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Original Research

Cloud Model Membership Degree of Rock Slope Stability Evaluation: Method and a Case Study

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Abstract

The ranking grade of rock engineering stability is generally determined by the maximum membership degree principle, and the value of membership degree is an exact numerical value. However, the membership degree of uncertainty with randomness and fuzziness has not been considered. The membership degree with uncertainty can describe the uncertainty effects of the evaluating factors on rock engineering stability and fully shows the degrees of one evaluating factor belonging to the ranking grades. In this study, a quantitative index representing the membership degree with uncertainty is proposed and obtained based on the membership degree of the cloud model. The membership degree of the cloud model is obtained based on normal cloud theory, the system and the ranking grade of evaluation factors and analytical hierarchy process (AHP) based on scale of the cloud model. An evaluation method for the membership degree of cloud model is conducted and applied in a rock slope project. The feasible and practical evaluation results is verified by comparing with that of AHP method, the extension theory and the numerical simulation. This proposed method can serve as a decision tool for stability evaluation in other similar projects.

Keywords: rock slope, stability evaluation, normal cloud theory, membership degree of cloud model, analytical hierarchy process (AHP) based on scale of the cloud model

Introduction

The stability evaluation of slope is an important basis for slope design and construction. Many researches on the analysis of slope stability generally used a deterministic approach [1-7]. Since the geological and environmental conditions are very complex, most project problems are filled with uncertainty. Generally, randomness and fuzziness are regarded as two important uncertainties. With respect to the application of fuzzy method, slope stability evaluation has been studied using fuzzy method [8-11], and it was also studied using the matter-element method [12, 13]. However, both randomness and fuzziness are not considered in the above studies.

In order to consider both randomness and fuzziness, the cloud model is regarded as one useful method to synthetically describe the uncertainty. The cloud model is a model of mutual conversion between uncertain description and quantitative values. On the basis of traditional probability theory and fuzzy set theory, randomness and fuzziness are combined through numerical characteristics [14]. With respect to the application of cloud model, Liu et al. presented

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the cloud model-based approach for comprehensive stability evaluation of complicated rock slopes of hydroelectric stations in mountainous area [15]. Wang et al. proposed a multi-index evaluation model for the rockfall risk assessment based on normal cloud model [16]. Gu et al. presented the rockfall risk assessment based on analytical hierarchy process (AHP)-normal cloud model [17]. Chen et al. evaluated the risk degree of landslide hazard based on the normal cloud model [18]. Wang et al. proposed a direct and quick method for analyzing slope stability by combining a multi-dimensional cloud model and set pair analysis of connection numbers theory [19]. The cloud model has also been applied in other projects safety evaluation. Zhang et al. established the risk assessment of existing pipelines in tunneling environments through the cloud model method [20]. In order to solve the uncertainty in the evaluation process of water inrush risk, Wang et al. established a novel comprehensive evaluation model based on the normal cloud theory [21]. Wang et al. presented a new evaluation method with dynamic feedback for the water inrush risk based on the normal cloud model [22]. Liu et al. established a classified prediction model of rockburst based on rough sets-normal cloud [23].

The above studies are the evaluation of the stability and safety of different projects by the cloud model. Generally, there are five steps for cloud model implementation in the above studies: (1) collecting data related to project safety; (2) ranking related factors; (3) obtaining the weight coefficient of the factors; (4) calculating the membership degree; (5) obtaining the ranking grade of engineering stability and safety according to the maximum membership degree principle. Obviously, the membership degree is related to the determining of ranking grade of engineering stability and safety, and the membership degrees in the above studies are deterministic. However, the membership degree with uncertainty has not been considered. The membership degree with uncertainty describes the uncertainty effects of the evaluating factors on engineering stability and safety, and fully shows the degrees of one evaluating factor belonging to the ranking grades. Therefore, the ranking grade of engineering stability and safety evaluated by establishing the membership degree with uncertainty will be more in line with the actual project.

In this study, a quantitative index representing the membership degree with uncertainty was proposed and obtained based on the membership degree of the cloud model. The membership degree of the cloud model was obtained based on normal cloud theory, the system and the ranking grade of evaluation factors and AHP based on scale of cloud model. Thus, the membership degree method of cloud model was established. Based on a rock slope of Road Construction Project in China, the membership degree method of cloud model was represented to evaluate the slope stability. The most significant factors to the slope stability and the ranking grade of slope stability were obtained. Finally, the evaluation results were compared with that of AHP method, the extension theory and the numerical simulation based on discrete element method.

Materials and Methods and Site Description

Membership Degree Method of Cloud Model for Engineering Stability Evaluation

A membership degree method of cloud model can be established based on normal cloud theory, the system and the ranking grade of evaluation factors and AHP based on scale of cloud model. The membership degree of cloud model can describe the uncertainty effects of the evaluating factors on engineering stability and fully show the degrees of one evaluating factor belonging to the ranking grades. A flowchart of the membership degree method of cloud model for engineering stability evaluation is shown in Fig. 1. The steps are as follows:

– Step 1: Obtaining membership degree of cloud model

Firstly, the environmental conditions of the project, geological conditions of the project and the project conditions are collected, then the system and ranking grade of evaluation factors can be determined, the evaluation factors values can be obtained according to the project site investigation and analysis. Secondly, the three numerical characteristics are calculated and the cloud droplets are generated by the normal cloud generator with MATLAB software. Thirdly, the membership degree of cloud model for each ranking grade is obtained based on the evaluation factors values and normal cloud theory.

– Step 2: Obtaining cloud model weight

The scales of AHP method all use identified values to represent the relative importance of two factors. The cloud model can be used to handle uncertainties in the AHP. The cloud model weight of each factor can be obtained based on the relative importance of two factors and normal cloud theory.

– Step 3: Obtaining integrated membership degree of cloud model and the ranking grade of engineering stability

The integrated membership degree of cloud model can be obtained based on the cloud model weight and the membership degree of cloud model, then the quantitative index representing the membership degree with uncertainty is calculated based on the three numerical characteristics of the integrated membership degree of cloud model. Finally, the ranking grade of engineering stability is determined by selecting the maximum value of the quantitative index v.

Detailed methods and processes for the membership degree method of cloud model are described in the following sections.

Normal Cloud Theory

Cloud Droplets and Cloud Model

Let U be a quantitative domain represented by an exact numerical value, and C be a qualitative concept on U. If the quantitative value $x \in U$ and x is a random occurrence of qualitative concept C, the membership degree $\mu(x) \in [0, 1]$ of x to C is a random number with stable tendency, then the distribution of x in the domain U is called a cloud, and each $(x, \mu(x))$ is called a cloud drop [24]. The parameter $\mu(x)$ can be estimated as:

$$\mu : U \rightarrow [0,1] \quad \forall x \in U \rightarrow \mu(x) \tag{1}$$

x obeys normal distribution based on the expected value Ex and the standard deviation En', and the normal random number En' obeys normal distribution based on the expected value En and the standard deviation He. If the membership degree $\mu(x)$ of x to C can be expressed as:

$$\mu(x) = e^{-\frac{(x-Ex)^2}{2(En')^2}} \tag{2}$$

Then, the distribution in the domain U is called as a normal cloud model. Normal cloud models represent a qualitative concept C through the three numerical characteristics of expected value, entropy and hyper entropy. Expected value Ex in a normal cloud model is the most representative point of the qualitative concept C. Entropy En represents a measure of the conceptually acceptable range of cloud droplets values in the domain U. Hyper entropy He is the entropy of entropy En, which represents the dispersion degree of cloud droplets.

Normal Cloud Generator

The forward cloud generator is a qualitative to quantitative mapping that generates cloud droplets according to the numerical characteristics of normal cloud models (Ex, En, He). The algorithm and calculation steps of forward cloud generator are as follows [16]:

(a) Calculate the expected value Ex and hyper entropy He of the numerical characteristics of the normal cloud model;

(b) Generate a normally random number En' according to expected value En and standard deviation He: $En \sim N(En, He^2)$;

(c) x_i is a quantitative value of the qualitative concept, generate a normally random number x_i with expected value Ex and standard deviation En': $x_i \sim N(EX, En'^2)$;

(d) Calculate the membership degree of quantitative values: $\mu(x_i) = e^{-\frac{(x_i-Ex)^2}{2(En')^2}}$;

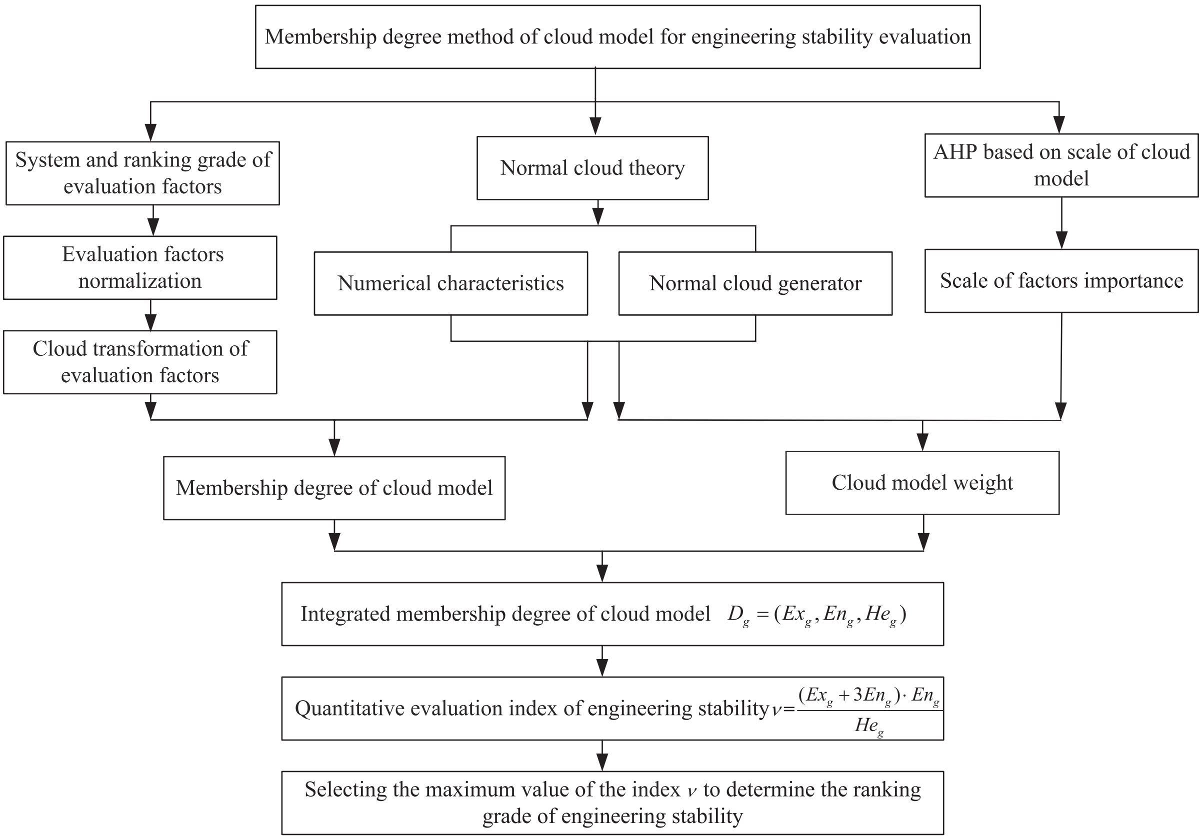


Fig. 1. Flowchart of the membership degree method of cloud model for engineering stability evaluation.

(e) Generate a cloud drop with the membership degree $\mu(x_i)$ and the normally random number x_i;

(d) Repeat steps (a) to (e) until the required cloud drops are generated.

Evaluation Factors Normalization and Cloud Transformation of Evaluation Factors Ranking Grade

The factors which have important influence on one engineering stability can be selected based on the investigation and analysis of the geological conditions, the environmental conditions and project conditions. The evaluation system of factors is established. Due to the units of the evaluation factors affecting the engineering stability are not uniform, the normalization method is adopted to make these factors dimensionless. After normalization, the change trends of the original value of the evaluation factors are not affected. If the greater the value of the evaluation factor, the lower the risk of engineering instability, then the evaluation factors normalization is conducted by Eq. (3).

$$C_{ij}' = \frac{C_{ij} - C_{ij\min}}{C_{ij\max} - C_{ij\min}} \tag{3}$$

Otherwise, if the smaller the value of the evaluation factor, the higher the risk of engineering instability, then the evaluation factors normalization is conducted by Eq. (4).

$$C_{ij}' = \frac{C_{ij\max} - C_{ij}}{C_{ij\max} - C_{ij\min}} \tag{4}$$

where $C_{ij\max}$ and $C_{ij\min}$ are the maximum value and the minimum value of the evaluation factor ij at an arbitrary ranking grade, respectively. C_{ij} is original value of the evaluation factor ij, C_{ij}' is normalization value of the evaluation factor ij.

The normalized values of the evaluation factors are transformed into the data form represented by the three numerical features of the cloud model. The three numerical features are obtained by Eq. (5)-(7). The process is called as cloud transformation of engineering stability ranking grade.

$$Ex_{ij} = \frac{B_{ij\max} + B_{ij\min}}{2} \tag{5}$$

$$En_{ij} = \frac{B_{ij\max} - B_{ij\min}}{6} \tag{6}$$

$$He_{ij} = k \tag{7}$$

where $B_{ij\max}$ and $B_{ij\min}$ are the maximum and the minimum of normalized value of the evaluation factor *ij* at an arbitrary ranking grade, respectively. He_{ij} can be set as an appropriate constant k (k<0.5), and k is generally 0.01 [16, 21]. (Ex_{ij}, En_{ij}, He_{ij}) represents the three numerical features of the cloud model of the evaluation factor *ij* at an arbitrary ranking grade.

Membership Degree of Cloud Model

After the evaluation factors are transformed into the normalized values (Eq. (3)-(4)), and the normalized values of the evaluation factors are transformed into the data form represented by the three numerical features of the cloud model (Eq. (5)-(7)). The membership degree $\mu(x)$ of the evaluation factors can be calculated by the normal cloud generator, i.e., Eq. (2). In general, when the entropy and hyper entropy are not equal to 0, the x generated by the algorithm is random and fuzzy. In the special case, when the entropy and hyper entropy are equal to zero, the x generated by the algorithm becomes an exact numerical value. In that sense, since certainty is a special case of uncertainty, the obtained membership degree $\mu(x)$ can be transformed into the cloud model membership degree based on the normal cloud theory.

Cloud Model Weight

AHP was proposed by American operational research scientist T.L. Saaty in the 1970s, it had been widely used in weight confirmation. The AHP method is used for the weight confirmation of each factor affecting engineering stability in this paper. Due to the factors have different influences on engineering stability and the scales of AHP method all use identified values to represent the relative importance of two factors [25]. In fact, the magnitude of importance is a random number with stable tendency, so the identified values based on the 1-9 scale in AHP method can not objectively represent the importance of pair-wise comparison. To overcome the shortcoming, a weight confirmation method based on cloud model is presented.

(a) Experts are invited to make the judgments of relative importance according to the main factors that affect the engineering stability. The judgments of relative importance of factors are shown as natural numbers, the classical Saaty scale uses a natural number between 1 and 9 [25], as shown in Table 1.

(b) The natural numbers for relative importance of factors are converted into the three numerical features (Ex_{ij}^0, En_{ij}^0, He_{ij}^0) of the cloud model by Eq. (8)-(10).

$$Ex_{ij}^0 = \frac{1}{N}\sum_{l=1}^{N} X_{ij,e} \tag{8}$$

$$En_{ij}^0 = \sqrt{\frac{\pi}{2}} \times \frac{1}{N}\sum_{l=1}^{N} \left| X_{ij,e} - Ex_{ij} \right| \tag{9}$$

Table 1. Scale for pair-wise comparison [25].

Intensity of relative importance	Definition
1	Equal importance
2	Equal to moderate importance
3	Moderate importance
4	Moderate to strong importance
5	Strong importance
6	Strong to very strong importance
7	Very strong importance
8	Very to extremely strong importance
9	Extreme importance
Reciprocals of above nonzero numbers	If factor C_{ij} is assigned one of the above numbers when compared to factor C_{mn}, then factor C_{mn} is assigned the number's reciprocal value when compared to factor C_{ij}.

$$He_{ij}^{0}=\sqrt{\left|\frac{1}{N-1}\sum_{e=1}^{N}(X_{ij,e}-Ex_{ij})^{2}-\left(En_{ij}\right)^{2}\right|} \tag{10}$$

Where *ij* denotes an arbitrary factor C_{ij}. *e* denotes an arbitrary expert, *N* denotes the number of the experts, and $X_{ij,e}$ denotes the judgment number of expert *e* on factor C_{ij}.

(c) Pair-wise comparison matrixes are constructed according to the AHP method. The root method is used for the weight computation of factors in an arbitrary comparison matrix [26], and its equations are as follows:

$$Ex_{ij}^{a}=\frac{(\prod_{l=1}^{m}Ex_{ij}^{0})^{\frac{1}{m}}}{\sum_{r=1}^{m}(\prod_{l=1}^{m}Ex_{ij}^{0})^{\frac{1}{m}}} \tag{11}$$

$$En_{ij}^{a}=\frac{(\prod_{l=1}^{m}Ex_{ij}^{0}\sqrt{\sum_{l=1}^{m}(\frac{En_{ij}^{0}}{Ex_{ij}^{0}})^{2}}\;)^{\frac{1}{m}}}{\sum_{r=1}^{m}(\prod_{l=1}^{m}Ex_{ij}^{0}\sqrt{\sum_{l=1}^{m}(\frac{En_{ij}^{0}}{Ex_{ij}^{0}})^{2}}\;)^{\frac{1}{m}}} \tag{12}$$

$$He_{ij}^{a}=\frac{(\prod_{l=1}^{m}Ex_{ij}^{0}\sqrt{\sum_{l=1}^{m}(\frac{He_{ij}^{0}}{Ex_{ij}^{0}})^{2}}\;)^{\frac{1}{m}}}{\sum_{r=1}^{m}(\prod_{l=1}^{m}Ex_{ij}^{0}\sqrt{\sum_{l=1}^{m}(\frac{He_{ij}^{0}}{Ex_{ij}^{0}})^{2}}\;)^{\frac{1}{m}}} \tag{13}$$

Where *m* denotes the number of factors in an arbitrary comparison matrix, *r* and *c* denote the number of rows and the number of columns in an arbitrary comparison matrix, respectively.

(d) In order to obtain the cloud model comprehensive weights of factors, the weights of factors obtained in an arbitrary comparison matrix are substituting into the computing equations for the cloud model, i.e., Eq. (14)-(19). The cloud model comprehensive weights of factors, (ΣEx_{ij}^{b}, ΣEn_{ij}^{b}, ΣHe_{ij}^{b}), are obtained.

$$Ex_{ij}^{b}=Ex_{i0}^{a}\times Ex_{ij}^{a} \tag{14}$$

$$En_{ij}^{b}=\left|Ex_{i0}^{a}\times Ex_{ij}^{a}\right|\times\sqrt{\left(\frac{En_{i0}^{a}}{Ex_{i0}^{a}}\right)^{2}+\left(\frac{En_{ij}^{a}}{Ex_{ij}^{a}}\right)^{2}} \tag{15}$$

$$He_{ij}^{b}=\left|Ex_{i0}^{a}\times Ex_{ij}^{a}\right|\times\sqrt{\left(\frac{He_{i0}^{a}}{Ex_{i0}^{a}}\right)^{2}+\left(\frac{He_{ij}^{a}}{Ex_{ij}^{a}}\right)^{2}} \tag{16}$$

$$\Sigma\,Ex_{ij}^{b}=\sum_{j=1}^{m}Ex_{ij}^{b} \tag{17}$$

$$\Sigma\,En_{ij}^{b}=\sqrt{\sum_{j=1}^{m}\left(En_{ij}^{b}\right)^{2}} \tag{18}$$

$$\Sigma\,He_{ij}^{b}=\sqrt{\sum_{j=1}^{m}\left(He_{ij}^{b}\right)^{2}} \tag{19}$$

Integrated membership degree of cloud model and quantitative evaluation index of engineering stability.

Combining with the cloud model comprehensive weights of factors, the integrated membership degree D_g of cloud model can be obtained by Eq. (20).

$$D_{g}=(Ex_{g},En_{g},He_{g})=\sum_{i,j=1}^{n}[(\Sigma\,Ex_{ij}^{b},\Sigma\,En_{ij}^{b},\Sigma\,He_{ij}^{b})\times(Ex_{ij},En_{ij},He_{ij})] \tag{20}$$

Where *g* denotes the ranking grade of engineering stability.

According to the numerical characteristics of normal cloud models (Ex_g, ΣEn_g, ΣHe_g), expected Ex_g is the expected of cloud drops distribution in the domain *U*, that is, the most representative points of the qualitative concept *g* on domain *U*. It's worth noting that the qualitative concept *g* denotes the ranking grade of engineering stability. Entropy En_g is the measurement of the qualitative concept *g*, and it reflects the range of values in the domain *U* that can be accepted by the qualitative concept *g*. According to the '3*En* rule' of the normal cloud model, the generated cloud drops are mainly focused on [Ex_g-3En_g, Ex_g+3En_g]. Contributions of cloud drops for the qualitative concept g are the most important within this interval [26]. Therefore, Ex_g and $3En_g$ reflect the most probable points of the qualitative concept g on domain U.

According to the analysis above, the index v_1 represents the certain measurement of range size that can be accepted by the qualitative concept g is expressed as follows:

$$v_1 = Ex_g + 3En_g \tag{21}$$

Hyper entropy He_g is the uncertain measurement of entropy En_g, and it represents the fuzziness and randomness measurement of entropy En_g, that is, it represents dispersion degree. Therefore, the index v_2 represents the uncertain measurement of the qualitative concept g is expressed as follows:

$$v_2 = \frac{He_g}{En_g} \tag{22}$$

The quantitative evaluation index of engineering stability v is expressed as follows:

$$v = \frac{v_1}{v_2} = \frac{(Ex_g + 3En_g) \cdot En_g}{He_g} \tag{23}$$

The greater the certain index v_1 is, the greater the range size that can be accepted by the qualitative concept g is. Similar to the maximum membership principle, the ranking grade of engineering stability g is determined by selecting the greater value v_1. The smaller the uncertain index v_2 is, the less the uncertainty measurement of the qualitative concept g is. That is, the less uncertainty there is in the evaluation result of engineering stability. Consider both certainty and uncertainty, if the value of uncertain index v_2 is small and the greater the certain index v_1 is, the quantitative evaluation index of engineering stability v increases. The ranking grades of engineering stability are determined by selecting the maximum value of the quantitative evaluation index v. The quantitative evaluation of engineering stability considering uncertainty is conducted. Factors with the three numerical features of the cloud model can also be compared by using the above method, such as factors weights, and the influence degree of each evaluation factor on the slope stability is referred to as v_w.

Site Description

Engineering Background

In this paper, the evaluation method for the membership degree of cloud model was conducted and applied in a rock slope project. The rock slope (Fig. 2) at K276+210-K276+330 of Road Construction Project in Suiman highway, P. R. China is chosen as a study area. Suiman highway is a national arterial highway in China, and it starts at Suifenhe in Heilongjiang Province and ends at Manchuria in Inner Mongolia Province. The total length of Suiman highway is 1680 km, the designed speed is 80 km/h.

The Geological and Environmental Conditions

According to the geotechnical investigation of the slope K276+210-K276+330 area, the formation lithology belongs to hard rock mass, whose rock density is 2.18 g/cm^3 and rock uniaxial compressive strength is 51 MPa. The structure characteristic of rock mass is not very good, the rock integrity coefficient is 0.38, the rock deformation modulus is 6.6 GPa and RQD is 59.

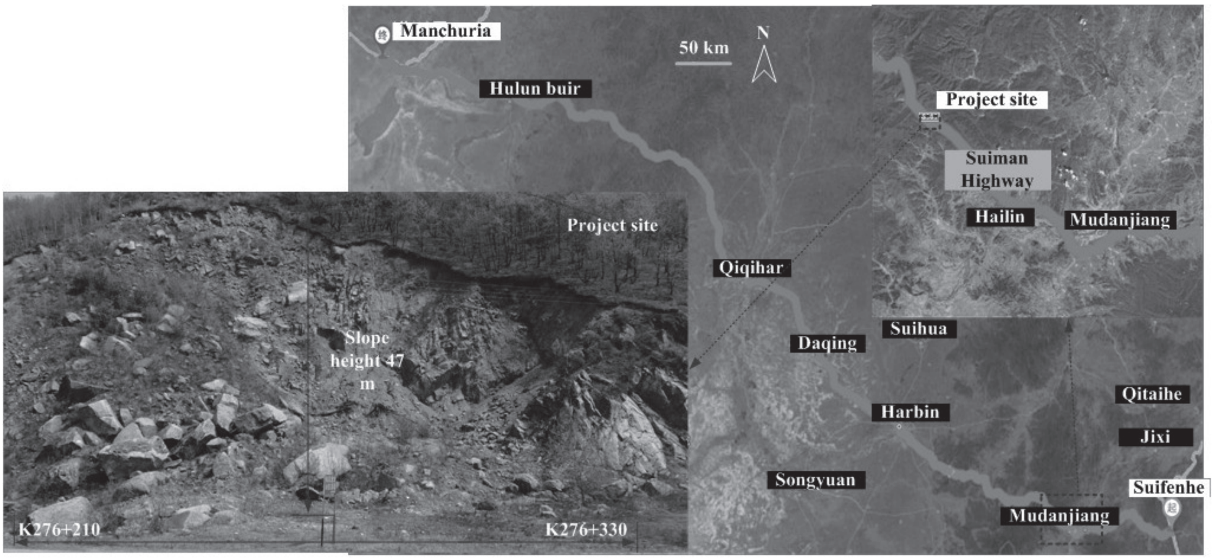


Fig. 2. The rock slope of K276+210-K276+330.

Table 2. The evaluation factors values of the rock slope.

Evaluation factors	Deformation Modulus (GPa)	Integrity index of rock mass	UCS (MPa)	RQD (%)	Friction angle (°)	Cohesion (MPa)	Slope height (m)	Slope angle (°)	Cumulative rainfall monthly (mm)	Seismic acceleration (g)
Value	6.6	0.38	51	59	18	0.035	47	23	90	0.07
Value with normalization	0.13	0.38	0.204	0.59	0.40	0.11	0.53	0.71	0.7	0.83

Table 3. Statistical results of geometric parameters of joints.

Joint set	Mean dip direction /(°)	Mean dip angle/(°)	Trace length/(m)		Number/ (count/m)
			Mean/(m)	Std./(m)	
1	66.66	70.53	21.15	7.82	25
2	150.19	68.24	19.89	6.52	16
3	240.37	61.75	24.23	9.55	19
4	333.50	72.37	18.26	6.14	30

The structural plane of rock mass is poor, the internal friction angle is 18°, and the cohesion is 0.035 MPa. These are the material properties of the rock mass of the slope area. The excavation height of the slope is 47 m and the slope gradient is 23°. The cumulative rainfall monthly is 90 mm. The slope K276+210-K276+330 area in Heilongjiang province has the basic seismic acceleration of 0.07 g. The factors which have important influence on the rock slope stability were selected based on the analysis of the geological conditions, the environmental conditions and the project conditions. The evaluation factors values and its normalization is shown in Table 2. The structural plane information in the K276+210-K276+330 rock slope is obtained by the collection system in ShapeMetriX3D [27], which is shown in Table 3.

Results and Discussions

Slope Stability Evaluation System and Ranking Grade of Evaluation Factors

There are many conditions that affect rock slope stability according to project site investigation and analysis, consisting of slope environment, slope geology and slope topography. In these project conditions, the main factors that affects the rock slope stability include ten indices. The evaluation system is shown in Fig. 3. Environmental conditions C_{10} are composed of seismic acceleration factor C_{11} and cumulative rainfall monthly factor C_{12}. Geological rock mass conditions C_{20} are composed of rock quality designation factor (RQD) C_{21}, uniaxial compressive strength factor (UCS) C_{22}, deformation modulus factor C_{23}, rock integrity coefficient factor C_{24}. Geological joints conditions C_{30} are composed of friction angle factor C_{31}, cohesion factor C_{32}. Geological topography conditions C_{40} are composed of slope angle factor C_{41} and slope height C_{42} factor. Since the stability of rock slope is commonly ranked into five grades [15], these factors can also be quantitatively classified into five grades, that is, extremely stable I, stable II, basically stable III, unstable IV, very unstable V, as shown in Table 4. Since the units of factors are not uniform, they are subjected dimensionless processing according to Eq. (3)-(4). The ranking grade of slope stability with the dimensionless factors are given in Table 4 with values in brackets. Based on the evaluation system and the evaluation factors values, the evaluation factors normalization is conducted by Eq. (3)-(4).

Integrated Membership Degree of Cloud Model

According to the ranking grade of evaluation factors and AHP based on scale of cloud model, five experts were invited to make the judgments of relative importance. The natural numbers for the judgments of relative importance can be transformed into the cloud model with the three numerical features according to Eq. (8)-(10), therefore, the comparison matrixes of the factors were obtained as following:

$$\begin{array}{ccccc} & C_{10} & C_{20} & C_{30} & C_{40} \\ C_{10} & (1,0,0) & (0.25,0.031,0.031) & (0.333,0.111,0.008) & (0.5,0.125,0.125) \\ C_{20} & (4,0.501,0.498) & (1,0,0) & (3,1.003,0.703) & (4,0.501,0.498) \\ C_{30} & (3,1.002,0.073) & (0.333,0.111,0.078) & (1,0,0) & (2,1.003,0.073) \\ C_{40} & (2,0.501,0.498) & (0.25,0.111,0.031) & (0.5,0.25,0.018) & (1,0,0) \end{array}$$

$$\begin{array}{ccccc} & C_{21} & C_{22} & C_{23} & C_{24} \\ C_{21} & (1,0,0) & (2,0.501,0.498) & (3,0.501,0.498) & (1,0.501,0.352) \\ C_{22} & (0.5,0.25,0.018) & (1,0,0) & (2,1.002,0.703) & (0.333,0.056,0.054) \\ C_{23} & (0.333,0.057,0.055) & (0.5,0.25,0.018) & (1,0,0) & (0.333,0.111,0.073) \\ C_{24} & (1,0.501,0.352) & (3,0.501,0.498) & (3,0.501,0.498) & (1,0,0) \end{array}$$

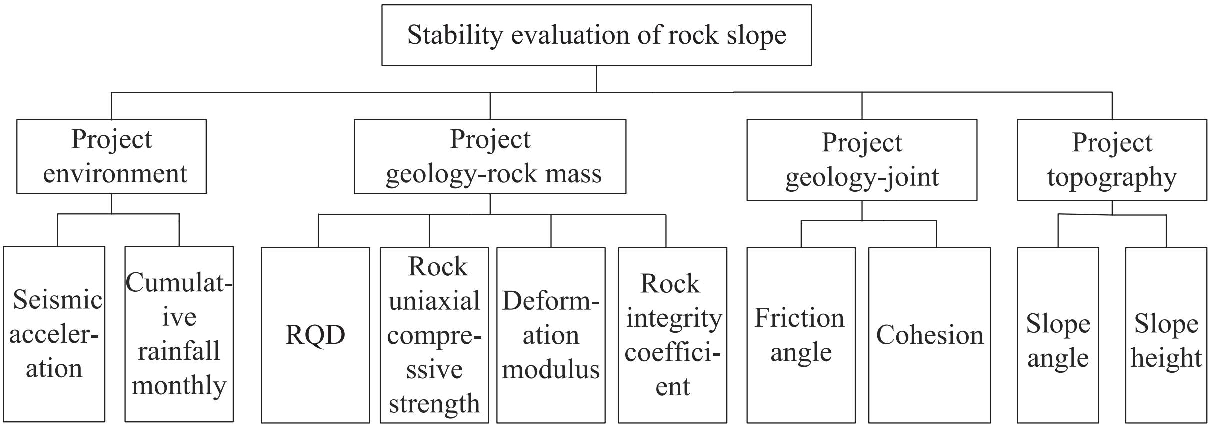


Fig. 3. The multi-level and multi-factor evaluation system.

Table 4. Ranking grade of evaluation factors (with dimensionless factors).

Ranking grade	V Very unstable	IV Unstable	III Basically stable	II Stable	I Extremely stable
Project environment C_{10}					
Seismic acceleration C_{11} (m/s^2)	0.20–0.40 (0.00~0.50)	0.15–0.20 (0.50~0.63)	0.10–0.15 (0.63~0.75)	0.05–0.10 (0.75~0.88)	0.00–0.05 (0.88-1.00)
Cumulative rainfall C_{12} (mm/month)	250~300 (0.00~0.17)	150~250 (0.17~0.50)	100~150 (0.50~0.67)	50~100 (0.67~0.88)	0~50 (0.88~1.00)
Project geology-rock mass C_{20}					
RQD C_{21}	0-25 (0.00-0.25)	25-50 (0.25-0.50)	50-75 (0.50-0.75)	75-90 (0.75-0.90)	90-100 (0.90-1.00)
UCS C_{22} (MPa)	0–25 (0.00-0.10)	25–50 (0.10-0.20)	50–100 (0.20-0.40)	100–150 (0.40-0.60)	150-250 (0.60-1.00)
Deformation modulus C_{23} (GPa)	0-1.3 (0.00-0.026)	1.3-6.0 (0.026-0.12)	6.0-20.0 (0.12-0.4)	20.0-33.0 (0.4-0.66)	33.0-50.0 (0.66-1)
Rock integrity coefficient C_{24}	0.00–0.15 (0.00~0.15)	0.15–0.35 (0.15~0.35)	0.35–0.55 (0.35~0.55)	0.55–0.75 (0.55~0.75)	0.75–1 (0.75~1.00)
Project geology-joint C_{30}					
Friction angle C_{31} (°)	0-13 (0.00-0.29)	13-21 (0.29-0.47)	21-29 (0.47-0.65)	29-37 (0.65-0.82)	37-45 (0.82-1)
Cohesion C_{32} (MPa)	0.00-0.05 (0.00-0.16)	0.05-0.08 (0.16-0.25)	0.08-0.12 (0.25-0.38)	0.12-0.22 (0.38-0.69)	0.22-0.32 (0.69-1.00)
Project topography C_{40}					
Slope angle C_{41} (°)	60–80 (0.00-0.25)	45–60 (0.25-0.44)	35–45 (0.44-0.56)	20–35 (0.56-0.75)	0–20 (0.75-1.00)
Slope height C_{42} (m)	80–100 (0.00~0.20)	60–80 (0.20~0.40)	45–60 (0.40~0.45)	30–45 (0.45~0.70)	0–30 (0.70~1.00)

$$\begin{array}{cc} & \begin{array}{cc} C_{11} & C_{12} \end{array} \\ \begin{array}{c} C_{11} \\ C_{12} \end{array} & \begin{bmatrix} (1,0,0) & (0.25,0.063,0.005) \\ (4,1.003,0.073) & (1,0,0) \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & \begin{array}{cc} C_{31} & C_{32} \end{array} \\ \begin{array}{c} C_{31} \\ C_{32} \end{array} & \begin{bmatrix} (1,0,0) & (2,0.501,0.498) \\ (0.5,0.125,0.125) & (1,0,0) \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & \begin{array}{cc} C_{41} & C_{42} \end{array} \\ \begin{array}{c} C_{41} \\ C_{42} \end{array} & \begin{bmatrix} (1,0,0) & (3,0.501,0.498) \\ (0.333,0.056,0.055) & (1,0,0) \end{bmatrix} \end{array}$$

The weights of factors in an arbitrary comparison matrix above were calculated by Eq. (11)-(13), and substituting into the computing equations for the cloud model, i.e., Eq. (14)-(19). The cloud model

Table 5. The cloud model comprehensive weights of factors and the importance v_w of the evaluation factors.

Evaluation factor	Seismic acceleration (g)	Cumulative rainfall monthly (mm)	Integrity index of rock mass	RQD (%)	UCS (MPa)
The comprehensive weights with cloud model	(0.018,0.005,0.006)	(0.074,0.085,0.086)	(0.205,0.192,0.210)	(0.167,0.149,0.185)	(0.090,0.085,0.084)
v_w	0.0303	0.325	0.717	0.495	0.351
Evaluation factor	Deformation modulus (GPa)	Friction angle (°)	Cohesion (MPa)	Slope angle (°)	Slope height (m)
The comprehensive weights with cloud model	(0.058,0.056,0.053)	(0.162,0.214,0.186)	(0.081,0.053,0.046)	(0.109,0.144,0.129)	(0.036,0.016,0.014)
v_w	0.233	0.925	0.277	0.604	0.094

comprehensive weights of factors affecting the slope stability were obtained, as shown in Table 5. According to the cloud model comprehensive weights of factors and Eq. (21)-(23), substitution the cloud model comprehensive weights into Eq. (21)-(23), the importance v_w of the evaluation factors on slope stability was obtained, as shown in Table 5.

The normal cloud model of evaluation factors of the rock slope can be set up using the normal cloud generator (Fig. 4). The abscissa represents the values of evaluation factors, and the ordinate represents the membership degree values of the cloud drops. According to Fig. 4, by substituting the values of the evaluation factors, the membership degree $\mu(x)$ of each evaluation factor was calculated. Based on the normal cloud model of evaluation factors, the cloud model comprehensive weights and the values of evaluation factors after normalization, the integrated membership degrees of the cloud model were obtained by Eq. (20). The results calculated are shown in Table 6. According to Eq. (21)-(23), the quantitative evaluation index of slope stability v can be obtained and the ranking grade of slope stability g is determined by selecting the maximum value v.

Results Analysis and Discussions

The results in Table 5 indicates that the influence degree of each evaluation factor on the slope stability is: friction angle>integrity index of rock mass>the slope angle>RQD>UCS>cumulative rainfall monthly >cohesion>deformation modulus>the slope height> seismic acceleration. The most significant factors are the joint friction angle and integrity index of rock mass. The integrity index of rock mass is influenced by the joint dip angle, joint dip direction, joint density and joint trace length. Therefore, the influence factor that controlled the rock slope stability is joint plane.

Table 6 shows the ranking grade of slope stability, the integrated membership degree of cloud model and the quantitative evaluation index of slope stability. The quantitative evaluation index indicates the possibilities of the slope stability in the corresponding ranking grade. The maximum value of the quantitative evaluation index v is 0.742 of ranking grade IV. Therefore, the slope stability is ranked IV grade, i.e., the status of slope is unstable. If only certainty is considered, the certain index v_1 can be used to evaluate slope stability, the maximum value v_1 is 0.652 of ranking grade III. The slope stability is ranked III grade, i.e., the status of slope is stable. Therefore, whether to consider the uncertainty of the membership degree has a significant influence on the evaluation result of the slope stability.

In order to validate the evaluation results, the evaluation of slope stability was conducted by using three-Dimensional Distinct Element Code (3DEC) simulation software. According to the project conditions, the geometry of simulated model and boundary conditions are shown in Fig. 5a). The volume represented by the red line is the excavated portion of the rock slope, and the slope angle difference before and after excavation is 6°. The displacement boundary conditions are assumed to be given by $u_x = u_y = 0$ on the four vertical planes of the model, and the displacement boundary conditions are assumed to be given by $u_x = u_y = u_z = 0$ on the bottom of the model [28]. The top surface of the model is stress free and has no displacement constraints. The self-weight stress is applied in the slope model.

Based on the above the conditions and the parameters (Table 2 and Table 3), the 3DEC simulation model is established, as shown in Fig. 5b), with 36, 487 grid points. In the 3DEC simulation model, the blocks obey the linear elastic model, and the joints obey the Coulomb slip model. Fig. 5c) shows the displacement distribution of the blocks after excavation. In the rectangle with a length of 90 m and a width of 50 m, the maximum displacement of the block is 2.08×10^{-1} m, and the mean displacement of all blocks is 1.16×10^{-1} m in the rectangle. As seen from Fig. 5d), the maximum joint shear displacement

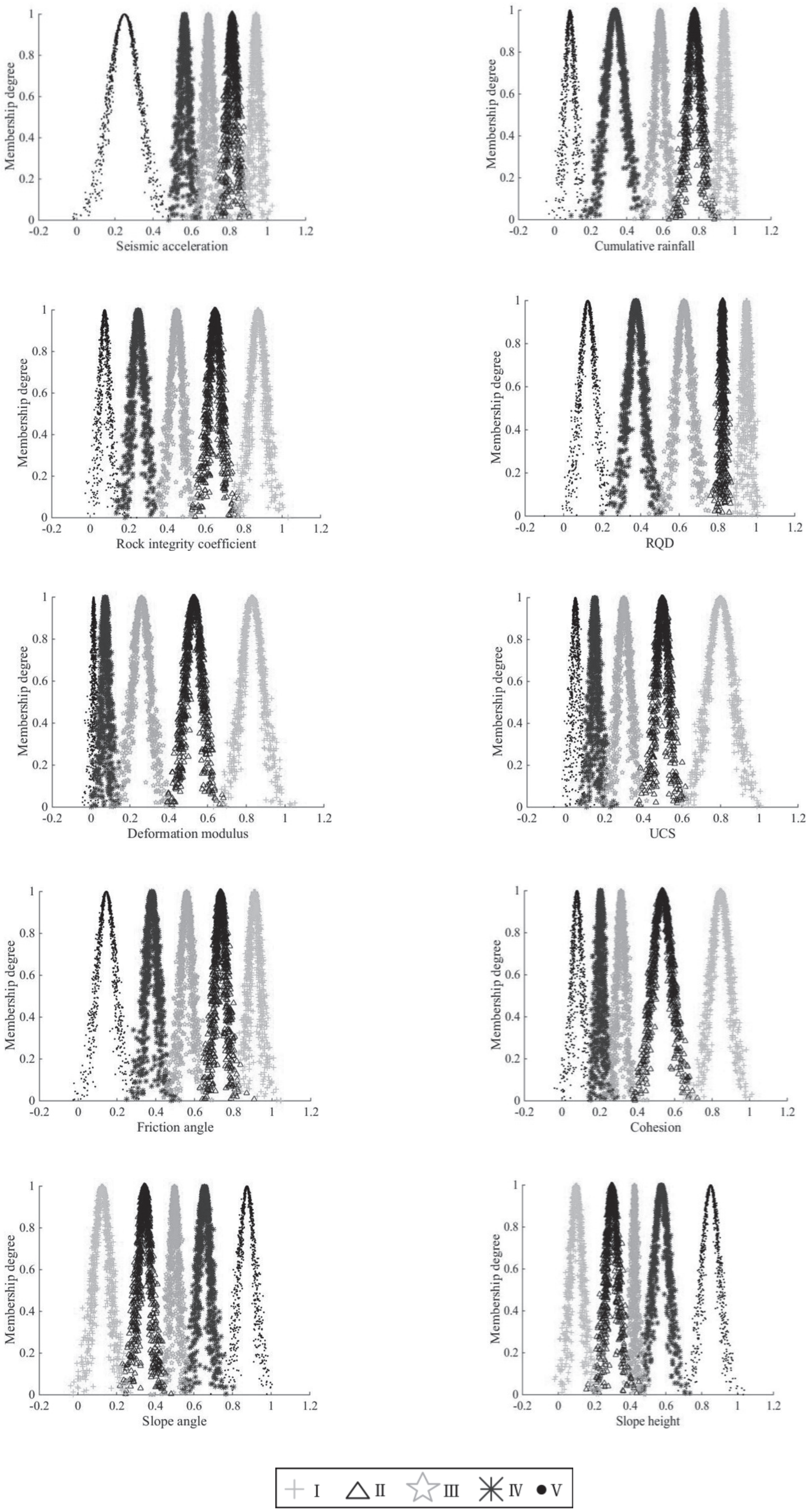
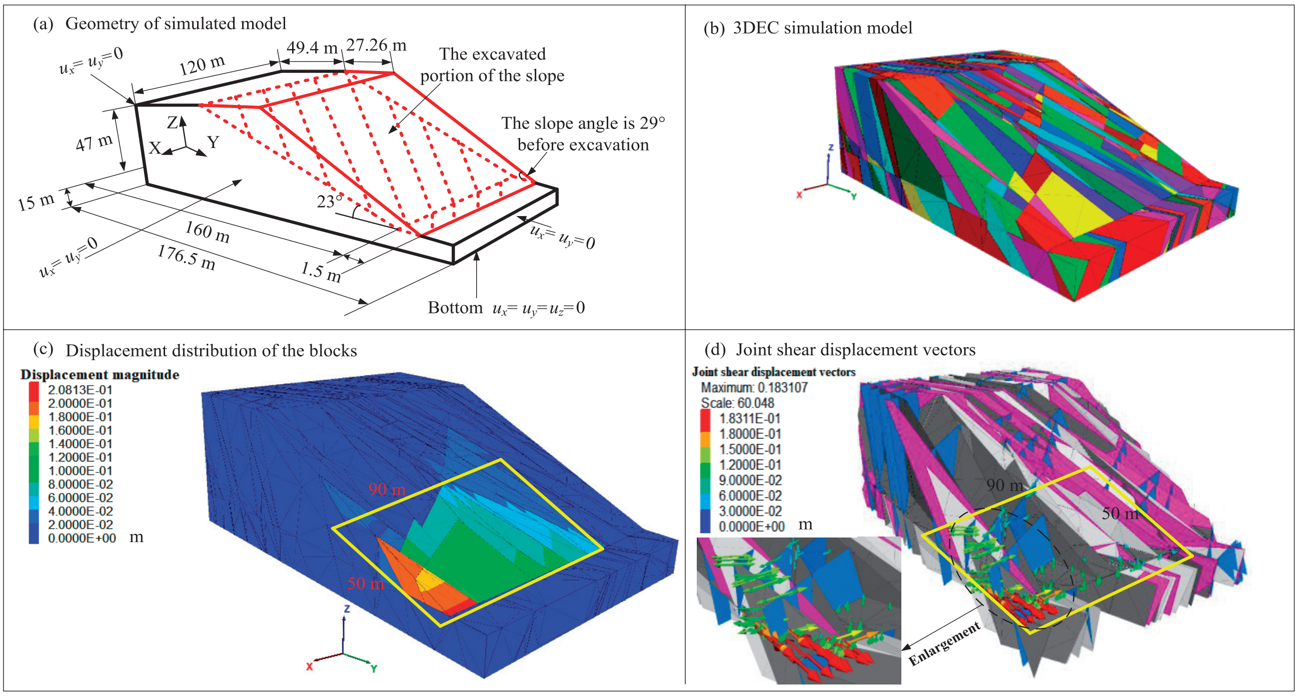


Fig. 4. Cloud for each evaluation factor established by normal cloud generator.

Table 6. The integrated membership degree of the cloud model and the quantitative evaluation index for slope stability.

Ranking grade of slope stability	V Very unstable	IV Unstable	III Basically stable	II Stable	I Extremely stable
The integrated membership degree D_g	(0.045,0.030,0.026)	(0.132,0.171,0.149)	(0.188,0.155,0.138)	(0.065,0.031,0.028)	$(5.41\times10^{-5},7.18\times10^{-5},6.45\times10^{-5})$
The certain index v_1	0.134	0.645	0.652	0.157	2.69×10^{-4}
The uncertain index v_2	0.871	0.868	0.893	0.911	0.897
The index of slope stability v	0.153	0.742	0.731	0.172	3×10^{-4}


(a) Geometry of simulated model

(b) 3DEC simulation model

(c) Displacement distribution of the blocks

(d) Joint shear displacement vectors

Fig. 5. Geometry of simulated model, 3DEC simulation model, displacement distribution of the blocks and joint shear displacement vectors.

vectors occur at the foot of the slope, the red arrow represents the direction of the joint shear displacement, and the maximum joint shear displacement is 1.83×10^{-1} m, and the mean joint shear displacement is 9.72×10^{-1} m in the rectangle. Therefore, the status of slope is unstable. The results were compared with that of AHP method and the extension theory [13, 25]. The slope stability is ranked III grade using AHP and the extension theory. From the analysis above, the evaluation results of slope stability obtained by the method is more in line with the actual project. The membership degree of cloud model can describe the uncertainty effects of the evaluating factors on slope stability, and the method can serve as a decision tool for the stability evaluation in uncertain condition.

Conclusions

In order to describe the uncertainty effects of the evaluating factors on engineering stability, so that the evaluation results of engineering stability are more in line with the actual project, there is a need to consider the membership degree with uncertainty. A quantitative index representing the membership degree with uncertainty in this paper was proposed and obtained based on the membership degree of the cloud model. The membership degree of the cloud model was obtained based on normal cloud theory, the system and the ranking grade of evaluation factors and AHP based on scale of cloud model. An evaluation method for the membership degree of cloud model was conducted and applied in a rock slope project. The results were compared with that of AHP method, the extension theory and the numerical simulation.

The most significant factors to the stability of rock slope are the joint friction angle and integrity index of rock mass. The status of rock slope is unstable after excavation, and there is need to take reinforcement measures, especially in the rectangular area of the slope. The membership degree method of the cloud model can describe the uncertainty effects of the evaluating factors on slope stability, and the method is verified to be a more competitive solution, where the uncertainty is incorporated in the evaluation system of the engineering stability. The method can serve as a decision tool for the stability evaluation in other similar projects, and to increase the likelihood of a successful project in uncertain condition.

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Conflict of Interest

The authors declare no conflict of interest.

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