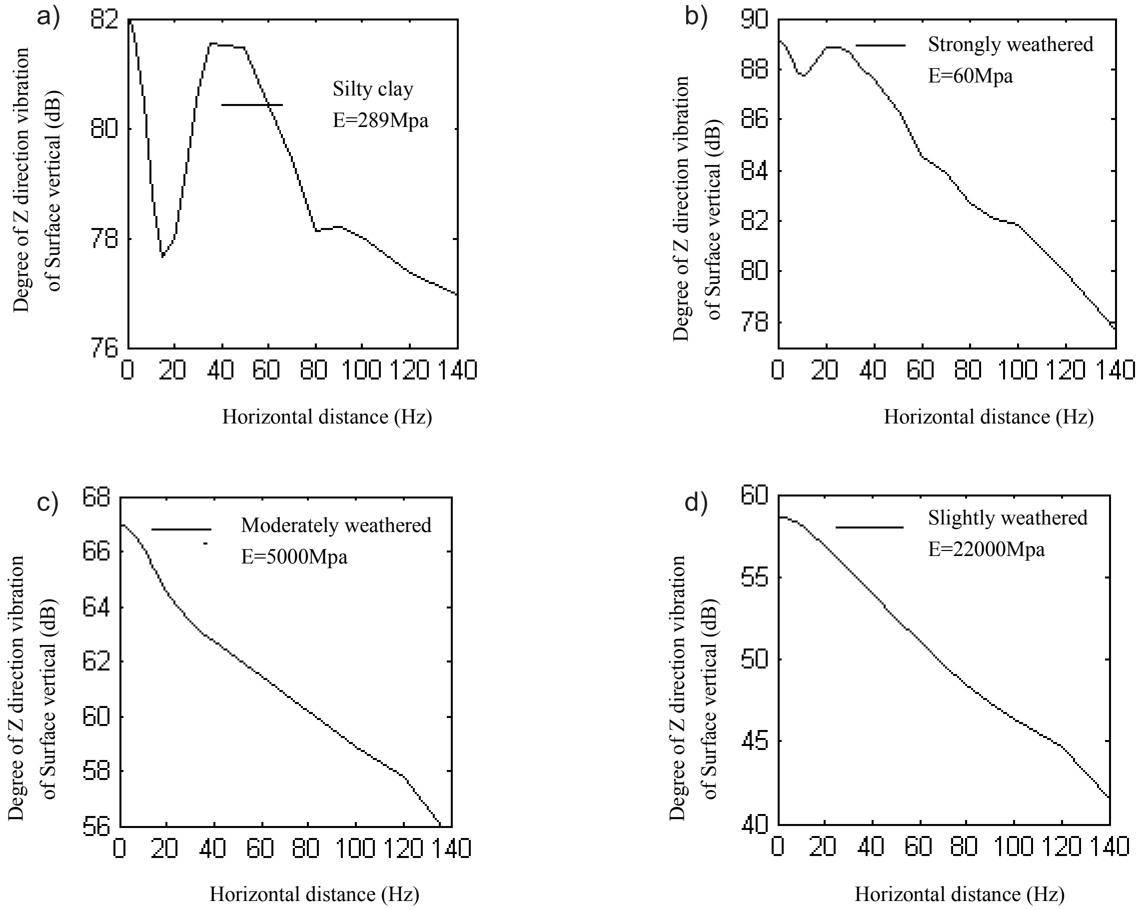

Fig. 10. The variation of degree of Z direction vibration of surface vertical with horizontal distance.

Table 2. Calculation conditions.

Conditions	Elastic modulus of rock stratum(MPa)		
	First layer	Second layer	Third layer
Condition 1	10	1,000	22,000
Condition 2	10	500	22,000
Condition 3	10	100	22,000
Condition 4	10	50	22,000
Condition 12	10	1,000	11,000
Condition 13	10	1,000	1,100

The Effects of Layered Medium's Elastic Modulus

When the rock's elastic modulus changes, the distribution of acceleration level and horizontal velocity on the free surface and below the centerline of the underground subway tunnel is shown in Figs. 11-14.

Obviously, with the increase of the second layer's elastic modulus, the vibration level on the free surface is reduced and the sphere of influence gets smaller, making the damping effect apparent.

The Impact of Tunnel Depth

As can be seen from Fig. 15, the shallower the tunnel depth, the bigger the sphere of vibrational influence. The vibrational response of the free surface increases with the decrease of subway tunnel depth [11].

Project Examples

Overview of Sensitive Points

(1) Ancient German architecture site
Name of Sensitive Points 1: Ancient German architecture
Line Mileage: K0 +825 ~ K0 +855
Protection level: excellent historical architecture
Location section: Railway Station – University Avenue Station
Horizontal distance from the line: 6 m
Tunnel depth: 20 m
Line velocity: 70 km/h

(2) Junan Road No. 3~7, Guangxi Road No. 9/11/13/24, Hunan Road No. 18
Name of Sensitive Points 2: Junan Lu No. 3~7, Guangxi Road No. 9/11/13/24, Hunan Road No. 18
Line Mileage: K0+700~K0+865
Protection level: environmental function zones

Location section: Railway Station – University Avenue Station
Horizontal distance from the line: 6 m
Tunnel depth: 20 m
Line velocity: 70 km/h

Degree of acceleration vibration (dB)

Case 13
Case 12
Case 1
Case 2
Case 3
Case 4

Distance of the symmetry center (m)

Fig. 11. The distribution of degree of acceleration vibration on free surface.

Degree of acceleration vibration (dB)

Case 13, E2=1000MPa, E3=1100MPa
Case 12, E2=1000MPa, E3=11000MPa
Case 1, E2=1000MPa, E3=22000MPa
Case 2, E2=500MPa, E3=22000MPa
Case 3, E2=100MPa, E3=22000MPa
Case 4, E2=50MPa, E3=22000MPa

Distance of the symmetry center (m)

Fig. 12. The distribution of degree of horizontal acceleration vibration on free surface.

Horizontal velocity (mm/s)

Case 1
Case 2
Case 3
Case 4
Case 12
Case 13

Distance of the symmetry center (m)

Fig. 13. The distribution of horizontal velocity on free surface.

Buried depth(m)

Case 13, E2=1000MPa, E3=1100MPa
Case 12, E2=1000MPa, E3=11000MPa
Case 1, E2=1000MPa, E3=22000MPa
Case 2, E2=500MPa, E3=22000MPa
Case 3, E2=100MPa, E3=22000MPa
Case 4, E2=50MPa, E3=22000MPa

Degree of acceleration vibration (dB)

Fig. 14. The distribution of degree of vertical acceleration vibration along the tunnel centerline.

Burial depth 15m
Burial depth 20m
Burial depth 25m
Burial depth 30m

Degree of vertical Z direction vibration (dB)

Horizontal distance to tunnel center (dB)

Fig. 15. Distribution of degree of vertical Z direction vibration on free surface as tunnel depth.

Table 3. Physical parameters of layers.

Soil layers	1	2	3	4
Layer names	Plain fill	Strongly weathered rock	Moderately weathered rock	Slightly weathered rock
Elastic Modulus (Mpa)	10	100	5000	22000
Poisson's ratio	0.31	0.26	0.25	0.22
Density (kg/m^3)	1.75e3	2.35e3	2.45e3	2.45e3

Geology and Physical Model

Analysis of Calculation Results

This calculation adopted Ansys. For finite element dynamic analysis, computational domain and cell size should meet the following conditions:

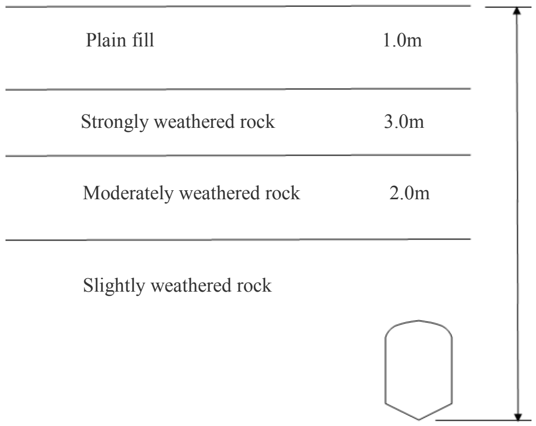

Fig. 16. The physical model of the stratum.

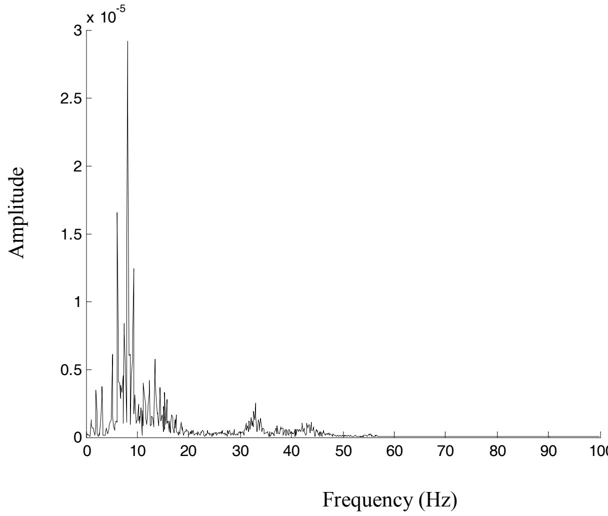

Fig. 18. Curve of horizontal velocity amplitude with frequency at Sensitive Point 1.

Horizontal Vibration Response at Sensitive Point 1

As can be seen from Figs. 17 and 18, when a train passes, the horizontal velocity increases and then minimizes gradually. The maximum horizontal velocity of sensitive point 1 reaches 0.15 mm/s. The frequency of horizontal response is mainly within the range of 10~30Hz,and the amplitude is relatively greater in the range of 5-10Hz.

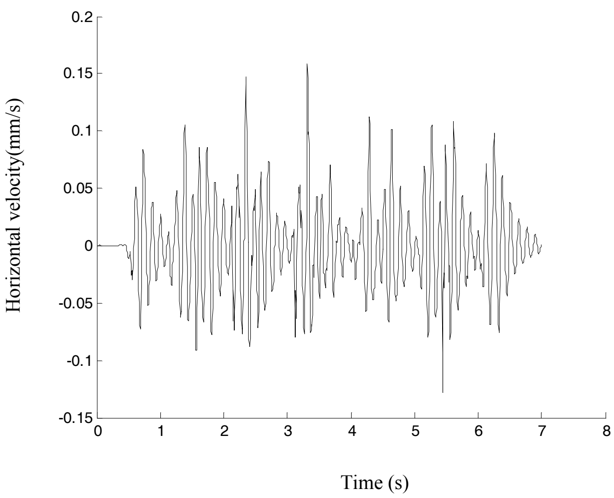

Fig. 17. Curve of horizontal velocity response with time at Sensitive Points 1.

Sensitive point 1 is an ancient German-style building, the allowed vibration speed is 0.45 mm/s, while the exceeding amount of vibration is 0.91 mm/s. Take the importance of heritage into account, damping measures need to be adopted to ensure the safety of cultural relics.

As can be seen from Fig. 19, within the range of 25 m, as the horizontal distance increases, the horizontal velocity gradually increases, and the maximum of horizontal velocity occurs at a distance of 25 m from the center of the tunnel, approximately 0.4 mm/s. In other distance segments, with the horizontal distance increases, the horizontal velocity has a downward trend in the mass.

Vertical Vibration Response at Sensitive Point 2

As can be seen from Figs. 20 to 23, when the subway train is drawing in, vertical velocity and acceleration increase gradually, and the maximum of vertical velocity and acceleration is 0.4 mm/s and 0.05 m/s^2, respectively. After the train passes by, vertical velocity and acceleration

Table 4. Calculation parameters and maximum structural velocity response at Sensitive Point 1.

Sensitive point number	Ancient architecture name	Distance from vibration source centerline (m)	Ground vibration velocity at the basis (mm/s)	Determination of Dynamic amplification factor						Model participation factors	Maximum structural response velocity (mm/s)
				Index of the vibration model	Structural natural frequency calculation coefficient	Structural natural frequency (Hz)	Ground vibration frequency (Hz)	Frequency ratio	Dynamic amplification factor		
118	German architecture (Guangxi Road No.9)	27.1	0.15	1	1.571	8.22	13.02	1.6	6	1.273	1.36
				2	4.712	32.65	13.02	0.6	7	-0.424	
				3	7.854	41.08	13.02	0.4	7	0.255	

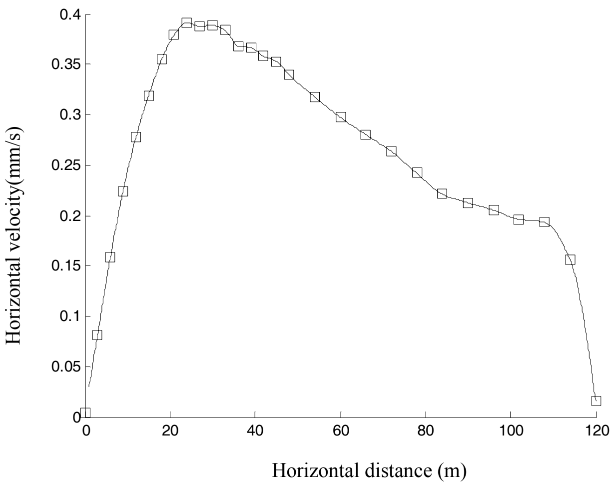

Fig. 19. Variation of the horizontal velocity amplitude with horizontal distance.

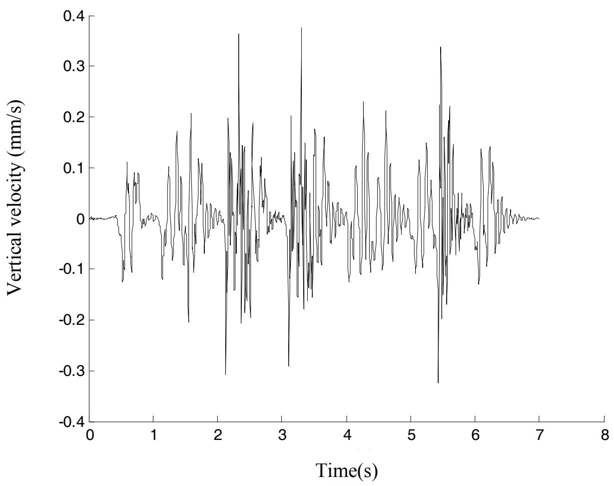

Fig. 20. Curve of vertical velocity response with time at Sensitive Point 2.

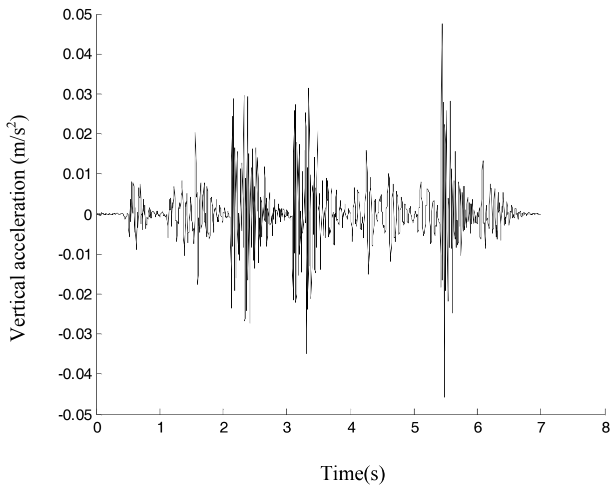

Fig. 21. Curve of vertical acceleration response with time at Sensitive Point 2.

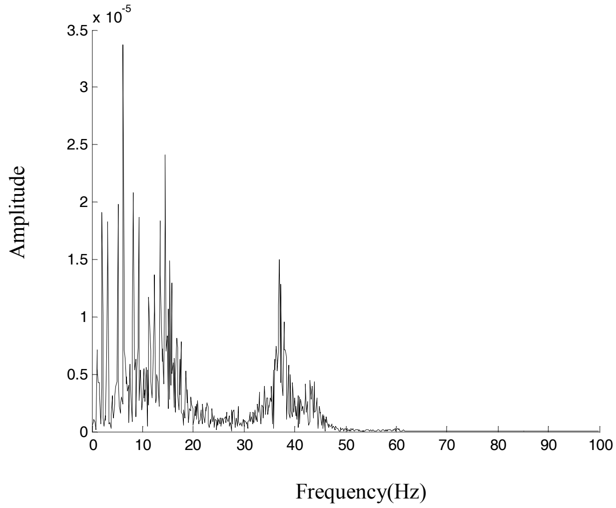

Fig. 22. Amplitude-frequency characteristic curve of vertical vibration velocity response at Sensitive Point 2.

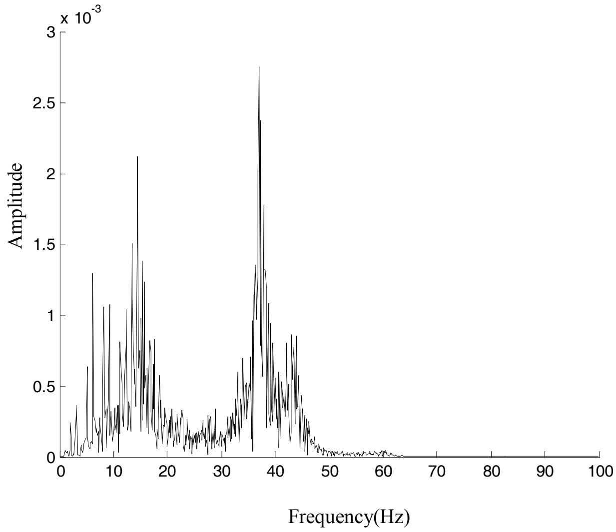

Fig. 23. Amplitude-frequency characteristic curve of vertical vibration acceleration response at sensitive point 2.

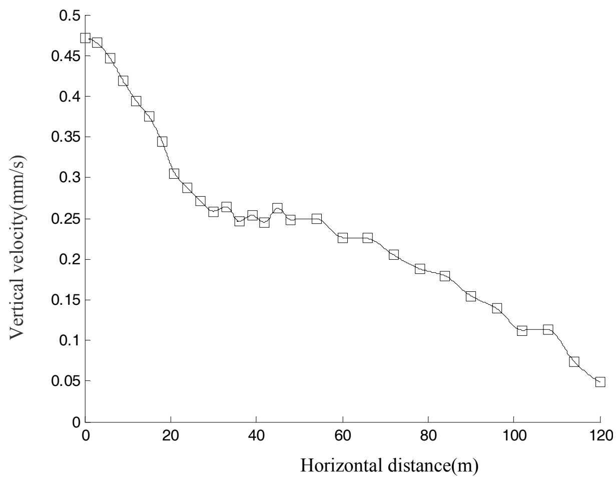

Fig. 24. Variation of vertical velocity amplitude with horizontal distance.

decay gradually. The responsive frequency of vertical velocity and acceleration are mainly within the range of 0~50Hz, vertical velocity response reaches its peak around 5Hz.

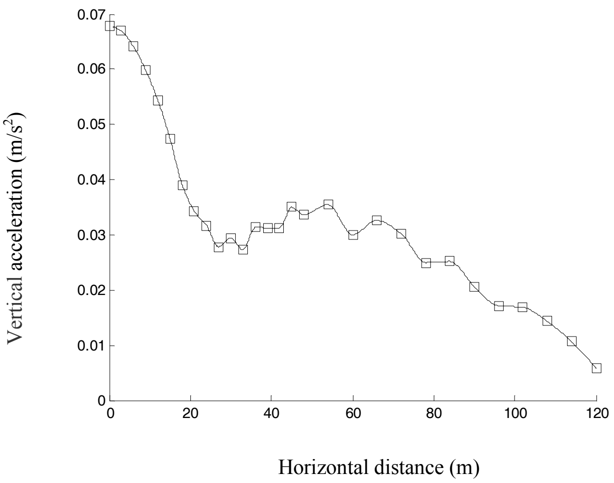

Fig. 25. Variation of vertical acceleration amplitude with horizontal distance.

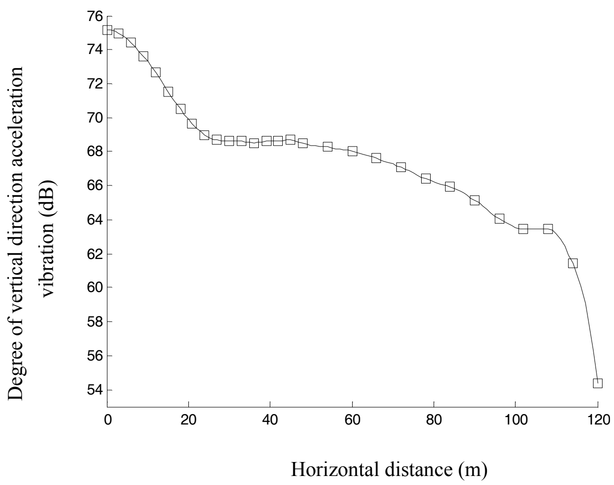

Fig. 26. Variation of vertical acceleration level with the horizontal distance.

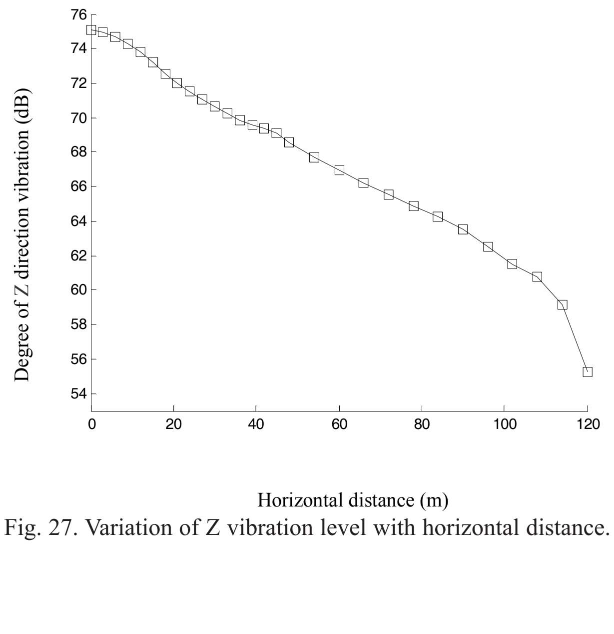

Fig. 27. Variation of Z vibration level with horizontal distance.

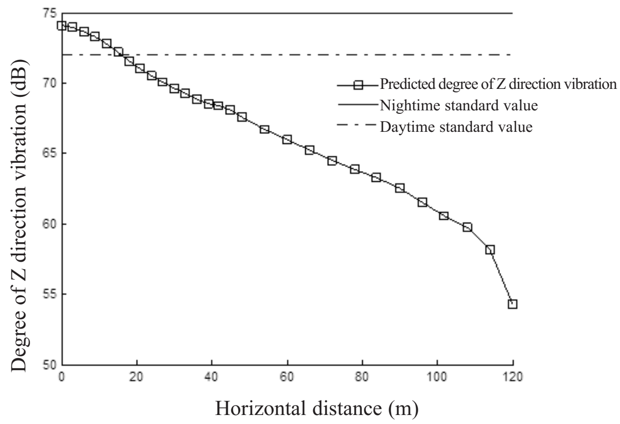

Fig. 28. The standard value of degree of Z direction vibration and its variation with horizontal distance.

As can be seen from Figs. 24 to 28, the maximums of vertical vibration velocity, acceleration, and vertical Z vibration levels all occurred at 0 m above from the tunnel, at respectively 0.46 mm/s, 0.07mm/s^2, and 74.8 dB. As the distance increases, the vertical vibration velocity, acceleration, and vertical Z vibration level all have a decreasing trend. In the range of 40 m-65 m, there exists an amplification area in vertical velocity. Sensitive Point 2 belongs to environmental function zone 4, where VLz is 74.6 dB, which is below the standard value daytime but higher than the nighttime standard value by 2.6 dB. Sphere of vibrational influence is 0 m in daytime and 15 m in nighttime, respectively.

Conclusions

(1) The greater the elastic modulus is, the faster the vibration transmission velocity will be. Increases in the elastic modulus of the rock will lead to the decrease of spatial attenuation coefficient of a stress wave's amplitude. The relationship between elastic modulus and temporal attenuation coefficient can be approximated as a linear relationship, while the relationship between elastic modulus and spatial attenuation coefficient can be approximated as an exponential relationship. Spatial attenuation is more sensitive to elasticity modulus than temporal attenuation. The greater the elastic modulus, the slower spatial attenuation will be.
(2) When a wave travels from hard rock to soft rock, the amplitude of the transmitted wave increases. It's easier for a vibration wave to spread from hard rock to soft rock.
(3) The frequency of horizontal vibration velocity and vertical vibration acceleration are mainly concentrated in the range of 10Hz or less. With the decrease of a layer's elastic modulus, the frequency corresponding to the maximum vibration amplitude gradually decreases and the mid-frequency range amplitude gradually disappears. The bigger the elastic modulus, the smaller the peak of the amplitude of vibration velocity, and there won't be an amplification area. But when the elastic modulus

small, an amplification area will appear outside the peak point of the horizontal velocity. The larger the elastic modulus, the smaller the peak of vertical Z vibration level and the range of vibrational impact. When the elastic modulus is small, an amplification area will appear outside the peak point of the vertical Z vibration level and the range of vibrational impact will be very large. Vibrational loading is mainly low-frequency vibration, so the better the rock's property, the larger the elastic modulus, the higher natural frequency, the better the filter effect, and the smaller the response on the surface.

(4) For multi-layer medium, the larger the elastic modulus of coating layer above the tunnel, the smaller the surface vibration. The deeper the tunnel is buried, the smaller the free surface's vibration will be. The amplitude of vertical Z vibration level decreases 0.3-0.4 dB for each additional depth of 1 m.

(5) Project example shows: the maximum horizontal velocity at Sensitive Point 1 is 0.15 mm/s, and the frequency is concentrated in the range of 0-10Hz.The maximum responsive velocity is 1.36 mm/s, which exceeds the standard value of 0.45 mm/s. Damping measures need to be adopted to ensure the safety of cultural relics. Horizontal response velocity tends to decrease with increasing distance. Vertical acceleration level at Sensitive Point 2 is 74.3dB, which is lower than the standard value of night-time environmental vibration; while Z vibration level is 74.6 dB, exceeding the standard value (72 dB) by 2.6 dB. The maximum value of Z vibration level is 74.8 dB, which occurs right above the tunnel center. Z vibration level tends to decrease with the increase of horizontal distance.

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References

1. JIA Y., GUO M., LIU W, ZHANG X., LIU W. Dynamic Effect of Train-Induced Vibration on Historic Buildings. Journal of Beijing Jiaotong University, **1**, 029, **2009**.
2. XIA H., CAO Y., DE GUIDO R., GEERT D. Environmental problems of vibrations induced by railway traffic. Frontiers of Architecture and Civil Engineering in China, **1**, (2), 142, **2007**.
3. FAHY F., GARDONIO P., HAMBRIC S. Sound and structural vibration. The Journal of the Acoustical Society of America, **122**, (2), 689, **2007**.
4. KOUROUSSIS G., CONTI C., VERLINDEN O. A numerical analysis of the influence of tram characteristics and rail profile on railway traffic ground-borne noise and vibration in the Brussels Region. Sci. Total Environ., **2013**.
5. LU F., KENNEDY D., WILLIAMS F W., LIN J.H. Sympletic analysis of vertical random vibration for coupled vehicle-track systems. J. Sound Vib., **317**, (1), 236, **2008**.
6. LEE S Y., CHENG Y C. Hunting stability analysis of high-speed railway vehicle trucks on tangent tracks. J. Sound Vib., **282**, (5), 881, **2005**.
7. SHEVTSOV I.Y., MARKINE V.L., ESVELD C. Optimal design of wheel profile for railway vehicles. Wear, **258**, (7), 1022, **2005**.
8. MA M., LIU W., JIN H., SUN X. Prediction Method of the Vibration Influence on Buildings Induced by Rail Transit. China Railway Science, **2**, 005, **2011**.
9. YUAN J., ZHU Y., WU M. Vibration Characteristics and Effectiveness of Floating Slab Track System. Journal of Computers, **4**, (12), 1249, **2009**.
10. JIA Y X., LIU W N., LIU W F., et al. Study of Vibration Effects on Historic Buildings Due to Moving Trains in Beijing [C]//9th International Symposium on Environmental Geotechnology and Global Sustainable Development, Hongkong. pp. 492-499, **2008**.
11. LI K., LIU W., LIU W., JIA Y. Tests and analysis of traffic-induced vibration effects on surrounding historic buildings. Journal of Beijing Jiaotong University, **1**, 018, **2011**.
12. LINYA L., RUI L., LINAN W. Assessment of Vibration Made by Central and Southern Shanxi Railway to the Red Flag Canal. **2011**.
13. NIE H., YAN W., GAO X., WANG W., FU Y. In situ experiment and analysis of adjacent buildings vibration induced by urban subway transit. Journal of Railway Science and Engineering, **1**, 012, **2008**.