Pol. J. Environ. Stud. Vol. 31, No. 5 (2022), 4859-4871
DOI: *10.15244/pjoes/150045* ONLINE PUBLICATION DATE: 2022-09-26

Original Research

Differential Game Model of Government-Enterprise Cooperation on Emission Reduction under Carbon Emission Trading Policy

Hao Sun[1], Guangkuo Gao[1]*, Zonghuo Li[2]

[1]Business School, University of Shanghai for Science and Technology, Shanghai 200093, China
[2]School of Politics and Public Administration, Soochow University, Suzhou, 215123, China

Received: 27 February 2022
Accepted: 14 May 2022

Abstract

In the context of green development, cooperation between government and enterprises in emission reduction is an effective way to improve the efficiency of enterprise emission reduction. In carbon trading policies, this paper assumes that the consumer demand influenced by product green level and enterprise goodwill, considering the continuous production in a multiple cycle under the condition of dynamic change characteristics, constructs the government and enterprise cooperation to reduce emissions of three kinds of differential game model, to explore the effective way of government and enterprise cooperation. The results show that cost-sharing contracts can achieve Pareto improvement of revenue for the government, enterprise under a certain condition; the revenue of the government, enterprises, and systems are optimal under collaborative cooperation. Finally, we prove the validity of the conclusion through an example analysis, and the sensitivity of related parameters is analyzed.

Keywords: carbon emissions permit trading, low-carbon emission reduction efforts, low carbon preference, cost-sharing, differential game

Introduction

With the rapid development of industrialization, people's living standards have been greatly improved. However, the consumption of a mass of fossil fuels has also led to a sharp increase in CO_2 emissions [1]. Intergovernmental Panel on Climate Change (IPCC) report: greenhouse gas emissions from human activities are the main cause of global warming and the frequency of extreme weather events [2]. Many countries and regions have promulgated various policies to limit carbon emissions, among which the most widely used policy is carbon emissions permit trading. Carbon emission trading refers to enterprises in the same industry, if the actual carbon emissions of the enterprise are lower than the government quota by means of increasing technological innovation, the enterprise can sell the extra quota in the carbon trading market to gain profits, if the carbon emission of an enterprise exceeds the quota, it needs to purchase the remaining quota

*e-mail: guangkuogao1011@163.com

of other enterprises in the market to maintain production [3]. Carbon emission trading can effectively control the production emissions of enterprises, but it is at the cost of reducing the production scale of enterprises and slowing down economic development [4]. Therefore, only by promoting enterprises to actively carry out green transformation and upgrading, can we achieve both economic growth and environmental protection.

In recent years, many scholars have studied enterprise emission reduction under carbon trading.

In terms of production and operation management of enterprises, Dobos used the dynamic Arrow-Karlin model to analyze the influence of the carbon emission trading mechanism on enterprise production and inventory strategy [5]. Du et al. studied the optimal production model of an enterprise with multiple emission access channels under the carbon emission trading machine [6]. Zhang and Xu proposed a linear solution with high complexity to analyze the optimal production strategy under the constraints of carbon cap and trade [7]. Xu et al. considered the influence of carbon cap-and-trade and deduced the optimal total emissions and optimal output of the product [8]. The above literature makes an important contribution to the application of carbon emission trading at the micro-level. However, they fail to consider the relationship between government and enterprises.

In terms of cooperative emission reduction, some scholars have studied the cooperative emission reduction of supply chain under the carbon trading mechanism. Such as, Yang et al. [9] studied supply chain pricing and carbon emission reduction decisions under vertical and horizontal cooperation considering the influence of carbon trading mechanism. Hou et al. [10] studied the investment decisions of manufacturers and retailers in emission reduction technologies under the mechanism of carbon trading and emission reduction cost sharing. Yin et al. [11] studied the carbon emission reduction and coordination strategy of the closed-loop supply chain of new energy vehicles.

Wang et al. [12] studied the two-way cost sharing and emission reduction decisions of manufacturers and retailers based on the carbon cap-and-trade system.

Research on the cooperation between government and enterprises on emission reduction mainly focuses on carbon subsidies. Li et al. studied the role of government subsidy policies in reducing carbon emissions [13]. Hafezalkotob et al. found that appropriate government subsidies can encourage manufacturers to carry out low-carbon production and reduce the environmental impact of the supply chain [14]. Zhang et al. studied the influence of government subsidy policies on manufacturers' emission reduction decisions under the carbon cap-and-trade system [15]. Cao et al. [16] studied the optimal production and carbon emission reduction level of manufacturers under carbon cap-and-trade and low-carbon subsidy policies. Different from the above studies, this paper studies the cooperation between government and enterprises from the aspects of green goodwill and cost sharing.

In the process of pushing enterprises to reduce emissions, consumers' low-carbon preference plays a key role [17]. Some scholars have studied consumers' low-carbon preferences, Wang et al. assumed that consumers had a preference for low-carbon products, studied the emission reduction investment behaviors and strategies of upstream and downstream enterprises [18]. Li et al. found that more and more consumers pay attention to enterprises' environmental records when making purchase decisions [19]. Fei et al. found the understanding degree of carbon label, the acceptability degree of carbon label, the credibility of carbon label, the reference group, and the price of low-carbon products are all important factors influencing consumers' willingness to buy low-carbon products [20]. Under the low carbon preference of consumers, the market demand mainly depends on the green goodwill and the green level of products of enterprises [21, 22]. However, the formation of green goodwill of enterprises requires not only needs the enterprises' own low-carbon emission reduction efforts but also needs the government's low-carbon efforts (low-carbon publicity). In the dual role of government and enterprises, the green goodwill of enterprises is constantly changing.

In reality, cooperation between government and enterprises to reduce emissions is a long-term and dynamic process. As an important dynamic game model, differential game can solve dynamic equilibrium results in continuous time [23]. The early application of differential game theory to the pursuit model in the military field, recently, more and more literature has applied this method to the field of management science, such as technological innovation management [24], environmental management [25], and production management [26]. Some researchers have developed game models in Low carbon emissions, mainly focused on the emission reduction of supply chain enterprises [27-30], but they do not involve the research on the government incentive for enterprise emission reduction. Therefore, assuming that consumer demand is affected by the green level of products and green goodwill of enterprises, and considering the carbon trading mechanism, this paper use the differential game model to analyzes the cooperative emission reduction between government and enterprises from the dynamic perspective.

The main contributions of this paper are as follows. (1) We link the demand of consumers with the green level of products and the green goodwill of enterprise, and fundamentally analyze the motivation of enterprise emission reduction. (2) The influence of carbon emission trading is considered in the process of enterprise emission reduction. (3) The differential game model is used to study the emission reduction efforts of governments and enterprises from a dynamic perspective.

Material and Methods

Problem Description

This paper takes the emission reduction system composed of the government and individual enterprises under the background of low carbon as the research object, studying the joint emission reduction problem between the government and enterprises. In the joint emission reduction process, the government shares part of the cost of low-carbon emission reduction for enterprises and publicizes their emission reduction behaviors (advertising effect), thus encouraging enterprises to actively invest in low-carbon emission reduction.

Considering that the government plays an important role in the process of low-carbon emission reduction, we take the government as the leading party of the Stackelberg differential game, and the enterprise is the follower. The decision-making process of governments and enterprises is as follows: First, the government decides the level of publicity effort and the proportion of emission reduction costs shared by enterprises. Then, enterprises determine low-carbon emission reduction efforts according to the government's decision.

In the whole process, the revenue of the enterprise is mainly brought by the increase of consumer demand caused by the improvement of green goodwill and the green level of the product. The revenue of the government mainly includes two parts. First, the social benefits of environmental improvement brought about by enterprise low-carbon emission reduction. Second, an increase in tax revenue resulting from the revenue increase of the enterprise. The decision-making process is shown in Fig. 1.

Conditional Hypothesis

Hypothesis 1 – The green goodwill of enterprises is related to the level of publicity efforts of the government and the level of low-carbon emission reduction efforts of enterprises, and it is a dynamic process. Drawing on Nerlove-Arrow's classic advertising goodwill model [31]. The change process of green goodwill can be expressed by the following differential equation:

$$\dot{R}(t) = \lambda_1 S_1(t) + \lambda_2 S_2(t) - \delta R(t)$$

Where $R(t)$ represents the green goodwill of the enterprise at moment t; $S_1(t)$ represents the level of propaganda efforts of the government t, $S_2(t)$ represent the low carbon emission reduction efforts of the government and enterprises at moment t; λ_1 and λ_2 respectively represent the influence coefficients of the emission reduction efforts of the government and enterprises on the green goodwill of the enterprise, δ represents the natural attenuation coefficient of enterprise green reputation; $\dot{R}(t)$ represents the change rate of the enterprise's green goodwill with time t.

Hypothesis 2 – Under the carbon emission trading mechanism, the flow of carbon trading funds of enterprises is expressed as follows:

$$T = p_e[eq_B - S_2(t) - U]$$

Where T represents the capital flow of carbon emissions permit trading, p_e represents the price of carbon emissions permit trading, e represents the carbon emission per unit product produced, q_B represents the output of the product, U represents the carbon quota obtained by the enterprise.

Hypothesis 3 – The green level of the product is consistent with the level of the enterprise's low-carbon emission reduction efforts. Consumers have green preference psychology, and their purchasing behavior is jointly influenced by the green goodwill of the enterprise and the green level of the product. The demand function of the enterprise's product can be expressed as follows:

Hypothesis 3 – Consumer demand is affected by the green goodwill of the enterprise and the green level of the product. We assumed that the green level

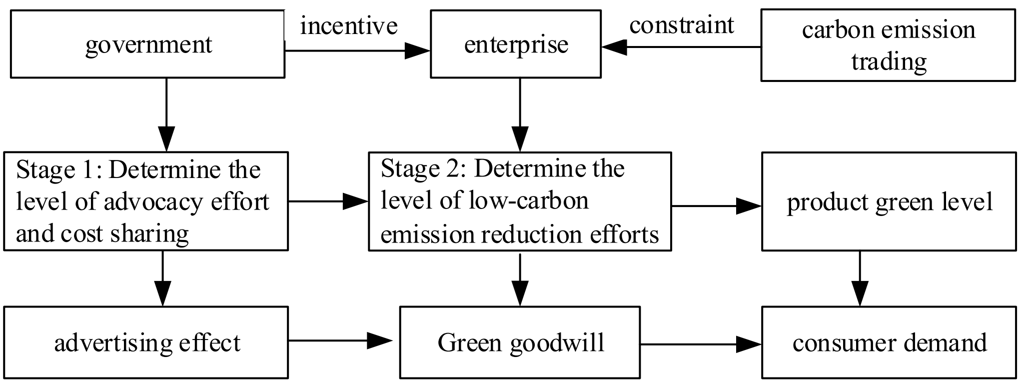

Fig. 1. Schematic diagram of government and enterprise cooperation in emission reduction.

of the product is consistent with the enterprise's emission reduction efforts. The consumer demand is expressed as:

$$Q(t) = \mu S_2(t) + \theta R(t)$$

Where, μ represents the influence coefficient of the green level of product on consumer demand, θ represents the influence coefficient of green goodwill on consumer demand.

Hypothesis 4 – Considering the convexity of effort costs, based on the effort cost in previous literature [32], we assume that the cost of government publicity efforts and the low carbon emission reduction costs of enterprises at moment *t* are as follows:

$$C_1(t) = \frac{k_1}{2} S_1^2(t),\ C_2(t) = \frac{k_2}{2} S_2^2(t)$$

Where C_1 and C_2 respectively represent the costs of the government and enterprises at the moment *t*; k_1 and k_2 represent the cost coefficients of the government and enterprises respectively.

Hypothesis 5 – The target revenue function of government and enterprise are expressed as follow:

$$\max_{S_1(t),L} J_1 = \int_0^{\infty} e^{-\rho t}[\pi_1 S_2(t) + \pi_2 Q(t) - C_1(t) - L(t)C_2(t)]dt \quad (1)$$

$$\max_{S_2(t)} J_2 = \int_0^{\infty} e^{-\rho t}[bQ(t) - p_e[eq_B - S_2(t) - U] - (1 - L(t))C_2(t)]dt \quad (2)$$

Where π_1 represents the influence coefficient of enterprise's emission reduction efforts on government revenue; π_2 and *b* represent the influence coefficient of consumer demand on government and enterprise revenue respectively; ρ represent discount rate. $L(t)$ represents the cost sharing ratio.

In the case of dynamic parameters in the model, it will be very difficult to solve, so this paper assumes that the parameters in the model are time-independent according to the processing method. In addition, for the convenience of writing, the unit of time *t* will be omitted in the following writing.

Results and Discussion

Model Analysis

Based on the description and hypothesis of the problem in the previous section, this section further analyzes whether the cost-sharing contract can make the optimal decision of the government and enterprises to reach the optimal level of cooperation. If it cannot reach the optimal level of cooperation, the paper analyzes whether the cost-sharing contract can make the optimal revenue of the government and enterprises realize pareto improvement and the degree of improvement, and to provide the decision-making basis for the future government-enterprise cooperation emission reduction.

Non-Cooperative Contract

Under the non-cooperative contract, the government and the enterprise make decisions independently to maximize their respective revenue. Learning from the literature [33], the objective function of the government and the enterprise can be expressed as:

$$\max_{S_1(t)} J_1^N = \int_0^{\infty} e^{-\rho t}[\pi_1 S_2 + \pi_2 Q - C_1]dt \quad (3)$$

$$\max_{S_2(t)} J_2^N = \int_0^{\infty} e^{-\rho t}[bQ - p_e(eq_B - S_2 - U) - C_2]dt \quad (4)$$

Proposition 1 – Under the non-cooperative contract, the optimal publicity effort level of government and the optimal emission reduction effort level of enterprises are:

$$\left(S_1^{N*}, S_2^{N*}\right) = \left(\frac{\lambda_1 \pi_2 \theta}{k_1(\rho+\delta)}, \frac{(\mu b + p_e)(\rho+\delta) + \lambda_2 b\theta}{k_2(\rho+\delta)}\right) \quad (5)$$

Proof We use the optimal control method to solve the equation [34], and the values V_1 and V_2 satisfy the HJB equation, we can get:

$$\rho V_1^N = \max_{S_1}\left\{\pi_1 S_2 + \pi_2 Q - \frac{1}{2}k_1 S_1^2 + V_1^{N'}(\lambda_1 S_1 + \lambda_2 S_2 - \delta R)\right\} \quad (6)$$

$$\rho V_2^N = \max_{S_2}\left\{bQ + p_e(eq_B - S_2 - U) - \frac{1}{2}k_2 S_2^2 + V_2^{N'}(\lambda_1 S_1 + \lambda_2 S_2 - \delta R)\right\} \quad (7)$$

We take the first derivative of (6) and (7) with respect to S_1 and S_2 respectively, we have:

$$S_1 = \frac{\lambda_1 V_1^{N'}}{k_1} \quad (8)$$

$$S_2 = \frac{\mu b + \lambda_2 V_2^{N'} + p_e}{k_2} \quad (9)$$

Where $V_1^{'} = \frac{\partial V_1^N}{\partial R}$, $V_2^{'} = \frac{\partial V_2^N}{\partial R}$.

Substituting equations (8) and (9) into equations (6) and (7) respectively, we can get:

$$\rho V_1^N = \left\{ \begin{array}{l} \dfrac{\pi_1\mu b + \pi_1\lambda_2 V_2^{N'} + \pi_1 p_e}{k_2} + \pi_2(\dfrac{\mu^2 b + \mu\lambda_2 V_2^{N'} + \mu p_e}{k_2} + \theta R) - \\ \dfrac{1}{2}\dfrac{(\lambda_1 V_1^{N'})^2}{k_1} + V_1^{N'}(\dfrac{\lambda_1^2 V_1^{N'}}{k_1} + \dfrac{\lambda_2\mu b + \lambda_2^2 V_2^{N'} + \lambda_2 p_e}{k_2} - \delta R) \end{array} \right\} \tag{10}$$

$$\rho V_2^N = \left\{ \begin{array}{l} b(\dfrac{\mu^2 b + \mu\lambda_2 V_2^{N'} + \mu p_e}{k_1} + \theta R) - p_e(eq_B - \dfrac{\mu b + \lambda_2 V_2^{N'} + p_e}{k_2} - U) - \\ \dfrac{(\mu b + \lambda_2 V_2^{N'} + p_e)^2}{2k_2} + V_2^{N'}(\dfrac{\lambda_1^2 V_1^{N'}}{k_1} + \dfrac{\lambda_2\mu b + \lambda_2^2 V_2^{N'} + \lambda_2 p_e}{k_2} - \delta R) \end{array} \right\} \tag{11}$$

According to the structure forms of (10) and (11), we can infer that the optimal linear function V_1, V_2 conforms to the solution conditions of the HJB equation [35]. Therefore, we assume that $V_1^N = c_1 R + c_2$, $V_2^N = d_1 R + d_2$, where c_1, c_2, d_1, d_2 is a constant. Substituting V_1^N, V_2^N into (10) and (11), we have:

$$\rho(c_1 R + c_2) = \left\{ \begin{array}{l} \dfrac{\pi_1\mu b + \pi_1\lambda_2 d_1 + \pi_1 p_e}{k_2} + \pi_2(\dfrac{\mu^2 b + \mu\lambda_2 d_1 + \mu p_e}{k_2} + \theta R) - \\ \dfrac{1}{2}\dfrac{(\lambda_1 c_1)^2}{k_1} + c_1(\dfrac{\lambda_1^2 c_1}{k_1} + \dfrac{\lambda_2\mu b + \lambda_2^2 d_1 + \lambda_2 p_e}{k_2} - \delta R) \end{array} \right\} \tag{12}$$

$$\rho(d_1 R + d_2) = \left\{ \begin{array}{l} b(\dfrac{\mu^2 b + \mu\lambda_2 d_1 + \mu p_e}{k_1} + \theta R) - p_e[eq_B - \dfrac{\mu b + \lambda_2 d_1 + p_e}{k_2} - U] - \\ \dfrac{1}{2}\dfrac{(\mu b + \lambda_2 d_1 + p_e)^2}{k_2} + d_1(\dfrac{\lambda_1^2 c_1}{k_1} + \dfrac{\lambda_2\mu b + \lambda_2^2 d_1 + \lambda_2 p_e}{k_2} - \delta R) \end{array} \right\} \tag{13}$$

By sorting out Equations (12) and (13), the coefficient of the optimal linear function is:

$$\left\{ \begin{array}{l} c_1 = \dfrac{\pi_2\theta}{\rho+\delta} \\ c_2 = \dfrac{1}{\rho}[\dfrac{\pi_1(\rho+\delta)(\mu b + p_e) + \pi_1\lambda_2 b\theta}{k_2(\rho+\delta)} + \pi_2\dfrac{(\mu^2 b + \mu p_e)(\rho+\delta) + \mu\lambda_2 b\theta}{k_2(\rho+\delta)} - \\ \dfrac{(\lambda_1\pi_2\theta)^2}{2k_1(\rho+\delta)^2} + \dfrac{\pi_2\theta}{\rho+\delta}(\dfrac{\lambda_1^2\pi_2\theta}{k_1(\rho+\delta)} + \dfrac{\lambda_2(\mu b + pe)(\rho+\delta) + \lambda_2^2 b\theta}{k_2(\rho+\delta)})] \end{array} \right. \tag{14}$$

$$\left\{ \begin{array}{l} d_1 = \dfrac{b\theta}{\rho+\delta} \\ d_2 = \dfrac{1}{\rho}[\dfrac{\mu^2 b^2(\rho+\delta) + \mu\lambda_2 b^2\theta + \mu p_e b(\rho+\delta)}{k_1(\rho+\delta)} - \\ p_e(eq_B - \dfrac{(p_e + \mu b)(\rho+\delta) + \lambda_2 b\theta}{k_2(\rho+\delta)} - U) - \dfrac{[(\mu b + p_e)(\rho+\delta) + \lambda_2 b\theta]^2}{2k_2(\rho+\delta)^2} \\ + \dfrac{b\theta}{\rho+\delta}[\dfrac{\lambda_1^2\pi_2\theta}{k_1(\rho+\delta)} + \dfrac{\lambda_2(\rho+\delta)(\mu b + p_e) + \lambda_2^2 b\theta}{k_2(\rho+\delta)}]] \end{array} \right. \tag{15}$$

Substituting (14) and (15) into linear function V_1, V_2, we can obtain the expression of the optimal revenue function as follows:

$$V_1^{N*} = \left\{ \begin{array}{l} \dfrac{\pi_2\theta}{\rho+\delta}R + \dfrac{1}{\rho}[\dfrac{\pi_1(\rho+\delta)(\mu b + p_e) + \pi_1\lambda_2 b\theta}{k_2(\rho+\delta)} + \\ \pi_2\dfrac{(\mu^2 b + \mu p_e)(\rho+\delta) + \mu\lambda_2 b\theta}{k_2(\rho+\delta)} - \dfrac{(\lambda_1\pi_2\theta)^2}{2k_1(\rho+\delta)^2} + \\ \dfrac{\pi_2\theta}{\rho+\delta}(\dfrac{\lambda_1^2\pi_2\theta}{k_1(\rho+\delta)} + \dfrac{\lambda_2(\mu b + pe)(\rho+\delta) + \lambda_2^2 b\theta}{k_2(\rho+\delta)})] \end{array} \right\} \tag{16}$$

$$V_2^{N*} = \left\{ \begin{array}{l} \dfrac{b\theta}{\rho+\delta}R + \dfrac{1}{\rho}[\dfrac{\mu^2 b^2(\rho+\delta) + \mu\lambda_2 b^2\theta + \mu p_e b(\rho+\delta)}{k_1(\rho+\delta)} - \\ p_e[eq_B - \dfrac{(p_e + \mu b)(\rho+\delta) + \lambda_2 b\theta}{k_2(\rho+\delta)} - U] - \\ \dfrac{[(\mu b + p_e)(\rho+\delta) + \lambda_2 b\theta]^2}{2k_2(\rho+\delta)^2} + \dfrac{b\theta}{\rho+\delta}[\dfrac{\lambda_1^2\pi_2\theta}{k_1(\rho+\delta)} + \\ \dfrac{\lambda_2(\rho+\delta)(\mu b + p_e) + \lambda_2^2 b\theta}{k_2(\rho+\delta)}]] \end{array} \right\} \tag{17}$$

Finally, substituting equations (16) and (17) into equations (8) and (9), we can get equation (5).

Corollary 1 – Under the non-cooperative contract, the optimal publicity effort level of government S_1 is negatively correlated with the government's emission reduction cost coefficient k_1, discount rate ρ and goodwill attenuation coefficient δ; and is positively correlated with the influence coefficient θ of green goodwill of enterprises on demand, influence coefficient λ_1 of publicity effort level of government on green goodwill, influence coefficient π_2 of consumer demand on government revenue.

Corollary 2 – The optimal emission reduction effort S_2 of the enterprise is negatively correlated with the cost coefficient k_2, the discount rate ρ and the goodwill attenuation coefficient δ; and is positively correlated with the influence coefficient μ of the green level of the products on consumer demand, the influence coefficient b of consumer demand on enterprise revenue, the carbon trading price p_e, the influence coefficient λ_2 of the enterprise's emission reduction efforts on green goodwill, and the influence coefficient θ of goodwill on demand.

According to proposition 1, the optimal publicity effort level of government and the optimal emission reduction efforts of enterprises are both positively correlated with their own revenue. This shows that both sides make decisions from the perspective of maximizing their own interests without considering the revenue of the system. When both sides formulate their own strategies based on the maximization of the revenue of the system, both the government and the enterprise will improve their revenue situation. We can see from corollary 2 that the enterprise's emission reduction effort level is negatively affected by its emission reduction cost coefficient. The emission reduction level of enterprises can be improved by reducing their emission reduction costs. Therefore, the government can encourage enterprises to invest in low-carbon emission reduction by sharing their emission reduction costs.

Cost Sharing Contract

Under the cost-sharing contract, the government chooses to share a certain proportion of emission reduction costs for enterprises to encourage enterprises to carry out low-carbon emission reduction. From the perspective of long-term dynamics, the decision of low-carbon emission reduction efforts between the government and enterprises constitutes a government-led Stackelberg game. The government first makes decisions on the level of optimal publicity effort and the proportion of cost sharing, and then the enterprises determine their own low-carbon emission reduction efforts according to the decision of the government. In this case, the revenue function of the government and enterprises can be expressed as:

$$\max_{S_1(t),L} J_1^D=\int_0^\infty e^{-\rho t}[\pi_1 S_2+\pi_2 Q-C_1-LC_2]dt \tag{18}$$

$$\max_{S_2(t)} J_2^D=\int_0^\infty e^{-\rho t}[bQ-p_e[eq_B-S_2-U]-(1-L)C_2]dt \tag{19}$$

Proposition 2 – Under the cost-sharing contract, the optimal equilibrium strategy between the government and enterprises is:

$$S_1^{D*}=\frac{\lambda_1\pi_2\theta}{k_1(\rho+\delta)} \tag{20}$$

$$S_2^{D*}=\begin{cases}\dfrac{[2(\pi_1+\pi_2\mu)+b\mu+p_e](\rho+\delta)+\lambda_2\theta(2\pi_2+b)}{2k_2(\rho+\delta)}, & 2B>A\\[2ex] \dfrac{(\mu b+p_e)(\rho+\delta)+\lambda_2 b\theta}{k_2(\rho+\delta)}, & 2B\le A\end{cases} \tag{21}$$

$$L^*=\begin{cases}\dfrac{2(\pi_1+\pi_2\mu+\dfrac{\lambda_2\pi_2\theta}{\rho+\delta})-(b\mu+p_e+\dfrac{\lambda_2 b\theta}{\rho+\delta})}{2(\pi_1+\pi_2\mu+\dfrac{\lambda_2\pi_2\theta}{\rho+\delta})+(b\mu+p_e+\dfrac{\lambda_2 b\theta}{\rho+\delta})}, & 2B>A\\[2ex] 0, & 2B<A\end{cases} \tag{22}$$

where $A=b\mu+p_e+\dfrac{\lambda_2 b\theta}{\rho+\delta}$, $B=\pi_1+\pi_2\mu+\dfrac{\lambda_2\pi_2\theta}{\rho+\delta}$.

Proof We use the optimal control method to solve the equation, the HJB equation of the enterprise can be expressed as:

$$\rho V_2^D=\max_{S_2}\begin{Bmatrix}b(\mu S_2+\theta R)-p_e(eq_B-S_2-U)-(1-L)\dfrac{k_2S_2^2}{2}+\\ V_2^{D'}(\lambda_1 S_1+\lambda_2 S_2-\delta R)\end{Bmatrix} \tag{23}$$

According to (22), by solving the first-order condition of S_2, we can obtain:

$$S_2^D=\frac{b\mu+p_e+\lambda_2 V_2^{D'}}{(1-L)k_2} \tag{24}$$

The government determine its level of publicity effort and cost sharing ratio according to the rational response of enterprises. At this time, the government's HJB equation can be expressed as:

$$\rho V_1^D=\max_{S_1,L}\left\{\pi_1 S_2+\pi_2(\mu S_2+\theta R)-\frac{k_1S_1^2}{2}-L\frac{k_2S_2^2}{2}+V_1^{D'}(\lambda_1 S_1+\lambda_2 S_2-\delta R)\right\} \tag{25}$$

Substituting Equation (24) into Equation (25), and solve the first derivative of publicity effort level S_1 and cost sharing ratio L, we can get:

$$S_1^D=\frac{\lambda_1 V_1^{D'}}{k_1} \tag{26}$$

$$L=\begin{cases}\dfrac{2(\pi_1+\pi_2\mu+V_1^{D'}\lambda_2)-(b\mu+p_e+V_2^{D'}\lambda_2)}{2(\pi_1+\pi_2\mu+V_1^{D'}\lambda_2)+(b\mu+p_e+V_2^{D'}\lambda_2)}, & 2B>A\\[2ex] 0, & 2B<A\end{cases} \tag{27}$$

where $A=b\mu+p_e+V_2^{D'}\lambda_2$, $B=\pi_1+\pi_2\mu+V_1^{D'}\lambda_2$.

Substituting (24), (26) and (27) for (23) and (25), we have:

$$\rho V_1^D=\begin{Bmatrix}\dfrac{\pi_1(2B+A)B}{2Ak_2}+\dfrac{\pi_2\mu(2B+A)B}{2Ak_2}+\pi_2\theta R-\dfrac{(\lambda_1V_1^{D'})^2}{2k_1}-\\ \dfrac{(2B-A)B^2}{8k_2A^2}+V_1^{D'}(\dfrac{\lambda_1^2V_1^{D'}}{k_1}+\dfrac{\lambda_2(2B+A)B}{2Ak_2}-\delta R)\end{Bmatrix} \tag{28}$$

$$\rho V_2^D=\begin{Bmatrix}\dfrac{b\mu(2B+A)B}{2Ak_2}+b\theta R-p_e(eq_B-\dfrac{(2B+A)B}{2Ak_2}-U)-\\ \dfrac{(2B+A)B^2}{4Ak_2}+V_2^{D'}(\dfrac{\lambda_1^2V_1^{D'}}{k_1}+\dfrac{\lambda_2(2B+A)B}{2Ak_2}-\delta R)\end{Bmatrix} \tag{29}$$

Similarly, let $V_1^D=p_1R+p_2$, $V_2^D=q_1R+q_2$, where p_1, p_2 and q_1, q_2 are constants. Substituting V_1^D, V_2^D into (28) and (29) respectively, we get:

$$\rho(p_1R+p_2)=\begin{Bmatrix}\dfrac{\pi_1(2B+A)B}{2Ak_2}+\dfrac{\pi_2\mu(2B+A)B}{2Ak_2}+\pi_2\theta R-\dfrac{(\lambda_1p_1)^2}{2k_1}-\\ \dfrac{(2B-A)B^2}{8k_2A^2}+p_1(\dfrac{\lambda_1^2p_1}{k_1}+\dfrac{\lambda_2(2B+A)B}{2Ak_2}-\delta R)\end{Bmatrix} \tag{30}$$

$$\rho(q_1R+q_2)=\left\{\begin{array}{l}\frac{b\mu(2B+A)B}{2Ak_2}+b\theta R-p_e(eq_B-\frac{(2B+A)B}{2Ak_2}-U)-\\ \frac{(2B+A)B^2}{4Ak_2}+q_1(\frac{\lambda_1^2p_1}{k_1}+\frac{\lambda_2(2B+A)B}{2Ak_2}-\delta R)\end{array}\right\} \tag{31}$$

The coefficients of the optimal linear function can be obtained as follows:

$$\begin{cases}p_1=\frac{\pi_2\theta}{\rho+\delta}\\ p_2=\frac{1}{\rho}[\frac{\pi_1(2B+A)B}{2Ak_2}+\frac{\pi_2\mu(2B+A)B}{2Ak_2}-\frac{(\lambda_1\pi_2\theta)^2}{2k_1(\rho+\delta)^2}-\frac{(2B-A)B^2}{8k_2A^2}+\\ \frac{\pi_2\theta}{\rho+\delta}(\frac{\lambda_1^2\pi_2\theta}{k_1(\rho+\delta)}+\frac{\lambda_2(2B+A)B}{2Ak_2})]\end{cases}$$

$$\begin{cases}q_1=\frac{b\theta}{\rho+\delta}\\ q_2=\frac{1}{\rho}[\frac{b\mu(2B+A)B}{2Ak_2}-p_e(eq_B-\frac{(2B+A)B}{2Ak_2}-U)-\frac{(2B+A)B^2}{4Ak_2}+\\ \frac{b\theta}{\rho+\delta}(\frac{\lambda_1^2\pi_2\theta}{k_1(\rho+\delta)}+\frac{\lambda_2(2B+A)B}{2Ak_2}]\end{cases}$$

where $A=b\mu+p_e+\frac{\lambda_2 b\theta}{\rho+\delta}$, $B=\pi_1+\pi_2\mu+\frac{\lambda_2\pi_2\theta}{\rho+\delta}$.

Substituting p_1, p_2 and q_1, q_2 nto the optimal linear function, we can get:

$$V_1^{D*}=\left\{\begin{array}{l}\frac{\pi_2\theta}{\rho+\delta}R+\frac{1}{\rho}[\frac{\pi_1(2B+A)B}{2Ak_2}+\frac{\pi_2\mu(2B+A)B}{2Ak_2}-\frac{(\lambda_1\pi_2\theta)^2}{2k_1(\rho+\delta)^2}-\\ \frac{(2B-A)B^2}{8k_2A^2}+\frac{\pi_2\theta}{\rho+\delta}(\frac{\lambda_1^2\pi_2\theta}{k_1(\rho+\delta)}+\frac{\lambda_2(2B+A)B}{2Ak_2})]\end{array}\right\} \tag{32}$$

$$V_2^{D*}=\left\{\begin{array}{l}\frac{b\theta}{\rho+\delta}R+\frac{1}{\rho}[\frac{b\mu(2B+A)B}{2Ak_2}-p_e(eq_B-\frac{(2B+A)B}{2Ak_2}-U)-\\ \frac{(2B+A)B^2}{4Ak_2}+\frac{b\theta}{\rho+\delta}(\frac{\lambda_1^2p_1}{k_1}+\frac{\lambda_2(2B+A)B}{2Ak_2}]\end{array}\right\} \tag{33}$$

Taking the derivative of (32) and (33) with respect to R, and substituting theirs into (24), (26) and (27), we can obtain (20), (21) and (22).

Corollary 3 – We can see from proposition 2 that only when $2B>A$, the government will share the cost of emission reduction for enterprises. The optimal proportion of government contribution L is positively correlated with π_1 and π_2, and negatively correlated with b.

Corollary 4 – Under the cost-sharing contract, the optimal publicity effort level of government S_1^D is negatively correlated with the discount rate ρ, the government's publicity effort cost coefficient k_1, the enterprise's green goodwill attenuation degree δ; and is positively correlated with the influence coefficient λ_1 of government's publicity effort on enterprise green goodwill, influence coefficient π_2 of consumer demand on government revenue and the influence coefficient $\theta$of green goodwill on consumer demand.

Corollary 5 – The optimal emission reduction effort level of the enterprise S_2^D is positively correlated with the influence coefficient b of consumer demand on enterprise revenue, influence coefficient π_2 of consumer demand on government revenue, influence coefficient π_1 of enterprise emission reduction on social benefit, carbon trading price p_e and the influence coefficient λ_2 of enterprise emission reduction effort level on enterprise green goodwill; and is negatively correlated with discount rate ρ, cost coefficient k_1 publicity effort level of government and attenuation coefficient δ of enterprise green goodwill.

Corollary 5 shows that when $2B>A$, government shares part of the cost of emission reduction for enterprises, which reduces the pressure of emission reduction and improves their optimal emission reduction efforts, which is consistent with the expectation above.

Cooperation Contract

In this section, we discuss the collaborative efforts of government and industry to reduce emissions. We assume that both government and business make their own decisions with the goal of maximizing the revenue of the system.

Proposition 3 – Under the cooperation contract, the optimal publicity effort level of the government and the optimal emission reduction effort level of the enterprise are:

$$S_1^{C*}=\frac{\lambda_1(\pi_2+b)\theta}{k_1(\rho+\delta)} \tag{34}$$

$$S_2^{C*}=\frac{(\rho+\delta)[\pi_1+(\pi_2+b)\mu+p_e]+\lambda_2(\pi_2+b)\theta}{k_2(\rho+\delta)} \tag{35}$$

Proof The objective function of system revenue can be expressed as:

$$\max_{S_1,S_2} J_3^C=\int_0^\infty e^{-\rho t}[\pi_1S_2+(\pi_2+b)Q-p_e(eq_B-S_2-U)-C_1-C_2]dt \tag{36}$$

At this point, the system optimal revenue function V_3^C satisfies the HJB equation, we have:

$$\rho V_3^C=\left\{\begin{array}{l}\max_{S_1,S_2}[\pi_1S_2+(\pi_2+b)Q-p_e(eq_B-S_2-U)-C_1-C_2+\\ V_3^{C'}(\lambda_1S_1+\lambda_2S_2-\delta R)]\end{array}\right\} \tag{37}$$

Solving the first-order conditions of S_1, S_2, we can get:

$$S_1 = \frac{\lambda_1 V_3^{C'}}{k_1} \tag{38}$$

$$S_2 = \frac{\pi_1 + (\pi_2 + b)\mu + p_e + \lambda_2 V_3^{C'}}{k_2} \tag{39}$$

where $V_3^{C'} = \frac{\partial V_3^C}{\partial R}$, substituting (35) and (36) into (34), we can get:

$$\rho V_3^C = \left\{ \begin{array}{l} \pi_1 \frac{\pi_1 + (\pi_2 + b)\mu + p_e + \lambda_2 V_3^{C'}}{k_2} + (\pi_2 + b)(\mu \frac{\pi_1 + (\pi_2 + b)\mu + p_e + \lambda_2 V_3^{C'}}{k_2} \\ + \theta R) - p_e(eq_B - \frac{\pi_1 + (\pi_2 + b)\mu + p_e + \lambda_2 V_3^{C'}}{k_2} - U) - \frac{(\lambda_1 V_3^{C'})^2}{2k_1} - \\ \frac{(\pi_1 + (\pi_2 + b)\mu + p_e + \lambda_2 V_3^{C'})^2}{2k_2} + V_3^{C'}(\frac{\lambda_1^2 V_3^{C'}}{k_1} + \\ \lambda_2 \frac{\pi_1 + (\pi_2 + b)\mu + p_e + \lambda_2 V_3^{C'}}{k_2} - \delta R) \end{array} \right\} \tag{40}$$

We assume that the linear optimal function V_3^C satisfies the solution of the HJB equation, let

$$V_3^C(R) = m_1 R + m_2 \tag{41}$$

where m_1, m_2 is a constant.

Similarly, by substituting (41) into (40), the coefficient of the optimal revenue function can be obtained:

$$\begin{cases} m_1 = \frac{(\pi_2 + b)\theta}{\rho + \delta} \\ m_2 = \frac{\pi_1 B_2}{k_2} + \frac{1}{\rho}[(\pi_2 + b)(\frac{\mu B_2}{k_2}) - p_e(eq_B - \frac{B_2}{k_2} - U) - \frac{(\lambda_1 m_1)^2}{2k_1} - \frac{B_2^2}{2k_2} + m_1(\frac{\lambda_1^2 m_1}{k_1} + \frac{\lambda_2 B_2}{k_2})] \end{cases}$$

where $B_2 = \pi_1 + (\pi_2 + b)\mu + p_e + \lambda_2 m_1$.

Substituting m_1, m_2 into Equation (41), we can obtain the optimal revenue function of the system as follows:

$$V_3^{C*} = \left\{ \begin{array}{l} \frac{(\pi_2 + b)\theta}{\rho + \delta} R + \frac{1}{\rho}[\frac{\pi_1 B_2}{k_2} + (\pi_2 + b)(\frac{\mu B_2}{k_2}) - p_e(eq_B - \frac{B_2}{k_2} - U) - \frac{(\lambda_1 m_1)^2}{2k_1} - \\ \frac{B_2^2}{2k_2} + m_1(\frac{\lambda_1^2 m_1}{k_1} + \frac{\lambda_2 B_2}{k_2})] \end{array} \right\} \tag{42}$$

Taking the derivative of Equation (42) with respect to R and substituting theirs in (38) and (39), we can get (34) and (35).

Corollary 6 – Under the government-enterprise cooperation emission reduction contract, the government's optimal level of propaganda efforts S_1^{C*} is positively correlated with the influence coefficient λ_1 of the government's propaganda efforts on enterprise green goodwill, the influence coefficient π_2 of consumer demand on government revenue, the influence coefficient b of consumer demand on enterprise revenue and the influence coefficient θ of green goodwill on consumer demand; is negatively correlated with discount rate ρ, the coefficient k_1 of the government's propaganda efforts cost and the coefficient δ of enterprise green goodwill attenuation.

Corollary 7 – The optimal emission reduction effort level S_2^{C*} is positively correlated with the influence coefficient b of consumer demand on enterprise revenue, the influence coefficient π_2 of consumer demand on government revenue, the influence coefficient π_1 of enterprise emission reduction on social benefit, carbon trading price p_e and the influence coefficient λ_2 of enterprise emission reduction effort level on enterprise green goodwill; is negatively correlated with discount rate ρ, the cost coefficient k_1 of Enterprise emission reduction efforts and the attenuation coefficient δ of enterprise green goodwill.

We can see from corollary 6 and 7, the cost coefficient of the government and enterprises has an important impact on their propaganda efforts and emission reduction efforts, which indicates that the improvement of the overall revenue of the system lies in the improvement of its own emission reduction efficiency. When the emission reduction cost is low, the system revenue can be better improved.

Comparison of Equilibrium Results

By comparing the equilibrium strategies and optimal returns of non-cooperative contracts, cost-sharing contracts and cooperative contracts, we can draw the following conclusions.

Proposition 4 – Under the cooperative contract, the propaganda efforts of government and emission reduction efforts of enterprises are reach the highest level.

Prove first, we compare the level of government propaganda efforts, according to equations (5), (20) and (34), we can get:

$$S_1^{N*} = S_1^{D*} = \frac{\lambda_1 \pi_2 \theta}{k_1(\rho + \delta)}, \quad S_1^{C*} = \frac{\lambda_1(\pi_2 + b)\theta}{k_1(\rho + \delta)}$$

Therefore $S_1^{N*} = S_1^{D*} < S_1^{C*}$.

Then, we compare the low carbon emission reduction efforts of enterprises, according to equations (5), (21) and (35), when $2B>A$, we can get:

$$S_2^{D*} - S_2^{N*} = \frac{2(\pi_1 + \pi_2\mu + \frac{\lambda_2\theta\pi_2}{\rho + \delta}) - (\frac{\lambda_2\theta b}{\rho + \delta} + \mu b + p_e)}{2k_2} = \frac{2B - A}{2k_2} > 0$$

$$S_2^{C*} - S_2^{D*} = \frac{(\rho + \delta)(b\mu + p_e) + \lambda_2\theta b}{2k_2(\rho + \delta)} > 0$$

So, when $2B>A$,we have $S_2^{C*} > S_2^{D*} > S_2^{N*}$.

where $A = b\mu + p_e + \frac{\lambda_2 b\theta}{\rho + \delta}$, $B = \pi_1 + \pi_2\mu + \frac{\lambda_2\pi_2\theta}{\rho + \delta}$.

According to proposition 4, under non-cooperation contract and cost-sharing contract, the level of propaganda efforts of the government remains unchanged, while the enterprise's emission reduction effort level increase, this is consistent with the conclusion of study [36]. Under the cooperation contract, both the government's publicity effort level and the enterprise's low-carbon emission reduction effort level are the highest.

Proposition 5 – Compared with non-cooperative contracts, the revenue of both government and enterprise are pareto improved under cost-sharing contracts.

Prove First, we compare the revenue of government, according to (16) and (32), we can get:

$$V_1^{D*} - V_1^{N*} = \frac{[(2\pi_1 + 2\mu b + p_e + \pi_2\mu)(\rho+\delta) + \lambda_2\theta(2\pi_2 - b)]^2}{8k_2\rho(\rho+\delta)^2} > 0$$

So, we have $V_1^{D*} - V_1^{N*} > 0$, that is $V_1^{D*} > V_1^{N*}$.

Then, we compare corporate earnings, from (17) and (33), we obtained:

$$V_2^{D*} - V_2^{N*} = \frac{2(\pi_1 + \pi_2\mu + \frac{\lambda_2\theta\pi_2}{\rho+\delta})(\mu b + p_e + \frac{\lambda_2\theta b}{\rho+\delta}) - (\frac{\lambda_2\theta b}{\rho+\delta} + \mu b + p_e)^2}{4k_2\rho}$$
$$= \frac{(2B - A)A}{4k_2\rho} > 0$$

So, when $2B>A$, we have $V_2^{D*} - V_2^{N*} > 0$, that is $V_2^{D*} > V_2^{N*}$.

According to proposition 5, under the cost-sharing contract, the revenue of both the government and the enterprise are greater than those of the non-cooperative contract. This shows that by sharing the cost of emission reduction for enterprises, the revenue of both sides have achieved pareto improvement. Cost-sharing contracts can coordinate supply chains significantly by changing enterprises' carbon reduction decisions and benefits (Wang et al., 2019).

Proposition 6 – Under the cooperative emission reduction contract, the revenue of the system is greater than that of the other two contracts.

Prove From proposition 5, we know that:

$$V_1^{D*} + V_2^{D*} > V_1^{N*} + V_2^{N*}$$

According to equations (42), (32) and (33), we can get:

$$V_3^{C*} - (V_1^{D*} + V_2^{D*}) = \frac{(b\theta\lambda_2)^2}{2k_2\rho(\rho+\delta)^2} + \frac{[(b\mu + p_e)(\rho+\delta) + \lambda_2 b\theta]^2}{8k_2\rho(\rho+\delta)^2} > 0$$

So, we have $V_S^{C*} > V_1^{D*} + V_2^{D*} > V_1^{N*} + V_2^{N*}$.

To sum up, when $2B>A$, cost-sharing contracts achieve pareto improvements in the revenue of all parties compared to non-cooperative contracts, this is consistent with the conclusion of study [37]. The optimal revenue of the system under the cost sharing contract are greater than those under the non-cooperation contract, while the optimal revenue of the system under the cooperation contract are greater than those under the cost sharing contract. The revenue distribution between the government and enterprises under the cooperation contract mainly depends on the bargaining power of both parties. If a reasonable revenue distribution mechanism can be established, it will be beneficial to both parties as well as the whole society.

Numerical Results

In this section, the effectiveness of the three contracts and the influence of key parameters are further analyzed through numerical simulation. We use real data for analysis to make the numerical experiment more convincing. We selected a chemical enterprise in Shanghai, which chose to invest in carbon capture and storage technology (CCS) to reduce carbon emission levels. The following data is obtained from the actual operation status of the enterprise, China Energy Statistical Yearbook, China Environmental Statistical Yearbook and some research on CCS technology investment. The specific data are as follows:

The price per unit of product is 1.5 yuan, the carbon trading price in the carbon trading market is 20-50 yuan/ton, which is mainly determined according to the actual trading price in the current carbon market; the carbon emission of each product produced by the enterprise is 0.02 kg; the production quantity is 2 million units, the carbon quota allocated by the government is 5 ton; Carbon emissions without CCS is 0.02 kg/unit; the cost coefficients of government and enterprise emission reduction efforts are 0.3 and 0.7 respectively; the influence coefficient of enterprise emission reduction on government revenue is 0.6, which is mainly determined through expert consultation; the influence coefficients of product green level and green goodwill on demand are 0.3 and 0.7 respectively; the influence coefficients of government and enterprise emission reduction efforts on enterprise green goodwill are 0.6 and 0.4 respectively; the influence coefficients of consumer demand on government and enterprise revenue are 0.4 and 0.7 respectively; we suppose that when the emission reduction efforts of both the government and the enterprise are zero, the green goodwill decay rate of the enterprise is 0.3; government and business have the same discount rate 0.6.

Fig. 2 and Fig. 3 compared the level of advocacy efforts of governments and the level of mitigation efforts of enterprises under the three contracts. Fig. 4 and Fig. 5 compared the revenue of government and enterprises under non-cooperative contracts and cost-sharing contracts. Fig. 6 compared the revenue of the system under the three contracts. Fig. 7 described the impact of relevant parameters on the cost-sharing ratio.

According to Fig. 2, we can see that the level of government propaganda efforts increases as the influence coefficient θ increases and decreases as

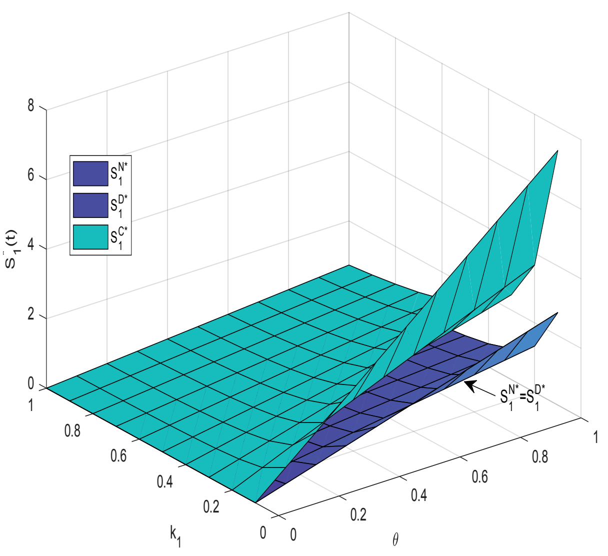

Fig. 2. comparison of government efforts to reduce emissions.

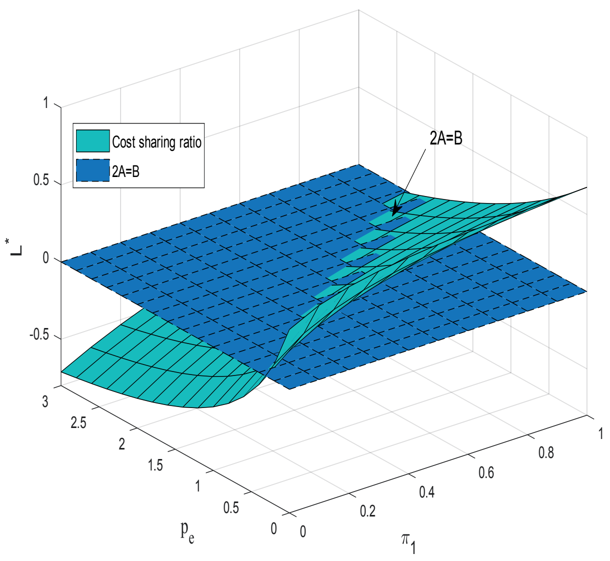

Fig. 4. The influence of coefficient p_e and π_1 on cost sharing ratio L.

the cost coefficient k_1 increase under the three contracts. The level of government advocacy efforts was the same in both the non-cooperation contract and cost-sharing contract, that is $S_1^{N*} = S_1^{D*}$. The level of government propaganda efforts under the contract of cooperation is the highest of the three contracts.

According to Fig. 3, we can see that the effort level of enterprise emission reduction increases with the increase of carbon trading price p_e and decreases with the increase of cost coefficient k_2 under three contracts. At the same time, we found that when the carbon trading price is low, the enterprise's emission reduction level under the cost-sharing contract is higher than that without the cost-sharing contract, that is $S_2^{D*}>S_2^{N*}$, however, when the carbon trading price exceeds a certain value (p_e satisfies 2B>A), the enterprise's emission reduction effort level under non-cooperative

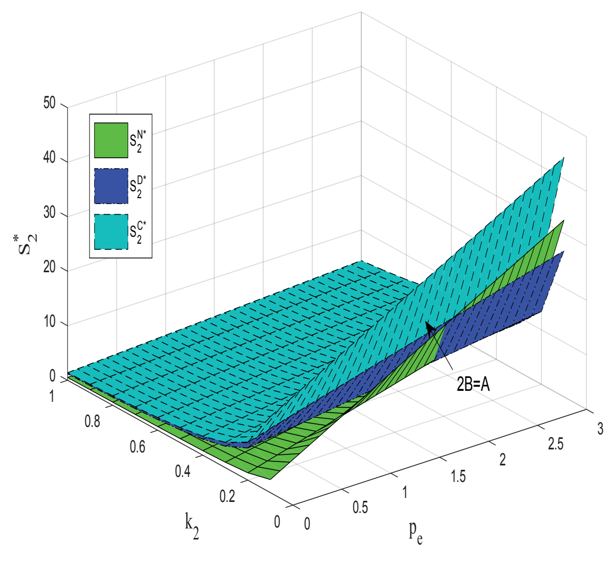

Fig. 3. Comparison of enterprise efforts to reduce emissions.

contract is higher than that under cost sharing contract, that is $S_2^{D*}<S_2^{N*}$. This shows that under certain conditions, government incentive measures will make enterprises realize expected earnings in advance, thus enterprises will scale back the level of their low-carbon emission reduction efforts.

The enterprise's low-carbon emission reduction efforts reach the optimal level under the cooperation contract, namely, $S_2^{C*}>S_2^{D*}>S_2^{N*}$. This shows that the cooperation contract can effectively improve the level of enterprise emission reduction efforts.

According to Fig. 4, we found that when p_e and π_1 satisfy $2B>A$, the value of L is positive, this indicates that only when p_e and π_1 satisfy $2B>A$, the government will choose to share the cost of emission reduction for enterprises. At the same time, we found that L increases gradually with the increase of π_1, this means that when the government gains more social revenue from enterprises' low-carbon emission reduction, the government will increase the share proportion of emission reduction costs to encourage enterprises to improve emission reduction level. In addition, we can also see that L decreases gradually with the increase of carbon trading price, this is mainly because when the trading price of carbon emission permits is high, enterprises will take the initiative to increase the carbon emission reduction level to save the cost of purchasing carbon emission permits, when the government realizes this, it will gradually reduce the cost sharing.

According to Fig. 5, we found that the government's revenue shows an increasing trend and gradually tends to be stable with the passage of time. This shows that as the government continues to invest in reducing emissions, the government's revenue has improved. At the same time, we found the government's revenue decreases gradually with the increase of attenuation coefficient. We can also see that cost-sharing contracts

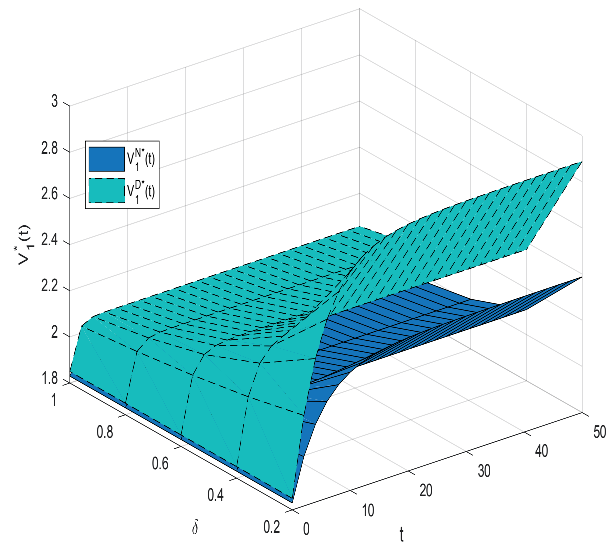

Fig. 5. Comparison of government revenue under different contracts.

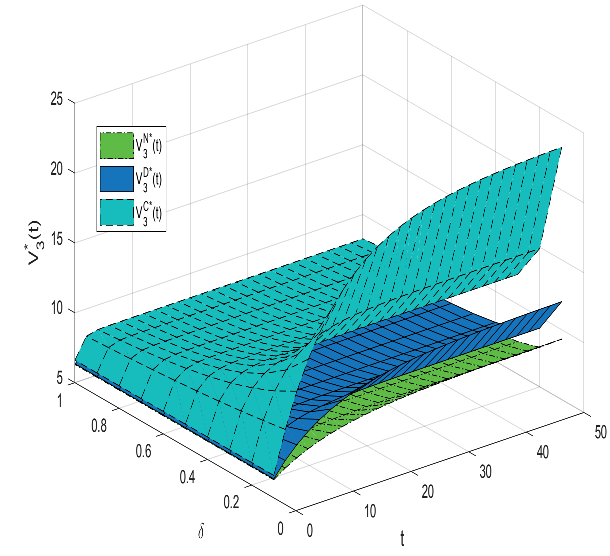

Fig. 7. Comparison of total returns under different contracts.

achieve pareto improvements in government revenue compared with non-cooperative contracts. This is mainly because the government shares the cost of emission reduction of enterprises, so enterprises will increase emission reduction level, and this will lead to increased consumer demand, which will lead to pareto improvements in government revenues.

According to Fig. 6, we found that the enterprise's revenue shows an increasing trend and gradually tends to be stable with the passage of time. This shows that with the improvement of enterprise emission reduction level, enterprise revenue has been improved. We can also see that under the cost sharing contract, the enterprise's revenue has achieved pareto improvement. This is mainly because the government shares the cost of emission reduction of enterprises, and enterprises will increase their emission reduction efforts to improve their green goodwill and green level of product, this will lead to increased consumer demand, which will lead to pareto improvement in enterprise's revenue.

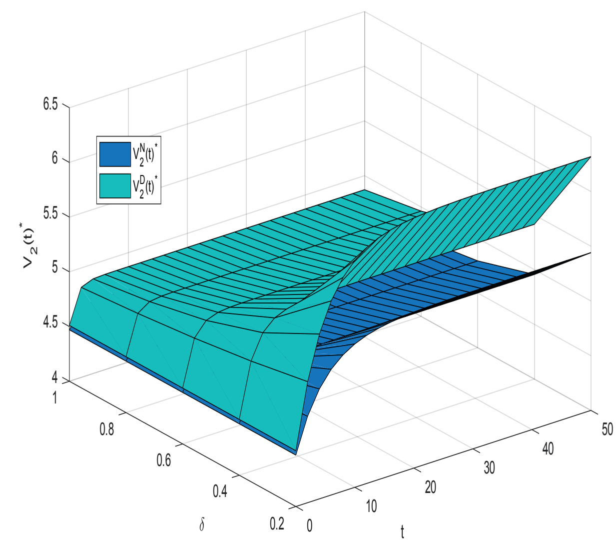

Fig. 6. Comparison of enterprise revenue under different contracts.

Similarly, we can see from Fig. 7, the revenue of the system also presents an increasing trend and eventually tends to be stable with the passage of time, however, it gradually decreases with the increase of attenuation coefficient. We can also see that the revenue of the emission reduction system is largest under the cooperative contract, while the revenue of the system is the smallest under the non-cooperative contract. This shows that cost sharing contract can realize pareto improvement of system revenue. The revenue of the system is higher than the other two contracts under the cooperation contract, it shows that stronger cooperation between government and enterprises can create more revenue.

The previous literature on government and corporate and cooperative emission reduction mainly focus on government subsidies under static conditions (13-16), without considering the dynamic relationship between consumer preferences and enterprises' emission reduction. The research results of this paper can effectively improve the efficiency of government-enterprise cooperation in emission reduction.

Conclusions and Suggestions

Conclusions

In recent years, the emission of greenhouse gases has led to the occurrence of climate warming and extreme weather. As the main body of carbon emission, enterprises should bear the main responsibility of climate change. In order to control the carbon emissions of enterprises, the government has made many policies, such as carbon trading, carbon subsidies. However, government incentives alone are not enough to drive companies to actively reduce emissions. consumer demand is also needed. In view of this, from the perspective of government cost sharing and consumer low-carbon preference, this paper constructs three differential game models of government and enterprise joint emission reduction and consumer demand-driven. Through solving and analyzing the model, the following conclusions can be obtained.

(1) Under certain conditions(2B>A), the government share part of the cost of emission reduction for enterprises, which can improve the revenue of both the government and enterprises. This also verifies the rationality of China's carbon emission subsidies for enterprises.

(2) In the absence of cost-sharing and cost-sharing contracts, governments and enterprises have different emission reduction efforts. The enterprise's emission reduction effort level will increase because the government shares a certain proportion of emission reduction costs, and the government's emission reduction effort level will remain unchanged.

(3) When the government and enterprises make joint decisions, the sum of the revenue of the government and enterprises is the largest, which also provides a theoretical basis for the coordination of government and enterprise emission reduction.

Suggestions

Based on the above analysis, we found that the cooperation between government and enterprises in emission reduction can promote the improvement of emission reduction efficiency. Therefore, we can draw the practical management enlightenment in this paper are as follows:

(1) Strengthen institutional design and build a long-term mechanism for cooperation on emission reduction. The government and enterprises sign clear cooperation agreements and put specific cooperation mechanisms into practice. To ensure positive revenue for the government and enterprises and enhance the stability of cooperation.

(2) Formulate preferential policies to reduce the cost of low-carbon emission reduction for enterprises. For example, developing green finance business and providing financial support to green transformation enterprises; establishing a flexible and diversified carbon tax system, and giving tax incentives and exemptions to enterprises that are good at low-carbon emission.

(3) Cultivate public awareness of environmental protection and encourage consumers to buy low-carbon products. Strengthen the publicity of green and low-carbon; organize green public welfare activities; provide subsidies to consumers who buy low-carbon products.

This paper considers that the cooperation contract can achieve the optimization revenue of the system, but does not consider the distribution and coordination of government and enterprise revenue under the cooperation contract, which needs further research in the future.

Acknowledgments

This research was funded by Shanghai Philosophy and Social Science Planning project (Grant No.2020BGL012).

Conflict of Interest

The authors declare no conflict of interest.

References

1. LIU Y., TAN X. J., YU Y., QI S. Z. Assessment of impacts of Hubei Pilot emission trading schemes in China-ACGE-analysis using Term CO_2 model. Applied Energy, **189**, 762, **2017**.
2. EDENHOFER O., PICHS-MADRUGA R., et al. IPCC special report on renewable energy sources and climate change mitigation. Prepared by Working Group III of the Intergovernmental Panel on Climate Change, Cambridge University Press, Cambridge, UK, **2011**.
3. ZANG J., WAN L., LI Z., WANG C., WANG, S. Does emission trading scheme have spillover effect on industrial structure upgrading? Evidence from the EU based on a PSM-DID approach. Environmental Science and Pollution Research, **27** (1), 12345, **2020**.
4. HUANG X., LING N.X. Differential game model of government and enterprise emission reduction based on emission right trading and emission reduction R&D subsidies. Journal of Systems Management, **29** (6), 1150, **2021**.
5. DOBOS I. The effects of emission trading on production and inventories in the Arrow-Karlin model. International Journal of Production Economics, **93**, 301, **2005**.
6. DU S., DONG J., LIANG L., et al. Optimal production policy with emission permits and trading. Chinese Journal of Management Science, **17** (3), 81, **2009**.
7. ZHANG B., XU L. Multi-item production planning with carbon cap and trade mechanism", International Journal of Production Economics, **144** (1), 118, **2013**.
8. XU X., ZHANG W., HE P., XU X. Production and pricing problems in make-to-order supply chain with cap-and-trade regulation. Omega, **66** (1), 248, **2017**.
9. YANG L, ZHANG Q, JI J. Pricing and carbon emission reduction decisions in supply chains with vertical and horizontal cooperation. International Journal of Production Economics, **191**, 286, **2017**.
10. HOU Q., GUAN Y., YU S. Stochastic differential game model analysis of emission-reduction technology under cost-sharing contracts in the carbon trading market. IEEE Access, **8**, 167328, **2020**.
11. YIN Y., LIU F. Carbon emission reduction and coordination strategies for new energy vehicle closed-loop

supply chain under the carbon trading policy. Complexity, e3720373, **2021**.
12. WANG Z., BROWNLEE A.E.I., WU Q. Production and joint emission reduction decisions based on two-way cost-sharing contract under cap-and-trade regulation [J]. Computers & Industrial Engineering, **146**, 106549, **2020**.
13. LI J., DU W., YANG F., et al. The carbon subsidy analysis in re-manufacturing closed-loop supply chain. Sustainability, **6** (6), 3861, **2014**.
14. HAFEZALOKTOB A., ALAVI A., MAKUI A. Government financial intervention in green and regular supply chains: Multi-level game theory approach. International Journal of Management Science and Engineering Management, **11** (3), 167, **2016**.
15. ZHANG S., WANG C., YU C. The evolutionary game analysis and simulation with system dynamics of manufacturer's emissions abatement behavior under cap-and-trade regulation. Applied Mathematics and Computation, **355**, 343, **2019**.
16. CAO K., XU X., WU Q., et al. Optimal production and carbon emission reduction level under cap-and-trade and low carbon subsidy policies. Journal of Cleaner Production, **167**, 505, **2017**.
17. LIU M., LI Z, ANWAR S., et al. Supply chain carbon emission reductions and coordination when consumers have a strong preference for low-carbon products, Environmental Science and Pollution Research, **28** (16), **2021**.
18. WANG Q.P., ZHAO D.Z., HE L.F. Strategies selection and behavior evolution of carbon-emission-reduction investment in supply chain firms. Journal of Management Engineering, **28** (30), 181, **2014**.
19. LI Q., LONG R., CHEN H. Empirical study of the willingness of consumers to purchase low-carbon products by considering carbon labels: A case study. Journal of Cleaner Production, **161**, 1237, **2017**.
20. FEI L., DONG S., XUE L., LIANG Q., YANG W. Energy consumption-economic growth relationship and carbon dioxide emissions in China. Energy policy, **39** (2), 568, **2011**.
21. BIANG J., ZHANG G., ZHOU G. Manufacturer vs. consumer subsidy with green technology investment and environmental concern. European Journal of Operational Research, **287** (3), **2020**.
22. ZHANG Z., YU L. Dynamic Optimization and Coordination of Cooperative Emission Reduction in a Dual-Channel Supply Chain Considering Reference Low-Carbon Effect and Low-Carbon Goodwill. International Journal of Environmental Research and Public Health, **18** (2), **2021**.
23. LI Y. Research on supply chain CSR management based on differential game[J]. Journal of Cleaner Production, **268** (8), 122171, **2020**.
24. NI J., ZHAO J., CHU L. K. Supply contracting and process innovation in a dynamic supply chain with information asymmetry. European Journal of Operational Research, **288** (2), 552, **2021**.
25. ZU Y., CHEN L., FAN Y. Research on low-carbon strategies in supply chain with environmental regulations based on differential game. Journal of Cleaner Production, **177**, 527, **2018**.
26. MAHMOODI A. Joint pricing and inventory control of duopoly retailers with deteriorating items and linear demand. Computers & Industrial Engineering, **132**, 36, **2019**.
27. WANG F., LI H., CAO Y. Knowledge sharing strategy and emission reduction benefits of low carbon technology collaborative innovation in the green supply chain. Frontiers in Environmental Science, **9**, **2022**.
28. HE L., YANG B., BIAN J. Differential game theoretic analysis of the dynamic emission abatement in low-carbon supply chains. Annals of Operations Research, **2021**.
29. XU C.Q., ZHAO D.Z., YUAN B.Y. Differential game model on joint carbon emission reduction and low-carbon promotion in supply chains. Journal of Management Sciences in China, **19** (2), 53, **2016**.
30. ZHOU Y.J., YE X. Differential game model of joint emission reduction strategies and contract design in a dual-channel supply chain. Journal of Cleaner Production, **190**, 592, **2018**.
31. NERLOVE M., ARROW KJ. Optimal advertising policy under dynamic situations. Economica, **29** (114), 129, **1962**.
32. GU D. A differential game approach to formation control. IEEE Transactions on Control Systems Technology, **16** (1), 85, **2007**.
33. LI Y. Research on supply chain CSR management based on differential game. Journal of Cleaner Production, **268**, 122171, **2020**.
34. BRASON A.E., HO YUQI. Apply Optimal Control: Optimization, Estimation, and Control [M]. Beijing: National Defence Industry Press, 141, **1982**.
35. JØRGENSEN S., TABOUBI S., ZACCOUR G. Retail promotions with negative brand image effects: Is cooperation possible?. European Journal of Operational Research, **150** (2), 395, **2003**.
36. CHEN X., WANG X., ZHOU M. Firms' green R&D cooperation behaviour in a supply chain: Technological spillover, power and coordination. International Journal of Production Economics, **218**, 118, **2019**.
37. WANG M., LI Y., LI M., et al. Will carbon tax affect the strategy and performance of low-carbon technology sharing between enterprises?. Journal of cleaner production, **210**, 724, **2019**.