APPENDIX

Appendix 1 : Proof proposition 1-3.

Proof proposition 1: The second derivative of Eq. 2 can be obtained by backward induction, $\partial \pi_r^2 / \partial p_n^2 = \partial \pi_r^2 / \partial p_r^2 = -2 < 0$. That is, the profit function π_r of the NEV power battery manufacturer is a strictly concave function with respect to p_n and p_r, and Eq. 2 has maximum values p_n and p_r. Substituting p_n and p_r into Eq. 1 respectively, the Hessian matrix of π_m^{MM} can be solved as follows:

$$H = \begin{bmatrix} \dfrac{\partial^2 \pi_m^{MM}}{\partial^2 \omega_n} & \dfrac{\partial^2 \pi_m^{MM}}{\partial \omega_n \partial \omega_r} & \dfrac{\partial^2 \pi_m^{MM}}{\partial \omega_n \partial \tau} \\ \dfrac{\partial^2 \pi_m^{MM}}{\partial \omega_r \partial \omega_n} & \dfrac{\partial^2 \pi_m^{MM}}{\partial^2 \omega_r} & \dfrac{\partial^2 \pi_m^{MM}}{\partial \omega_r \partial \tau} \\ \dfrac{\partial^2 \pi_m^{MM}}{\partial \tau \partial \omega_n} & \dfrac{\partial^2 \pi_m^{MM}}{\partial \tau \partial \omega_r} & \dfrac{\partial^2 \pi_m^{MM}}{\partial^2 \tau} \end{bmatrix} = \begin{bmatrix} -1 & k & 0 \\ k & -1 & 0 \\ 0 & 0 & -2c_l \end{bmatrix}$$

For Eq. 1 to have a maximum, that is, π_m^{MM} is a strictly concave function with respect to τ, ω_n and ω_r, the Hessian matrix of π_m^{MM} must be negative definite. It refers to the research of Hong et al. (2012), we use d_i to denote the i ($i = 1,2,3$) order principal minor of this matrix, then, $d_1 = -1 < 0$, $d_2 = 1 - k^2 > 0$, $d_3 = 2c_l(k^2 - 1) < 0$. Let $\partial \pi_m^{MM} / \partial \omega_n = 0$, $\partial \pi_m^{MM} / \partial \omega_r = 0$ and $\partial \pi_m^{MM} / \partial \tau = 0$ be equal to 0, solve the system of equations simultaneously, and obtain τ^{MM*}, ω_n^{MM*} and, ω_r^{MM*}, and then solve for p_n^{MM*} and p_r^{MM*}. Substituting all the above equilibrium results into equations Eq. 1 and Eq. 2 respectively, π_r^{MM*} and π_m^{MM*} can be obtained. Thus, Proposition 1 is proved.

Proof proposition 2: from Eq. 6, the second derivative $\partial \pi_r^2 / \partial_\tau^2 = -2c_l < 0$, and $\partial \pi_r^2 / \partial p_n^2 = \partial \pi_r^2 / \partial p_r^2 = -2 < 0$. It follows that the profit function π_r of the NEV power battery retailer is strictly concave with respect to p_n, p_r and τ, and Eq. 6 has maximum values p_n, p_r and τ. Substituting p_n, p_r and τ into Eq. 5 respectively, the Hessian matrix of π_m^{MR} can be solved as follows:

$$H=\begin{bmatrix} \dfrac{\partial^2 \pi_m^{MR}}{\partial^2 \omega_n} & \dfrac{\partial^2 \pi_m^{MR}}{\partial \omega_n \partial \omega_r} \\ \dfrac{\partial^2 \pi_m^{MR}}{\partial \omega_r \partial \omega_n} & \dfrac{\partial^2 \pi_m^{MR}}{\partial^2 \omega_r} \end{bmatrix} = \begin{bmatrix} -1 & k \\ k & -1 \end{bmatrix}$$

Due to the first-order ordinal principal subformula $|H_1| = -1 < 0$ and the second-order ordinal principal subformula $|H_2| = 1 - k^2 > 0$, it can be found that the Hessian matrix of π_m^{MR} is negative definite, in which case Eq. 5 has maximum values ω_n and ω_r. Let's set $\partial \pi_m^{MR} / \partial \omega_n = 0$, $\partial \pi_m^{MR} / \partial \omega_r = 0$, the system of simultaneous equations, we can solve for ω_n^{MR*} and ω_r^{MR*}, and then we can solve for p_n^{MR*}, p_r^{MR*} and τ^{MR*}. We can find π_r^{MR*} and π_m^{MR*} by substituting the above equilibrium results into Eq. 5 and Eq. 6, respectively. Proposition 2 is proved.

Proof proposition 3: The second derivative $\partial \pi_T^{\ 2} / \partial_\tau^2 = -2c_l < 0$, can be obtained from Eq. 11, which shows that the profit function π_t of the third-party recycler is a strictly concave function with respect to the recycling rate τ of used power batteries, and Eq. 11 has a maximum value τ. We set $\partial \pi_T^{MT} / \partial \tau = 0$ to solve the optimal solution τ^* of the recovery rate of waste power batteries. Substitute τ^* into Eq. 9. The manufacturer's profit function becomes:

$$\pi_m = (\omega_n - c_n)q_n + (\omega_r - c_r)q_r + \frac{(\Delta - b)(b - p_c)(S_0 + p_c v)^2}{2c_l}$$

This calculation procedure is the same as in MM model, from which the equilibrium solutions of ω_n^{MT*}, ω_r^{MT*}, p_n^{MT*} and p_r^{MT*} can be derived. By substituting the above results into Eq. 9, Eq. 10 and Eq. 11, π_t^{MT*}, π_r^{MT*} and π_m^{MT*} can be obtained. Proposition 3 is proved.

Appendix 2 : Proof corollary 1-9.

Proof corollary 1:

$$\omega_n^{MM*} - \omega_n^{MR*} = \frac{-(c_n - c_n k^2 + g\varphi k + g - g\varphi)}{2(k^2-1)} - \frac{-(c_r + gk + g\varphi - c_r k^2 - gk\varphi)}{2(k^2-1)} = 0;$$

Similarly, $\omega_n^{MM*} = \omega_n^{MR*} = \omega_n^{MT*}$, $\omega_r^{MM*} = \omega_r^{MR*} = \omega_r^{MT*}$, $p_n^{MM*} = p_n^{MR*} = p_n^{MT*}$, $p_r^{MM*} = p_r^{MR*} = p_r^{MT*}$. Corollary 1 is proved.

Proof corollary 2, For the sake of comparison, in the above three modes, we mainly analyze the case where the price of recycling waste power batteries from consumers is consistent, that is, $p_{c1} = p_{c2} = p_{c3}$. So, it can be calculated that:

$$\tau^{MM*} - \tau^{MR*} = \frac{(\Delta - p_{1c})(vp_{c1} + S_0) - (b - p_{2c})(vp_{2c} + S_0)}{2c_l}$$

$$\tau^{MR*} - \tau^{MT*} = -\frac{(p_{2c} - p_{3c})(S_0 - bv + vp_{2c} + vp_{3c})}{2c_l}$$

Due $p_{c1} = p_{c2} = p_{c3}$, $\Delta > b > p_c$, we can get $\tau^{MM*} - \tau^{MR*} > 0$, and it can be inferred that $\tau^{MM*} > \tau^{MR*}$, $\tau^{MR*} - \tau^{MT*} = 0$ and $\tau^{MR*} = \tau^{MT*}$. Corollary 2 is proved.

Proof corollary 3: $\pi_m^{MM*} - \pi_m^{MR*} = \dfrac{(A-b)^2 + (b-p_c)^2 (S_0 + p_c v)^2}{4c_l}$

$\pi_m^{MR*} - \pi_m^{MT*} = 0$.

From this we can conclude that $\pi_m^{MM*} > \pi_m^{MR*} = \pi_m^{MT*}$, and Corollary 3 is proved.

Proof corollary 4: $\pi_r^{MM*} - \pi_r^{MR*} = \dfrac{(b-p_c)^2 (S_0 + p_c v)^2}{4c_l}$

$$\pi_r^{MR*} - \pi_r^{MT*} = -\frac{(b-p_c)^2 (S_0 + p_c v)^2}{4c_l}$$

$$\pi_r^{MM*} - \pi_r^{MT*} = 0$$

Thus, $\pi_r^{MM*} > \pi_r^{MR*}, \pi_r^{MR*} < \pi_r^{MT*}$, and then $\pi_r^{MM*} = \pi_r^{MT*} > \pi_r^{MR*}$, and Corollary 4 is proved.

Proof corollary 5: $\pi^{MM*} - \pi^{MR*} = \dfrac{(A-b)^2 (S_0 + p_c v)^2}{4c_l} > 0$

Similarly, it follows that, $\pi^{MM*} > \pi^{MT*} > \pi^{MR*}$, Corollary 5 is proved.

Proof corollary 6, here, the first-order partial derivatives of ω_n, ω_r, p_n and p_r with respect to φ and k can be obtained:

$$(1)\begin{cases}\dfrac{\partial\omega_n}{\partial\varphi}=\dfrac{-g}{2(k+1)}<0\\ \dfrac{\partial p_n}{\partial\varphi}=\dfrac{-3g}{4(k+1)}<0\\ \dfrac{\partial\omega_r}{\partial\varphi}=\dfrac{g}{2(k+1)}>0\\ \dfrac{\partial p_r}{\partial\varphi}=\dfrac{3g}{4(k+1)}>0\end{cases};\ (2)\begin{cases}\dfrac{\partial\omega_n}{\partial k}=\dfrac{g[2k(1-\theta)+\theta+k^2\theta]}{2(k^2-1)^2}>0\\ \dfrac{\partial p_n}{\partial k}=\dfrac{3g[2k(1-\theta)+\theta+k^2\theta]}{4(k^2-1)^2}>0\\ \dfrac{\partial\omega_r}{\partial k}=\dfrac{g[2k\theta+(1-\theta)(k^2+1)]}{2(k^2-1)^2}>0\\ \dfrac{\partial p_r}{\partial k}=\dfrac{3g[2k\theta+(1-\theta)(k^2+1)]}{4(k^2-1)^2}>0\end{cases}$$

Corollary 6 is proved.

Proof corollary 7, taking the first partial derivatives of π_r^{MM*}, π_r^{MR*} and π_r^{MT*} with respect to π_r^{MT*} and π_r^{MT*}. We can obtain,

$$\frac{\partial\pi_r^{MM*}}{\partial\varphi}=\frac{\partial\pi_r^{MT*}}{\partial\varphi}=\frac{g[(1+\ k)(c_n-c_r)+g(2\varphi-1)]}{8(k+1)}$$

$$\frac{\partial\pi_r^{MR*}}{\partial\varphi}=\frac{g[(1+k)+g(2\varphi-1)]}{8(k+1)}$$

$$\frac{\partial\pi_r^{MM*}}{\partial k}=\frac{\partial\pi_r^{MT*}}{\partial k}=\frac{1}{8}(-c_nc_r-\frac{g^2\left(\left(-1+\varphi\right)\varphi(1+k^2)-k\left(1+2\varphi(\varphi-1)\right)\right)}{\left(-1+k^2\right)^2}$$

$$\frac{\partial\pi_r^{MR*}}{\partial k}=\frac{1}{8}\left(-c_nc_r-\frac{g^2\left(\left(-1+\varphi\right)\varphi+k^2\left(-1+\varphi\right)\varphi+k\left(-1+2\varphi-2\varphi^2\right)\right)}{\left(-1+k^2\right)^2}\right)$$

$$\frac{\partial\pi_r^{MM*}}{\partial k}=\frac{\partial\pi_r^{MT*}}{\partial k}=\frac{1}{8}(-c_nc_r-\frac{g^2\left(\left(-1+\varphi\right)\varphi(1+k^2)-k\left(1+2\varphi(\varphi-1)\right)\right)}{\left(-1+k^2\right)^2}$$

Using the same method, we can find the first partial derivatives of π_m^{MM*}, π_m^{MR*} and π_m^{MT*} with respect to φ and k, respectively.

$$\frac{\partial\pi_m^{MM*}}{\partial\varphi}=\frac{g\left(\left(c_n-c_r\right)\left(1+k\right)+g\left(-1+2\varphi\right)\right)}{4\left(1+k\right)}$$

$$\frac{\partial\pi_m^{MM*}}{\partial k}=\frac{1}{4}\left(-c_nc_r-\frac{g^2\left(\left(-1+\varphi\right)\varphi+k^2\left(-1+\varphi\right)\varphi-k\left((1-\varphi)^2+\varphi^2\right)\right)}{\left(-1+k^2\right)^2}\right)$$

$$\frac{\partial\pi_m^{MR*}}{\partial\varphi}=\frac{\partial\pi_m^{MT*}}{\partial\varphi}=\frac{g\left(c_n-c_r-g+c_nk-c_rk+2g\varphi\right)}{4\left(1+k\right)}$$

$$\frac{\partial \pi_m^{MR*}}{\partial k} = \frac{\partial \pi_m^{MT*}}{\partial k} = \frac{1}{8}\left(-c_n c_r - \frac{g^2\left(\left(-1+\varphi\right)\varphi + k^2\left(-1+\varphi\right)\varphi + k\left(-1+2\varphi-2\varphi^2\right)\right)}{\left(-1+k^2\right)^2} \right)$$

We can figure out by extrapolation that when $k > \frac{g(1-2\varphi)-\Delta}{\Delta}$, we get $\frac{\partial \pi_m^{MM*}}{\partial \varphi} > 0$; when $k < \frac{g(1-2\varphi)-\Delta}{\Delta}$, we can get $\frac{\partial \pi_r^{MR*}}{\partial \varphi} > 0$, $\frac{\partial \pi_r^{MT*}}{\partial \varphi} > 0$. Because of $c_n - c_r > 1$. Combining $\frac{g(1-2\varphi)-\Delta}{\Delta}$ and $\frac{g(1-2\varphi)-\Delta}{\Delta}$ leads to the result (1) in Corollary 7. Similarly, we can calculate when $k < \frac{g - 2g\varphi + 2g\varphi^2 + \sqrt{g^2\left(1-2\varphi\right)^2 - 4c_n c_r\left(-1+\varphi\right)\varphi}}{2g\left(-1+\varphi\right)\varphi}$, it can be concluded $\frac{\partial \pi_m^{MM*}}{\partial k} > 0$, $\frac{\partial \pi_m^{MR*}}{\partial k} > 0$, $\frac{\partial \pi_m^{MT*}}{\partial k} > 0$; $\frac{\partial \pi_r^{MM*}}{\partial k} > 0$, $\frac{\partial \pi_r^{MR*}}{\partial k} > 0$, $\frac{\partial \pi_r^{MT*}}{\partial k} > 0$; On the contrary, $\frac{\partial \pi_m^{MM*}}{\partial k} < 0$, $\frac{\partial \pi_m^{MR*}}{\partial k} < 0$, $\frac{\partial \pi_m^{MT*}}{\partial k} < 0$; $\frac{\partial \pi_r^{MM*}}{\partial k} < 0$, $\frac{\partial \pi_r^{MR*}}{\partial k} < 0$, $\frac{\partial \pi_r^{MT*}}{\partial k} < 0$. The proof of (2) in Corollary 7 is completed.

Proof corollary8: $\tau^{MRT*} - \tau^{MM*} = 1 - \frac{\left(\Delta + b - 2p_c\right)\left(S_0 + p_c v\right)}{2c_l} > 0$.

That is, $\tau^{MRT*} > \tau^{MM*}$, Corollary 8 is proved.

Proof Corollary 9:

$\pi_m^{MRT*} - \pi_m^{MM*} = \frac{\left(S_0 + p_c v\right)\left(2\Delta\left(2c_l + p_c\left(S_0 + p_c v\right)\right) - 4bc_l - \left(\Delta^2 + p_c^{\ 2}\right)\left(S_0 + p_c v\right)\right)}{4c_l} > 0$, it can be concluded $\pi_m^{MRT*} > \pi_m^{MM*}$. Combined with Corollary 3, we know that $\pi_m^{MRT*} > \pi_m^{MM*} > \pi_m^{MR*} = \pi_m^{MT*}$. $\pi_r^{MRT*} - \pi_r^{MR*} = -\frac{\left(c_l - \left(b - p_c\right)\left(S0 + p_c v\right)\right)^2}{c_l} < 0$, it can be concluded $\pi_r^{MRT*} < \pi_r^{MR*}$. Combined with Corollary 4, $\pi_r^{MM*} = \pi_r^{MT*} > \pi_r^{MR*} > \pi_r^{MRT*}$. $\pi_\tau^{MRT*} - \pi_\tau^{MT*} = 0$, it can be concluded $\pi_\tau^{MRT*} = \pi_\tau^{MT*}$. $\pi^{MR*} - \pi^{MRT*} > 0$, it can be concluded $\pi^{MR*} > \pi^{MRT*}$, combined with the conclusion of Corollary 5 $\pi^{MM*} > \pi^{MT*} > \pi^{MR*} > \pi^{MRT*}$. The proof of Corollary 9 is completed.