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Original Research

The Assessment of Water Quality in the Ningxia Section of the Yellow River Using Intuitionistic Fuzzy Sets -TOPSIS Model

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Abstract

The level assessment of water quality has great significance for the prevention of water pollution; Its assessment is influenced by many factors. To assess the level of water quality in the Ningxia section of the Yellow River accurately, the intuitionistic fuzzy sets -TOPSIS model is introduced at first; secondly, the decisive matrix about the intuitionistic fuzzy sets is established, and the index weight coefficients considering the uncertainty of assessment indices are determined by using the Entropy weight method, then the weighed decisive matrix is obtained. Finally, the degree of membership at different levels of the water level is determined. Based on the ranking sequence of degree of membership, the water quality level corresponding to the maximum degree of membership is the final assessment level. The conclusions are drawn that the accuracy of level estimation of water quality in the Ningxia section of the Yellow River is very high using the intuitionistic fuzzy sets model in comparison with the current specifications, and the method is feasible for the level assessment of water quality, so it provides a new method and thoughts to assess the level of water quality in the future.

Keywords: assessment, water quality, Intuitionistic Fuzzy Sets -TOPSIS Model

Introduction

The water resource is an important natural resource, it is indispensable for human survival and social development [1], and it is the important basis of sustainable economic development. However, with the development of the global economy and dramatically increasing populations, the insufficiency of water resources and the deterioration of water quality has become a serious issues [2]. Especially, the discharges of domestic sewage in the city, the agricultural sewage, and industrial wastewater have affected the water quality seriously [3]. The crisis of water resources becomes serious gradually [4], so the investigations on

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the level assessment of water quality in different rivers have great significance to prevent the deterioration of water quality [5].

The method of the water quality has been investigated by many researchers [6] in many countries. The water quality Index method was suggested by Horton, et al in America in 1965, the eight indices of water quality were provided in the method [7]. Then Nemerow water quality index method was provided by Nemerow [8-9]; the water quality assessment of rivers under the influence of industrial and city sewage was performed by Surendra Sutha [10]. Since the 1990s, many mathematical methods and models are applied to assess the water quality widely, and good effects are obtained. Such as the neural network evaluation method [11], the matter element analysis [12], the comprehensive water quality identification index method [13], the projection pursuit method [14], and the grey index method [15], etc. These methods improve the development of water quality assessment enormously. However, there are still some shortcomings and insufficiency [16]. For example, lack of compatibility, large error, strong subjectivity, and complex calculative process.

To overcome the above shortcomings, the Intuitionistic Fuzzy Sets -TOPSIS model is used to assess the water quality in the paper. Relatively to the traditional fuzzy mathematical method, the non-membership function is added in the intuitionistic fuzzy sets [17] so the fuzzy concept can be described the further. And TOPSIS Model has direct geometric significance, it is characterized as the sufficient usage of original datum, little information loss, and wide application [18-19], so it is an efficient multiple attribute decision-making method [20-21]. When the intuitionistic fuzzy sets theory is combined with the TOPSIS model, a new model is established. In comparison with the traditional methods, a new model has higher efficiency.

The paper is organized as follows: In Section 1, the engineering overview in the study area is introduced at first. In Section 2, a new risk assessment method of water quality assessment is introduced based on the Intuitionistic Fuzzy Sets -TOPSIS model. In Section 3, the Intuitionistic Fuzzy Sets-TOPSIS model is established for the water quality assessment in the Ningxia Section of the Yellow River, and the assessment results of the proposed model are discussed. In Section 4, conclusions are drawn.

Materials and Methods

Study Area

Ningxia Hui Autonomous Region is located in the east of northwest regions, in China, and upper and middle reaches of the Yellow River watershed. Its territory is narrow and long, the distance between the south and north is about 465 km, and the distance from east to west is 45-250 km^2; the total area is 6.64×10^4 km^2. The Yellow River is important to surface water resource in Ningxia Hui Autonomous Region. The total length of the Ningxia section in the Yellow River is 397 km, which accounts for 7.2% of the total length. It is winded in the Weining plain and Yinchuan plain, its entrance is located in south long beach, Zhongwei City, and the exit is located in Mahuang gully, Shizuishan city. The width of the river is 1500~1700 m. The natural total fall is 197 m, the annual average inflow of water is 317×10^9m^3, the average annual outflow of water is 294×10^9 m^3, and the average peak discharge is 3570 m^3/s; its location is plotted in Fig. 1.

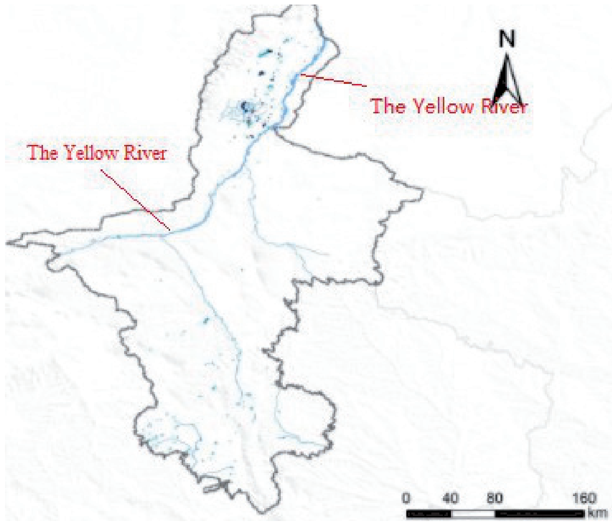

Fig. 1. The location of Ningxia section of Yellow River.

The Determination of Assessment Index about the Water Quality

The variables of water quality are effects by many factors, so the construction of relevant index systems is essential to assess the level of water quality correctly. Based on the monitoring data of water quality, in combination with relevant affirmative principles of water quality parameters, the dissolved oxygen (DO), the permanganate index (COD_{Mn}), Chemical Oxygen Demand (COD), Five-day biochemical oxygen demand (BOD_5), Ammonia nitrogen, Total phosphorus, and Fluoride are selected as the assessment index of water quality. And according to effective <<Environmental Quality Standard of surface water>>, the seven assessment indices of water quality can be classified into five levels as shown in Table 1, level I (very good), level II (good), level III (common), level IV (bad) and level IV (worse).

The Construction of the Model Frame

The level assessment of water quality has important significance for the prediction of the water quality.

Table 1.The classification standard of water quality.

Index \ Level	I	II	III	IV	V
DO	≥7.5	[6 7.5)	[5 6)	(3 5)	≤3
COD_{Mn}	≤2	(2 4]	(4 6]	(6 10]	(10 15]
COD	≤10	(10 15]	(15 20]	(20 30]	(30 40]
BOD_5	≤2	(2 3]	(3 4]	(4 6]	(6 10]
Ammonia nitrogen	≤0.15	(0.15 0.5]	(0.5 1]	(1 1.5]	(1.5 2]
Total phosphorus	≤0.20	(0.02 0.1]	(0.1 0.2]	(0.2 0.3]	(0.3 0.4]
Fluoride	≤0.25	(0.25 0.5]	(0.5 0.75]	(0.75 1]	(1 1.5]

To assess the level of water quality, a new model is suggested based on the combination of intuitionistic fuzzy sets and the TOPSIS model, its calculative frame is plotted in Fig. 2.

In Fig. 2, complete assessment index system are built up at first; then the intuitionistic fuzzy sets and TOPSIS model are respectively applied to determine the degree of membership of different indices and ideal solutions at different levels. Secondly, the decisive matrix about the water quality is determined based on the above outcomes. Thirdly, the pasting schedule can be solved according to the decisive matrix. Finally, the level of water quality can be determined based on the magnitude of the pasting schedule.

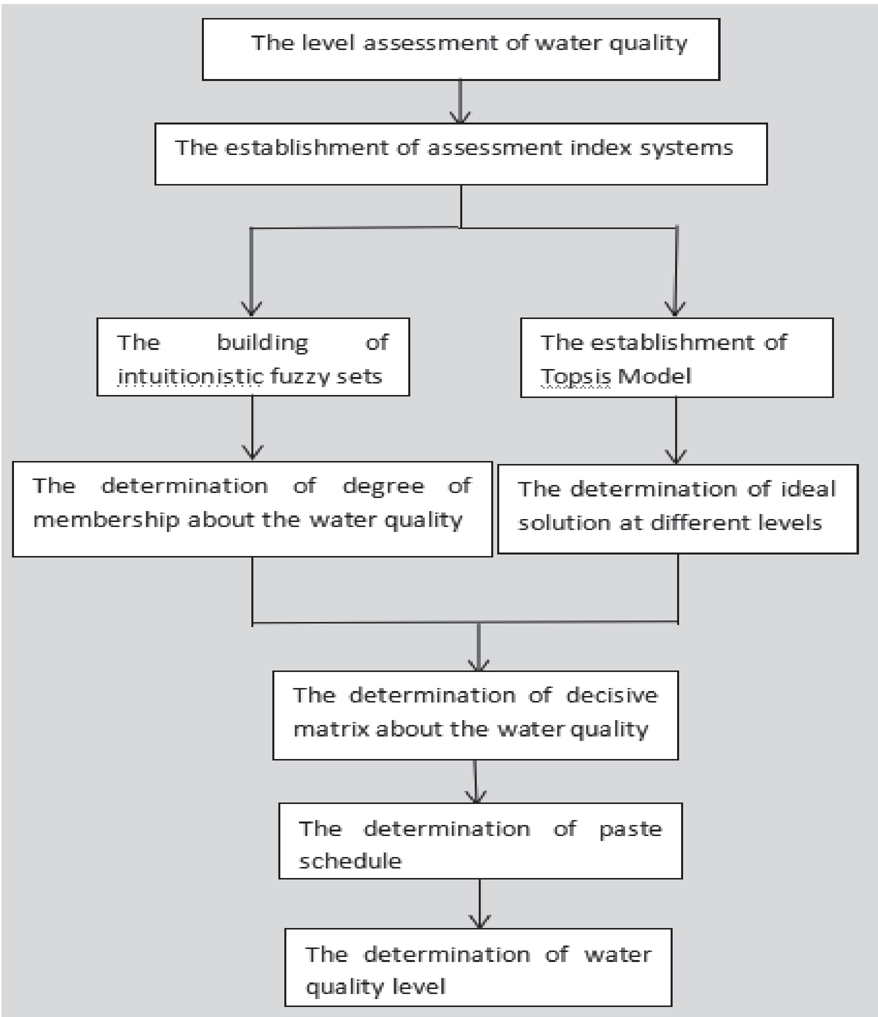

Fig. 2. The level assessment process of water quality.

The Entropy Weight Theory

(1) assuming that there are m cases of water quality and n assessment indices, so the original matrix can be expressed as:

$$X=\begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{n1} & x_{n2} & \dots & x_{nm} \end{bmatrix} \quad (1)$$

(2) normalization treatment is conducted for the main indices x_{ij}, its expressions are described as follows:

The positive indicator:

$$x_{ij}^{'}=\frac{x_{ij}-\min\{x_{ij}, \quad \dots \quad x_{nj}\}}{\max\{x_{1j}, \quad \dots \quad x_{nj}\}-\min\{x_{ij}, \quad \dots \quad x_{nj}\}} \quad (2)$$

The negative indicator:

$$x_{ij}^{'}=\frac{\min\{x_{1j}, \quad \dots \quad x_{nj}\}-x_{ij}}{\max\{x_{1j}, \quad \dots \quad x_{nj}\}-\min\{x_{ij}, \quad \dots \quad x_{nj}\}} \quad (3)$$

Where, i is assessment scheme; j is assessment index; x_{ij} is the corresponding magnitude of the *jth* assessment index in the *ith* scheme (i = 1, 2, 3 ..., m; j = 1, 2, 3 ..., n.

(3) the determination of proportion about the *jth* evaluation index in the *ith* scheme, it is expressed as:

$$\mathrm{b}_{ij}=\frac{x_{ij}}{\sum_{i=1}^{n} x_{ij}} \quad (4)$$

(4) the entropy of *jth* evaluation index is shown as follows:

$$\mathrm{s}_{j}=-k\sum_{i=1}^{n} b_{ij}\ln(b_{ij}) \quad (5)$$

(5) the weight of *jth* evaluation index is depicted as:

$$\omega_{\mathrm{j}}=\frac{1-s_{j}}{n-\sum_{j=1}^{n} s_{j}} \quad (6)$$

The Establishment of Decisive Matrix about the Intuitionistic Fuzzy Sets

The intuitionistic fuzzy sets are established based on the fuzzy sets theory. It is suggested by Atanassov [22] that the uncertainty systems can be described in a comprehensive, delicate, and flexible manner, so the intuitionistic fuzzy can be defined as follows: let x to be a nonempty set, if x is corresponding to two nonempty mappings μ_{A}:→[0,1] and ν_{A}:→[0,1], when $x \in X \rightarrow \mu_{A}(x) \in [0,1]$ and $x \in X \rightarrow \nu_{A}(x) \in [0,1]$, and they meet with the conditions $0< \mu_{A} + \nu_{A} <1$, then A is called as an intuitionistic fuzzy set of the nonempty set x which is determined by μ_{A} and ν_{A}, namely:

$$A=\{\langle x,\mu_{A}(x),\nu_{A}(x)| x\in X\rangle\} \quad (7)$$

Where, μ_{A} and ν_{A} are respectively the membership degree and non-membership degree of the element $x \in A$; if A and B are respectively intuitionistic fuzzy sets, then their product can be expressed as follows:

$$AB=\{\langle x,\mu_{A}(x)\mu_{B}(x),\nu_{A}(x)+\nu_{B}(x)-\nu_{A}(x)\nu_{B}(x)\rangle / x\in X\} \quad (8)$$

To establish the decisive matrix of fuzzy sets, it is assumed that the multiple attribute decisive sets of water quality assessment are S and attribute sets are X, namely:

$$S=\{\mathrm{s}_{1},s_{2},\dots,s_{k},\dots,s_{K}\} \quad (9)$$

$$X=\{x_{1},x_{2},\dots,x_{k},\dots,x_{K}\} \quad (10)$$

Where, the magnitudes of attributes x_{n} (n = 1, 2, ..., N) of scheme s_{k} (k = 1, 2, ..., K) are expressed as A_{nk}; the matrix of all attribute value can be recorded as $F(A_{\mathrm{nk}})_{N\times K}$.

To construct the intuitionistic fuzzy matrix, the membership degree and non-membership degree of nth attribute value x_{n} of the assessment samples of water quality in the corresponding level interval $[\underline{S_{\mathrm{k}}},\overline{S_{k}}]$ are transformed into an intuitionistic fuzzy number $A_{\mathrm{nk}} = \langle\mu_{nk}, \nu_{nk}\rangle$. The corresponding determined model about the membership and non-membership function can be depicted as:

$$\mu_{\mathrm{nk}}=\exp\left[-\frac{(x_{n}-c_{\mu k})^{2}}{2\sigma_{\mu k}^{2}}\right] \quad (11)$$

$$\nu_{\mathrm{n}k}=1-\exp\left[-\frac{(x_{n}-c_{\gamma k})^{2}}{2\sigma_{\gamma k}^{2}}\right] \quad (12)$$

$$\mathrm{c}_{uk}=c_{\gamma k}=\frac{\underline{S_{k}}+\overline{S_{k}}}{2} \quad (13)$$

$$\sigma_{\mu\mathrm{k}}^{2}=-\frac{(\overline{S_{k}}-c_{\mu k})^{2}}{2\ln\frac{1-\alpha}{2}} \quad (14)$$

$$\sigma_{\gamma\mathrm{k}}^{2}=-\frac{(\overline{S_{k}}-c_{\gamma k})^{2}}{2\ln\left(\alpha+\frac{1-\alpha}{2}\right)} \quad (15)$$

Where, $c_{\mu k}$, $c_{\gamma k}$, $\sigma_{\mu k}$, $\sigma_{\gamma k}$ are respectively calculative parameters; α is the intuitionistic fuzzy hesitating degree, it is adopted for 0.2; to eliminate the influence of the minimum $S_{\min}$ and maximum $S_{\max}$ of the index assessment standard on the assessment results, when $\underline{S_k} = S_{\min}$ is selected as boundary value, then $\overline{S_k} = S_{\max}$, $c_{\mu k} = c_{\gamma k} = \overline{S_k}$, $\sigma_{\mu k}^2 = -\frac{(\underline{S_k} - c_{\mu k})^2}{2\ln\frac{1-\alpha}{2}}$, $\sigma_{\gamma k}^2 = -\frac{(\underline{S_k} - c_{\gamma k})^2}{2\ln\left(\alpha + \frac{1-\alpha}{2}\right)}$.

Based on the intuitionistic fuzzy number $A_{nk} = \langle \mu_{nk}, \nu_{nk} \rangle$, the decisive matrix of intuitionistic fuzzy sets about the samples can be expressed as:

$$F_P = \begin{bmatrix} (\mu_{11}, \nu_{11}) & (\mu_{12}, \nu_{12}) & \dots & (\mu_{1K}, \nu_{1K}) \\ (\mu_{21}, \nu_{21}) & (\mu_{22}, \nu_{22}) & \dots & (\mu_{2K}, \nu_{2K}) \\ \dots & \dots & \dots & \dots \\ (\mu_{N1}, \nu_{N1}) & (\mu_{N2}, \nu_{N2}) & \dots & (\mu_{NK}, \nu_{NK}) \end{bmatrix} \quad (16)$$

Intuitionistic Fuzzy Sets -TOPSIS Model

Intuitionistic fuzzy sets -TOPSIS model is applied to solve the problem of priority ranking of multi-attribute decision-making objects [23]. And the risk assessment of water quality can be transformed into the probability ranking problem of different levels.

Its specific procedure is listed as follows:

(1) the influential factors of water quality are analyzed in details at first, and the assessment indices and classification standards of water quality can be established.

(2) the determination of weight coefficients.

Based on the intuitionistic fuzzy sets theory, the intuitionistic fuzzy numbers are applied to express the important degree of the index weights; and the entropy weight theory is used to solve the weight coefficients of different indices.

It is assumed that the weights of membership function of assessment indices about water quality are $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n)$, weights of non-membership function are $\beta = (\beta_1, \beta_2, ..., \beta_n)$, then its combination weights coefficients are:

$$\omega_n = \langle \chi_n, \gamma_n \rangle = \langle \min(\alpha_n, \beta_n), 1 - \max(\alpha_n, \beta_n) \rangle \quad (17)$$

Where, ω_n is the combination weights of intuitionistic fuzzy sets, χ_n, γ_n are respectively the important and non-important degree of assessment indices, and $0 \le \chi_n + \gamma_n \le 1$.

(3) based on Eqs (16) and (17), the determination of weighted decisive matrix of intuitionistic fuzzy sets about water quality can be obtained as follows:

$$\overline{F_P} = \omega_n F = \langle \chi_n \mu_{nk}, \gamma_n + \nu_{nk} - \gamma_n \nu_{nk} \rangle_{n \times k} \quad (18)$$

(4) in the evaluation systems, the corresponding plus ideal solution B+ and negative ideal solution B at different stable levels of assessment indices are calculated:

$$B^+ = \left[\langle \mu_1^+, \nu_1^+ \rangle, \langle \mu_2^+, \nu_2^+ \rangle, \dots \langle \mu_n^+, \nu_n^+ \rangle \right]$$
$$B^- = \left[\langle \mu_1^-, \nu_1^- \rangle, \langle \mu_2^-, \nu_2^- \rangle, \dots \langle \mu_n^-, \nu_n^- \rangle \right] \quad (19)$$

Where,
$\mu_n^+ = \max_{1\le k\le n}(\overline{\mu_{nk}}); \nu_n^+ = \max_{1\le k\le n}(\overline{\nu_{nk}}); \mu_n^- = \min_{1\le k\le n}(\overline{\mu_{nk}}); \nu_n^- = \max_{1\le k\le n}(\overline{\nu_{nk}}); n = 1,2,...,n$

(5) The determination of degree of membership about the water quality.

The Euclidean distance of plus and negative ideal solutions are calculated as follows:

$$D(s_k, B^+) = \sqrt{\frac{1}{2}\sum_{n=1}^{N}\left[(\overline{\mu_{nk}} - \mu_n^+)^2 + (\overline{\nu_{nk}} - \nu_n^+)^2 + (\mu_n^+ + \nu_n^+ - \overline{\mu_{nk}} - \overline{\nu_{nk}})^2 \right]} \quad (20)$$

$$D(s_k, B^-) = \sqrt{\frac{1}{2}\sum_{n=1}^{N}\left[(\overline{\mu_{nk}} - \mu_n^-)^2 + (\overline{\nu_{nk}} - \nu_{nk}^-)^2 + (\mu_n^- + \nu_n^- - \overline{\mu_{nk}} - \overline{\nu_{nk}})^2 \right]} \quad (21)$$

$$\eta_k = \frac{D^2(s_k, B^-)}{D^2(s_k, B^-) + D^2(s_k, B^+)} \quad (22)$$

Where, $D(s_k, B^+)$ and $D(s_k, B^-)$ are respectively the Euclidean distance of plus and negative ideal solutions of intuitionistic fuzzy sets at scheme S_k; η_k is the degree of membership at scheme S_k .

(6) The levels of water quality can be determined based on the magnitudes of pasting schedule.

Once the degrees of membership about the water quality are determined using the Eqs (20)-(22), their magnitudes are ranked from big to small, finally, the maximum degree of membership is selected as the assessment levels of corresponding level to water quality.

Results and Discussion

The Level Assessment Model about the Water Quality

According to the field investigations, the monitoring datum of water quality at four cross- sections in the Ningxia section of the Yellow River is selected as the assessment contents. The location of the four cross-sections is shown in Table 2.

The monitoring datum of water quality in the Ningxia section of Yellow River in 2012 is selected as study object, to investigate the variations of water quality in one year, the monitoring datum in 2012 is shown in Table 3.

Based on Table 1, according to Eqs (11)-(15), the parameters of membership and non-membership

Table 2. The locations of monitoring cross section.

Cross section name	Length (km)	Coordinates	Notes
Yesheng road bridge H_1	78	106⁰12’98”,38⁰4’44.7”	State-controlled
Yingu road bridge H_2	78	106⁰24’40.2”,38⁰21’31.8”	Area-controlled
Pingluo Yellow River bridge H_3	78	106⁰39’59”,38⁰48’44.7”	State-controlled
Mahuang gully H_4	96	106⁰47’23.2”,39⁰20’59.9”	State-controlled

Table 3.The monitoring datum in the flood season.

Cross section number	DO	COD_{Mn}	COD	BOD_5	Ammonia nitrogen	Total phosphorus	Fluoride
H_1	7.317	4.458	21.375	1.708	0.278	0.113	0.283
H_2	7.167	4.425	23.808	2.192	0.328	0.121	0.317
H_3	6.938	5.738	32.363	3.256	0.715	0.144	0.456
H_4	7.217	4.958	26.725	2.642	0.385	0.143	0.348

Table 4 The parameters of membership and non-membership function.

Level / Index	I	II	III	IV	V
DO	$c_{\mu1}=c_{\gamma1}=8.25$ $\sigma^2_{\mu1}=0.3069$ $\sigma^2_{\gamma1}=0.5506$	$c_{\mu2}=c_{\gamma2}=6.75$ $\sigma^2_{\mu2}=0.3069$ $\sigma^2_{\gamma2}=0.5506$	$c_{\mu3}=c_{\gamma3}=5.5$ $\sigma^2_{\mu3}=0.1364$ $\sigma^2_{\gamma3}=0.2447$	$c_{\mu4}=c_{\gamma4}=4$ $\sigma^2_{\mu4}=0.5457$ $\sigma^2_{\gamma4}=0.9788$	$c_{\mu5}=c_{\gamma5}=1.5$ $\sigma^2_{\mu5}=1.2278$ $\sigma^2_{\gamma5}=2.2023$
COD_{Mn}	$c_{\mu1}=c_{\gamma1}=1$ $\sigma^2_{\mu1}=0.5457$ $\sigma^2_{\gamma1}=0.9788$	$c_{\mu2}=c_{\gamma2}=3$ $\sigma^2_{\mu2}=0.5457$ $\sigma^2_{\gamma2}=0.9788$	$c_{\mu3}=c_{\gamma3}=5$ $\sigma^2_{\mu3}=0.5457$ $\sigma^2_{\gamma3}=0.9788$	$c_{\mu4}=c_{\gamma4}=8$ $\sigma^2_{\mu4}=2.1827$ $\sigma^2_{\gamma4}=3.9152$	$c_{\mu5}=c_{\gamma5}=12.5$ $\sigma^2_{\mu5}=3.4105$ $\sigma^2_{\gamma5}=6.1175$
COD	$c_{\mu1}=c_{\gamma1}=5$ $\sigma^2_{\mu1}=13.642$ $\sigma^2_{\gamma1}=24.4702$	$c_{\mu2}=c_{\gamma2}=6.75$ $\sigma^2_{\mu2}=0.3069$ $\sigma^2_{\gamma2}=6.1175$	$c_{\mu3}=c_{\gamma3}=17.5$ $\sigma^2_{\mu3}=3.4105$ $\sigma^2_{\gamma3}=6.1175$	$c_{\mu4}=c_{\gamma4}=25$ $\sigma^2_{\mu4}=13.642$ $\sigma^2_{\gamma4}=24.4702$	$c_{\mu5}=c_{\gamma5}=35$ $\sigma^2_{\mu5}=13.642$ $\sigma^2_{\gamma5}=24.4702$
BOD_5	$c_{\mu1}=c_{\gamma1}=1$ $\sigma^2_{\mu1}=0.5457$ $\sigma^2_{\gamma1}=0.9788$	$c_{\mu2}=c_{\gamma2}=2.5$ $\sigma^2_{\mu2}=0.1364$ $\sigma^2_{\gamma2}=0.2447$	$c_{\mu3}=c_{\gamma3}=3.5$ $\sigma^2_{\mu3}=0.1364$ $\sigma^2_{\gamma3}=0.2447$	$c_{\mu4}=c_{\gamma4}=5$ $\sigma^2_{\mu4}=0.5457$ $\sigma^2_{\gamma4}=0.9788$	$c_{\mu5}=c_{\gamma5}=8$ $\sigma^2_{\mu5}=2.1827$ $\sigma^2_{\gamma5}=3.9152$
Ammonia nitrogen	$c_{\mu1}=c_{\gamma1}=0.0075$ $\sigma^2_{\mu1}=0.0031$ $\sigma^2_{\gamma1}=0.0055$	$c_{\mu2}=c_{\gamma2}=0.325$ $\sigma^2_{\mu2}=0.0167$ $\sigma^2_{\gamma2}=0.03$	$c_{\mu3}=c_{\gamma3}=0.75$ $\sigma^2_{\mu3}=0.0341$ $\sigma^2_{\gamma3}=0.0612$	$c_{\mu4}=c_{\gamma4}=1.25$ $\sigma^2_{\mu4}=0.0341$ $\sigma^2_{\gamma4}=0.0612$	$c_{\mu5}=c_{\gamma5}=1.75$ $\sigma^2_{\mu5}=0.0341$ $\sigma^2_{\gamma5}=0.0612$
Total phosphorus	$c_{\mu1}=c_{\gamma1}=0.01$ $\sigma^2_{\mu1}=5.46\times10^{-5}$ $\sigma^2_{\gamma1}=9.79\times10^{-5}$	$c_{\mu2}=c_{\gamma2}=0.06$ $\sigma^2_{\mu2}=8.731\times10^{-4}$ $\sigma^2_{\gamma2}=0.0016$	$c_{\mu3}=c_{\gamma3}=0.15$ $\sigma^2_{\mu3}=0.0014$ $\sigma^2_{\gamma3}=0.0024$	$c_{\mu4}=c_{\gamma4}=0.25$ $\sigma^2_{\mu4}=0.0014$ $\sigma^2_{\gamma4}=0.0024$	$c_{\mu5}=c_{\gamma5}=0.35$ $\sigma^2_{\mu5}=0.0014$ $\sigma^2_{\gamma5}=0.0024$
Fluoride	$c_{\mu1}=c_{\gamma1}=0.125$ $\sigma^2_{\mu1}=0.0085$ $\sigma^2_{\gamma1}=0.0153$	$c_{\mu2}=c_{\gamma2}=0.375$ $\sigma^2_{\mu2}=0.0085$ $\sigma^2_{\gamma2}=0.0153$	$c_{\mu3}=c_{\gamma3}=0.625$ $\sigma^2_{\mu3}=0.0085$ $\sigma^2_{\gamma3}=0.0153$	$c_{\mu4}=c_{\gamma4}=0.875$ $\sigma^2_{\mu4}=0.0085$ $\sigma^2_{\gamma4}=0.0153$	$c_{\mu5}=c_{\gamma5}=1.25$ $\sigma^2_{\mu5}=0.0314$ $\sigma^2_{\gamma5}=0.0612$

function of the intuitionistic fuzzy sets can be shown in Table 4.

To reflect the characteristics of intuitionistic fuzzy sets, the membership function and non-membership function are respectively plotted in Figs 3 and 4 as follows:

According to Eqs (7) and (8), in combination with Figures 3 and 4, the H_1 cross-section in the flood season is selected as an example to assess, based on Eq. (16), the decisive matrix F of intuitionistic fuzzy sets about H_1 cross-section can be depicted as follows:

$$F=\begin{bmatrix} (0.2422,0.5464) & (0.5923,0.2532) & (0,0.9988) & (0,0.9964) & (0,0.9995) \\ (0,0.9978) & (0.1426,0.6624) & (0.764,0.1393) & (0.0565,0.7985) & (0,0.9949) \\ (0,0.9958) & (0,0.9984) & (0.1107,0.7069) & (0.6178,0.2335) & (0.0011,0.9775) \\ (0.6317,0.225) & (0.1003,0.7224) & (0,0.9986) & (0,0.9961) & (0.0001,0.9938) \\ (0.0013,0.9764) & (0.936,0.0361) & (0.0381,0.838) & (0,0.9996) & (0,1) \\ (0,1) & (0.2002,0.5843) & (0.6133,0.2481) & (0.0012,0.98) & (0,1) \\ (0.2303,0.5577) & (0.6078,0.2416) & (0.001,0.9781) & (0,1) & (0,0.9995) \end{bmatrix}$$

According to Eq. (17), the intuitionistic weights can be obtained as follows according to the intuitionistic fuzzy numbers.

$$\omega=\begin{bmatrix}(0.1281,0.8769) & (0.1243,0.8377) & (0.1142,0.8507) & (0.1164,0.8461) & (0.1826,0.7611)\\ (0.1196,0.8674) & (0.1256,0.771) & & & \end{bmatrix}$$

Constituting the expression of matrix F and ω into Eq. (18), the weighted intuitionistic fuzzy sets can be expressed as follows:

$$F_1=\begin{bmatrix}(0.031,0.9443) & (0.0758,0.9093) & (0,0.9999) & (0,0.9996) & (0,0.9999)\\ (0,0.9996) & (0.0177,0.9192) & (0.095,0.8603) & (0.007,0.9673) & (0,0.9992)\\ (0,0.9994) & (0,0.9998) & (0.0126,0.9562) & (0.0706,0.8856) & (0.0001,0.9966)\\ (0.0735,0.8807) & (0.0177,0.9573) & (0,0.9998) & (0,0.9994) & (0,0.999)\\ (0.0002,0.9944) & (0.1709,0.7697) & (0.007,0.9613) & (0,0.9999) & (0,1)\\ (0,1) & (0.0239,0.9449) & (0.0001,0.9974) & (0.0001,0.9973) & (0,1)\\ (0.0289,0.8987) & (0.0763,0.8263) & (0,1) & (0,1) & (0,0.9999)\end{bmatrix}$$

Based on the Eq. (19), the minus and plus ideal solutions of corresponding levels in H_1 cross section can be expressed as follows:

$$B^{+}=[(0.0735,0.8807)\ \ (0.1709,0.7697)\ \ (0.095,0.8603)\ \ (0.0706,0.8856)\ \ (0.0001,0.9966)]$$

$$B^{-}=[(0,1)\ \ (0,0.9998)\ \ (0,1)\ \ (0,1)\ \ (0,1)]$$

According to Eqs (20)-(22), the Euclidean distance and degree of membership of minus and plus ideal solutions corresponding to the intuitionistic fuzzy sets of different levels for H_1 cross-section can be obtained as follows:

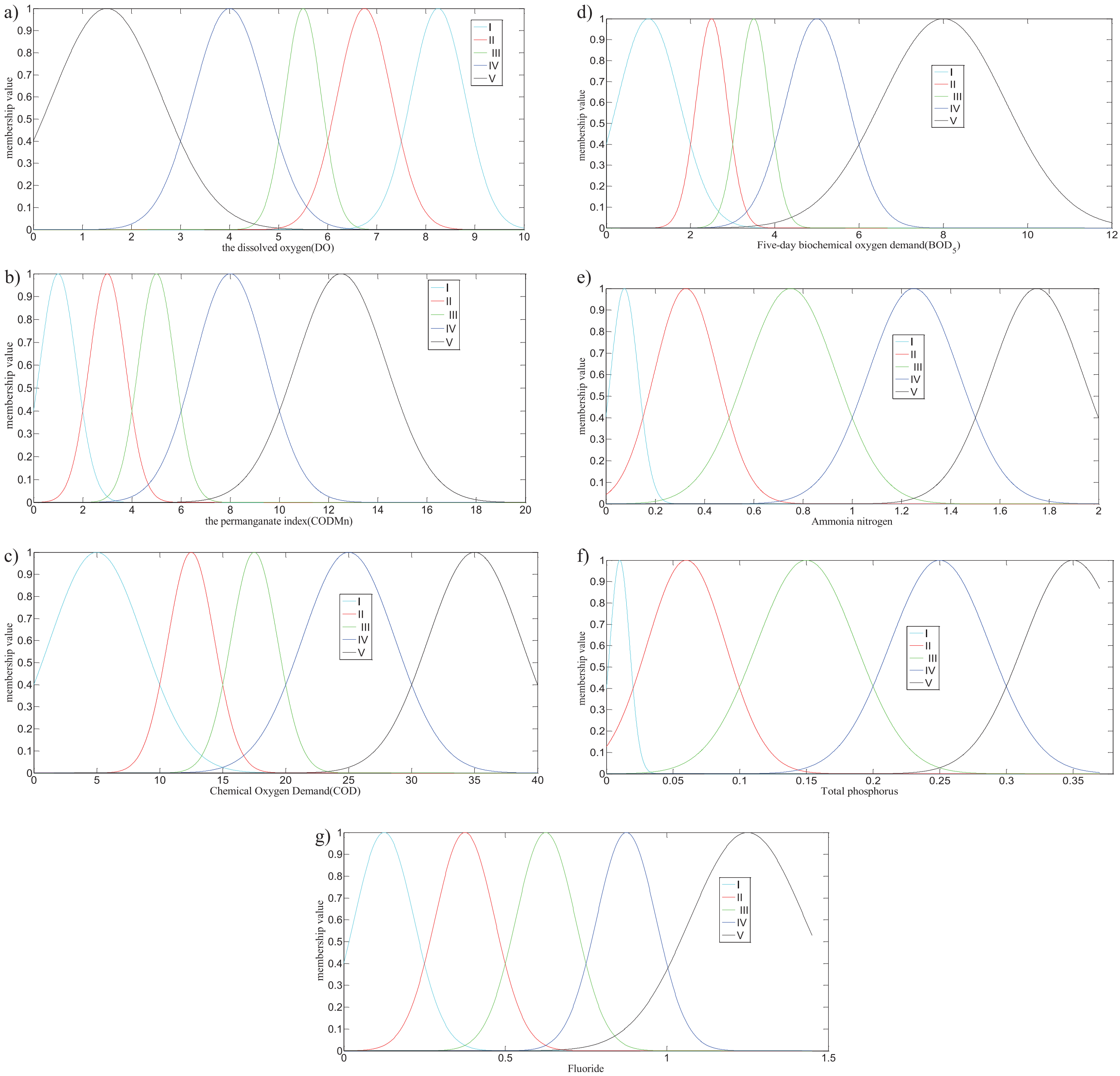

Fig. 3. The membership function of different intuitionistic fuzzy sets: a) the dissolved oxygen (DO), b)the permanganate index (COD_{Mn}), c) Chemical Oxygen Demand (COD), d) Five-day biochemical oxygen demand (BOD_5), e) Ammonia nitrogen, f) Total phosphorus, g) Fluoride.

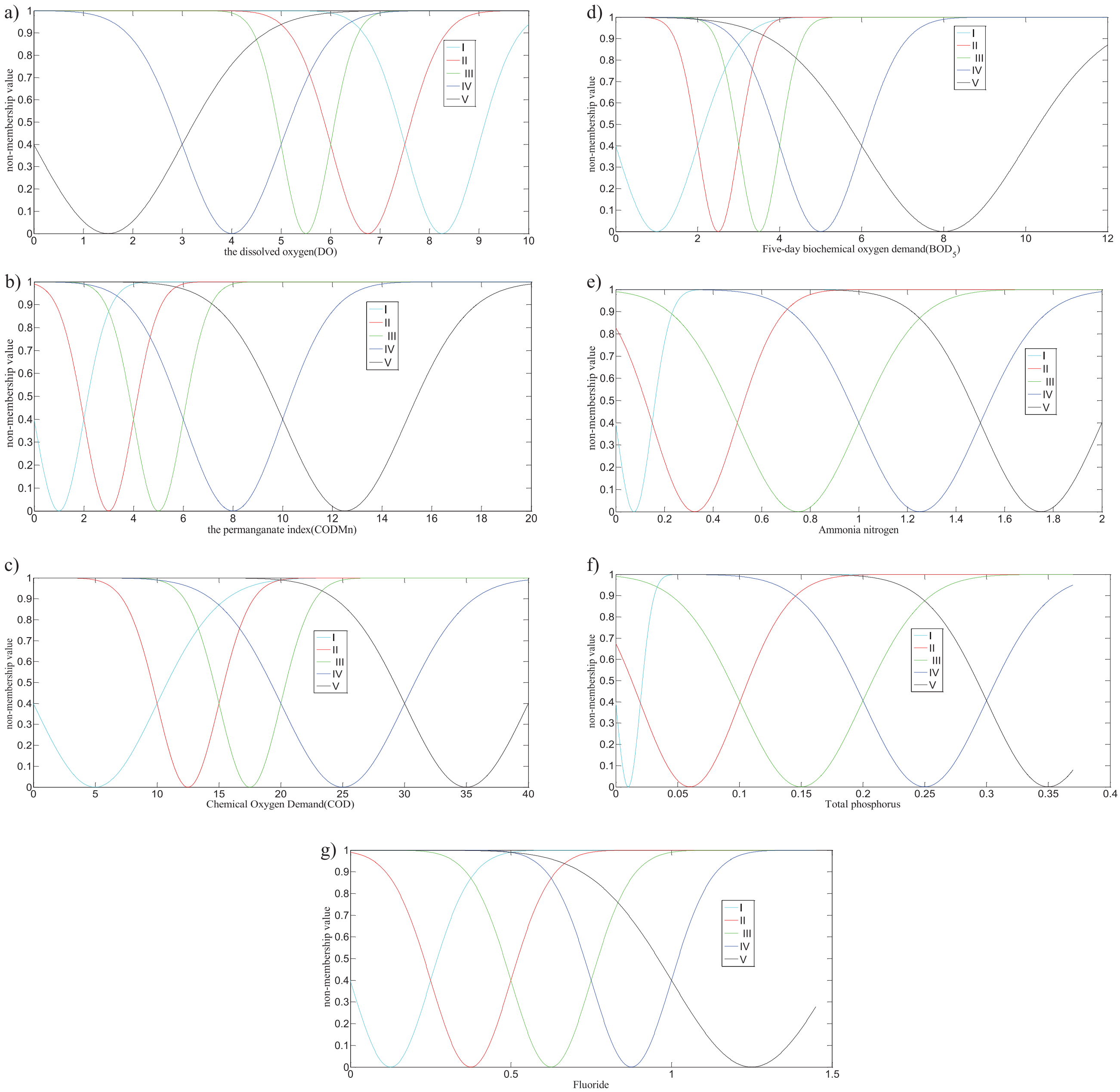

Fig. 4. The non-membership function of different intuitionistic fuzzy sets: a) the dissolved oxygen (DO), b) the permanganate index (COD_{Mn}), c) Chemical Oxygen Demand (COD), d) Five-day biochemical oxygen demand (BOD_5), e) Ammonia nitrogen, f) Total phosphorus, g) Fluoride.

$$D(t_1, B^+) = 0.2235,\ D(t_1, B^-) = 0.1463,\ \eta_1 = 0.2999$$
$$D(t_2, B^+) = 0.3798,\ D(t_2, B^-) = 0.2846,\ \eta_2 = 0.3598$$
$$D(t_3, B^+) = 0.2776,\ D(t_2, B^-) = 0.1345,\ \eta_3 = 0.1902$$
$$D(t_4, B^+) = 0.2344,\ D(t_2, B^-) = 0.1044,\ \eta_4 = 0.1591$$
$$D(t_5, B^+) = 0.0075,\ D(t_5, B^-) = 0.0036,\ \eta_5 = 0.1881$$

Similar to H_1 cross-section, the Euclidean distance and degree of membership of minus and plus ideal solutions corresponding to the intuitionistic fuzzy sets of different levels for H_2, H_3 and H_4 cross-sections are respectively in Table 5.

It can be found in Table 5 that the water quality in the Ningxia section of the Yellow River can be generally divided into five levels from low to high. The final quality level of H_1 and H_2 cross-sections are both II. One of the H_3 and H_4 cross-sections are both III. It means that the water quality of H_1 and H_2 cross-sections are good, and the water quality of H_3 and H_4 cross-sections are both common, so no measures need to be taken for H_1, H_2,H_3, and H_4 cross sections in 2013. These conclusions demonstrate that the water quality in the Ningxia Section of the Yellow River is good. But the necessary measure should be taken to prevent the deterioration of water quality, especially for H_3 and H_4 cross-sections.

Table 5. The assessment of water quality and comparison.

Cross section number	The assessment level					The text method	Current specifications	Cloud model
	I	II	III	IV	V			
H_1	0.2999	0.3598	0.1902	0.1591	0.1881	II	II	II
H_2	0.3695	0.4015	0.2873	0.1491	0.1538	II	II	III
H_3	0.2055	0.2472	0.4551	0.4214	0.1459	III	III	III
H_4	0.3033	0.3948	0.5253	0.1486	0.0345	III	III	III

According to the comparative analysis of the assessment results in Table 5, the results assessed by using three different methods are consistent for 4 different cross-sections. Its accuracy arrives at 100% in the text method, which is higher than the results from the cloud model [24] So the conclusions are drawn that it is feasible to estimate the level of water quality by using the intuitionistic fuzzy sets -TOPSIS model. The model not only achieves accurate results but also provides more details about the assessment of water quality. For example, the COD of the H_2 cross-section is 22.808, which should belong to level IV according to Table 1. In addition, the degree of membership of the other indices obtained by the TOPSIS model belongs to level II, and the level probability of water quality in the H_2 cross-section at the levels II is bigger than that of level I, IV, III, and V. So it only belongs to level II and almost impossibly belongs to levels I, IV, III and V. The conclusion is consistent with current specifications. Furthermore, the level of water quality at the H_2 cross-section is more likely to be level II than that of H_1, H_3, and H_4 cross- sections, because the maximum degree of membership of water quality at the H_2 cross- section for level II (0.4015) is higher than that of H_1 (0.3598), H_3 (0.2472), and H_4(0.3948). In total, the results based on the intuitionistic fuzzy sets -TOPSIS model not only reflect the water quality level accurately but also further determine the level ranking of water quality for different cross-sections at the same level.

Discussion

By comparing this approach with the conventional cloud model, and actual investigation, the advantages of the suggested method can be summarized as follows:

(1) Their judgments under inherent uncertainty in the proposed model can be conveyed. More significantly, the degree of indeterminacy can be handled adequately in the evaluation.

(2) In comparison with the traditional fuzzy mathematical method, the proposed model has sufficient usage of original datum, minor information loss, and wider application. And it can precisely determine which indexes require more to be supported.

(3) Relatively to the traditional assessment method, the proposed method not only can deal with vague information but also ease our workload, and the efficiency and accuracy can be improved.

Conclusions

Considering the dissolved oxygen (DO), the permanganate index (COD_{Mn}), Chemical Oxygen Demand (COD), Five-day biochemical oxygen demand (BOD_5), Ammonia nitrogen, Total phosphorus, as well as fluoride, a new evaluation method is introduced in this paper to assess the level of water quality in Ningxia Section of the Yellow River using the Intuitionistic Fuzzy Sets-TOPSIS model. The decisive matrix about the water quality is established at first. Then the combination weighting coefficients of the different indices are obtained by using the entropy weighting method. Finally, the level of water quality at different cross-sections is determined by using the degree of membership.

The proposed method is applied to assess the level of water quality in the Ningxia Section of the Yellow River. Finally, its result is compared with that of the current specifications and cloud model, and it is found that by three various methods are almost the same; its accuracy is 100%. The final quality level of H_1 and H_2 cross-sections are both II. One of the H_3 and H_4 cross-sections are both III. It means that the water quality of H_1 and H_2 cross-sections are good, and the water quality of H_3 and H_4 cross-sections are both common, so no measures need to be taken in the Ningxia section of the Yellow River in 2013. In all, the results of the proposed Intuitionistic Fuzzy Sets-TOPSIS model are highly consistent with the current specifications, it not only reflects the water quality level accurately but also further determines the level ranking of water quality for different cross- sections at the same level. Besides, it can provide a new method and thoughts for the level assessment of water quality in the future.

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