

Original Research

Fuzzy Correlation Evaluation of Forest Ecological Carrying Capacity Considering the Interaction between Attributes

Jing Lu¹, Jian Lin^{1,2*}, Ying Zhao¹, Yan Huang³

¹College of Computer and Information Sciences, Fujian Agriculture and Forestry University, Fuzhou, Fujian, 350002, China

²Business School, Sichuan University, Chengdu, Sichuan, 610064, China

³College of Economics and Management, Fujian Agriculture and Forestry University, Fuzhou, Fujian, 350002, China

Received: 3 November 2022

Accepted: 2 February 2023

Abstract

Forest ecological carrying capacity can measure the balance between environmental pollution and forest ecosystems in the process of social development. This paper first selects 17-year panel data of five provinces in southeastern China, and uses the generalized trapezoidal fuzzy number to describe and express them based on the flat peak characteristic of the data itself; Secondly, the generalized Shapley value with the introduction of λ -fuzzy measure is used to measure the correlation between attribute indicators, and the indicator weight is determined by the attributes contribution; Then, an optimal fuzzy measure linear programming model is established based on the similarity, which is used to determine the generalized Shapley value of each attribute indicator. Finally, the λ -Shapley-Choquet integral operator is used for information aggregation and the centroid method is used to rank the comprehensive evaluation values. The results show that the forest ecological carrying capacity of five provinces during 2004-2020 is ranked from high to low as Fujian Province, Jiangxi Province, Zhejiang Province, Guangxi Province, and Guangdong Province. Based on this, it is proposed that strengthening the forest's ecological carrying capacity in southeastern China should be carried out in parallel from two aspects: enriching forest resources and reducing environmental pollution.

Keywords: ecological carrying capacity, generalized trapezoidal fuzzy information, resource and environmental pressure, correlation evaluation

Introduction

Rapid economic development will inevitably pollute the resources and environment, and affect the stability of the ecological environment. When the resource and environmental pressure brought by human society exceeds its tolerable threshold, the forest ecosystem loses its automatic adjustment function, resulting in an imbalance of the entire ecosystem, which in turn hinders economic and social development. Economic development cannot be at the expense of the ecological environment. How to achieve a balance between economic development and ecological environment has become an important issue in environmental protection. In 1987, the World Commission on Environment and Development published a report entitled "Our Common Future", which proposed the strategic idea of "sustainable development". To achieve economic development while preserving the natural resources and the environment, such as forests, oceans, fresh water, atmosphere and land, on which human beings depend for their survival, so that future generations can develop sustainably [1].

As the largest ecosystem on land, the forest not only has a complete and powerful biological chain and ecological functions, but also provides continuous driving force for human socio-economic development. Tree leaves have the ability to purify air pollution, at the same time, tree roots can also maintain water and soil, prevent wind and fix sand. Forests have a certain absorption and purification effect on environmental pollution [2]. At present, human society is committed to the rapid development of the economy and technology, but it is also faced with many contradictions such as resources, ecological environment, and social and economic development. The deterioration of the ecological environment and the over-exploitation of resources should arouse our great attention. If the economic and social development is carried out at the cost of environmental damage, the forest regulation ability will be lost, which will have a serious impact on the balance of the ecosystem, thereby hindering the sustainable development of the economy and society, and greatly affecting the balance of the ecological environment. Forest's ecological carrying capacity refers to the ability of forest ecosystems in a certain period to carry social development and other influencing factors in the development process [3]. As an important part of sustainable forestry theory, the forest ecology carrying capacity is an essential indicator for evaluating the sustainable development of regional forests [4].

In the evaluation of the regional forest's ecological carrying capacity, Li Yan [5] calculated the index of forest's ecological carrying capacity based on the PSR model, and studied the evolution law of the trajectory of the center of gravity of forest's ecological carrying capacity in Anhui Province from 2001 to 2016; Liao [3] used the entropy weight TOPSIS method to study the evolution law and dynamic relationship of forest's

ecological carrying rate; Jiang et al. [6] used the literature retrieval method, Delphi method, principal component analysis method, and AHP to construct the evaluation index system of forest ecological carrying capacity; Tang [2] proposed a framework and method for constructing a forest resource carrying capacity indicator, the ideal forest ecological security indicator was simulated by the forest ecological location coefficient, and using the difference between it and the forest ecological security index as the forest resource carrying capacity index for the cross-sectional data of 1086 counties constituting the Yangtze River Economic Belt in China in 2015; Mi Yun et al. [7] measured the current situation of environmental pollution through the entropy weight TOPSIS method, and analyzed the temporal and spatial distribution of forest ecological carrying capacity in Shaanxi Province from 2008 to 2017; Cui et al. [8] used the entropy weight TOPSIS method to calculate the forest ecological carrying capacity and environmental pollution pressure index, then studied the evolution law of the forest ecological carrying rate; Song [9] put forward the forest environmental carrying capacity evaluation system from many aspects, and used the proposed depth learning model to comprehensively evaluate and predict the forest environmental carrying capacity of 40 cities in the Yangtze River Delta region of China; Moukhtar [10] quantified the environmental carrying capacity of the petrified forest protectorate in eastern Greater Cairo from three levels: physical carrying capacity(PCC), real carrying capacity(RCC) and effective carrying capacity(ECC); Zhu and Zhang [11] used the entropy weight method to determine the indicator weight, and analyzed the change of forest ecological carrying capacity in Changbai County, Jilin Province. Liao [12] developed a "direction-speed-pattern" tri-dimensional framework, and assessed ecological carrying capacity(ECC) by incorporating the resource provision capacity and environmental support capacity. Wu [13] construct a comprehensive evaluation index system of the resource and environment carrying capacity (RECC), and analyzed the factors influencing the RECC, the overall level, the spatial difference, and the carrying status by using the TOPSIS model based on the entropy weight method.

In general, the existing research literature combines the entropy weight method with the TOPSIS model to construct the entropy weight TOPSIS model to measure the current situation of environmental pollution, and then evaluate the forest's ecological carrying capacity. Although these methods can reflect the importance of each indicator and reflect the dynamic evolution trend of the evaluation system, they do not take into account the interactions between the indicators.

At present, the traditional multi-attribute decision-making and evaluation methods have been studied and discussed by many scholars, and the related theories and expanded concepts tend to be perfected, but there is still a big drawback: the traditional multi-attribute decision-

making and evaluation methods assume that the importance of evaluation indicators is independent of each other, which essentially corresponds to an additive measure [14-17]. Nevertheless, the assumption that the importance of evaluation indicators is independent of each other does not always hold. Due to the possible interaction between indicators, the combination of different indicators will produce different effects [18]. The additive measure only considers the individual weights of the evaluation indicators, and does not consider the combined weights between different indicators, so the traditional multi-attribute decision-making and evaluation methods will be limited in practical applications. Therefore, this paper uses a new method that can consider the interaction between different indicators for the evaluation of forest ecological carrying capacity. The generalized trapezoidal fuzzy number and its related concepts will be introduced first, and the parameter estimation method of the generalized trapezoidal fuzzy number is defined; 12 evaluation indicators were selected to construct the indicator evaluation system of forest ecological carrying capacity, and the generalized trapezoidal fuzzy number was used to describe the initial data, and then normalized to eliminate the influence of dimension; To measure the degree of the interrelationship among attribute indicators, the generalized Shapley value with λ -fuzzy measure is used, and the optimal fuzzy measure linear programming model is established based on similarity, the λ -Shapley-Choquet integral operator is used for information aggregation, and the centroid method is used to rank the comprehensive evaluation values.

Data and Methods

Expression Form of Data

At present, most studies on the evaluation of forest ecological carrying capacity are based on the calculation of the ratio of forest ecological carrying capacity to environmental pollution pressure, to calculate the forest ecological carrying capacity, which is expressed in real values. The method is usually to carry out year-by-year analysis or use ArcGIS software for visualization processing, few of them only use numerical values to completely express the data of forest ecological carrying capacity in a certain place [7]. Most of the analysis methods are to make the load rate data over the years into a change trend chart and perform descriptive analysis through the change of the trend chart. The analysis results rely on the change of the line chart, which has certain objectivity limitations. If one wants to analyze the forest ecological carrying capacity of a certain area over the years, although year-by-year analysis can ensure accuracy, this method is not suitable for large-scale data. With the increase in the number of years, the year-by-year analysis method becomes very complicated and has high repeatability;

if the mean and mode of the data over the years are used, the accuracy of the data will be greatly reduced and the information expression will be distorted. With the gradual improvement of the uncertainty theory, the research results of fuzzy numbers such as probabilistic linguistic fuzzy numbers, Pythagorean fuzzy numbers, intuitionistic fuzzy numbers, dual fuzzy numbers and so on are getting better and better [19-21]. In terms of big data description, some authors use interval numbers to describe the data over the years, but interval fuzzy numbers only describe the situation of two endpoints, without considering that different data have different distribution characteristics in the interval, and can't reflect the situation of data in the interval, and are vulnerable to the impact of outliers. Some studies use triangular fuzzy numbers to describe the data over the years. However, due to the limitation of the membership function of triangular fuzzy numbers, the most probable value is only one point. For a series of parameter sets with flat overall distribution and no obvious peak value that widely exists in practice, triangular fuzzy numbers will bring large simulation errors, and even affect the final decision results. Based on inheriting the advantages of triangular fuzzy numbers, generalized trapezoidal fuzzy numbers represent the most probable values in the form of intervals, which can better fit the parameters with relatively wide peak values, make up for the shortcomings of triangular fuzzy simulation, and have greater advantages in data representation. At present, no literature uses generalized trapezoidal fuzzy numbers to represent the forest ecological carrying capacity data of a region over the years. According to the characteristics of the data itself, this paper combines the two and applies the form of fuzzy number to the information expression of forest ecological carrying capacity, which can avoid data redundancy and express the original information of data as much as possible. The generalized trapezoidal fuzzy number contains the maximum, minimum, most probable interval, and the probability of the most probable interval of the data, which can describe the data more completely. Therefore, this paper analyzes the forest's ecological carrying capacity based on the generalized trapezoidal fuzzy number.

The Definition and Operations of Generalized Trapezoidal Fuzzy Number

Fuzzy numbers are widely used in multi-attribute decision-making and evaluation research. Among them, generalized trapezoidal fuzzy numbers are often used to deal with fuzzy multi-attribute evaluation problems because it can express the initial information more completely [22]. The generalized trapezoidal fuzzy number \tilde{A} is a fuzzy set defined on real numbers which is expressed as $\tilde{A} = (a, b, c, d; \omega)$, where $0 < \omega \leq 1$, a, b, c, d are all real numbers. If its membership function $\mu_{\tilde{A}}(x)$ is expressed as

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} \cdot \omega, x \in [a, b) \\ \omega, x \in [b, c] \\ \frac{d-x}{d-c} \cdot \omega, x \in (c, d] \\ 0, \text{otherwise} \end{cases} \tag{1}$$

Then the fuzzy number \tilde{A} is a generalized trapezoidal fuzzy number, where ω is the peak value of the generalized trapezoidal fuzzy number \tilde{A} , if $\omega = 1$, then \tilde{A} is a normalized trapezoidal fuzzy number, denoted as $\tilde{A} = (a, b, c, d)$; if $a = b$ and $c = d$, then \tilde{A} is a generalized interval fuzzy numbers; if $b = c$, then \tilde{A} is a generalized triangular fuzzy numbers, and when $b = c = d$, \tilde{A} is called a generalized left triangular fuzzy number, when $a = b = c = d$ and $\omega = 1$, generalized trapezoidal fuzzy number \tilde{A} degenerates into a real number.

According to the extension principle proposed by Zadeh, the operations of generalized trapezoidal fuzzy numbers is defined as follows [23].

Let two generalized trapezoidal fuzzy numbers $\tilde{A}_1 = (a_1, b_1, c_1, d_1; \omega_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2; \omega_2)$, that is

$$(1) \tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \min(\omega_1, \omega_2));$$

$$(2) \tilde{A}_1 \otimes \tilde{A}_2 = (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2, d_1 \times d_2; \min(\omega_1, \omega_2));$$

$$(3) \lambda \tilde{A}_1 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1; \omega), \lambda \geq 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1; \omega), \lambda < 0 \end{cases}$$

$$(4) (\tilde{A}_1)^\xi = (a_1^\xi, b_1^\xi, c_1^\xi, d_1^\xi; \min(\omega_1, \omega_2)), \xi \text{ is an arbitrary constant.}$$

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1; \omega_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2; \omega_2)$ denote two generalized trapezoidal fuzzy numbers, then their similarity $Sim(\tilde{A}_1, \tilde{A}_2)$ is defined as follows [24]:

$$Sim(\tilde{A}_1, \tilde{A}_2) = (1 - \frac{1}{4}((a_1 + b_1 + c_1 + d_1) - (a_2 + b_2 + c_2 + d_2))) \times (1 - |x_{\tilde{A}_1}^* - x_{\tilde{A}_2}^*|)^{Z(W_{\tilde{A}_1}, W_{\tilde{A}_2})} \times \frac{\min(y_{\tilde{A}_1}^*, y_{\tilde{A}_2}^*)}{\max(y_{\tilde{A}_1}^*, y_{\tilde{A}_2}^*)} \tag{2}$$

where

$$W_{\tilde{A}_1} = d_1 - a_1, W_{\tilde{A}_2} = d_2 - a_2$$

$$Z(W_{\tilde{A}_1}, W_{\tilde{A}_2}) = \begin{cases} 1, & \text{if } W_{\tilde{A}_1} + W_{\tilde{A}_2} > 0 \\ 0, & \text{if } W_{\tilde{A}_1} + W_{\tilde{A}_2} = 0 \end{cases}$$

$$y_{\tilde{A}_i}^* = \begin{cases} \frac{(c_i - b_i + 2) \cdot \omega_i}{d_i - a_i}, & \text{if } a_i \neq d_i \\ \frac{\omega_i}{2}, & \text{if } a_i = d_i \end{cases}$$

$$x_{\tilde{A}_i}^* = \frac{y_{\tilde{A}_i}^* \cdot (c_i + b_i) + (d_i + a_i)(\omega_i - y_{\tilde{A}_i}^*)}{2\omega_i}, i = 1, 2.$$

In 2006, Wang defined the centroid sorting indicator of fuzzy numbers on the basis of the concept of centroid of fuzzy numbers proposed by Cheng according to the geometric characteristics of centroid [25]. Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1; \omega_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2; \omega_2)$ denote two generalized trapezoidal fuzzy numbers, and the abscissa and ordinate coordinates of their centroids are:

$$\bar{x}(A_i) = \frac{1}{3}(a_i + b_i + c_i + d_i - \frac{c_i d_i - a_i b_i}{(c_i + d_i) - (a_i + b_i)}) \tag{3}$$

$$\bar{y}(A_i) = \frac{\omega_i}{3} (1 + \frac{c_i - b_i}{(c_i + d_i) - (a_i + b_i)}) \tag{4}$$

the centroid sorting indicator is:

$$D(A_i) = \bar{x}(A_i) \cdot \bar{y}(A_i), i = 1, 2. \tag{5}$$

If $D(A_1) > D(A_2)$, then it is considered that \tilde{A}_1 takes precedence over \tilde{A}_2 , denoted as $\tilde{A}_1 \succ \tilde{A}_2$; if $D(A_1) < D(A_2)$, then it is considered that \tilde{A}_1 is inferior to \tilde{A}_2 , denoted as $\tilde{A}_1 \prec \tilde{A}_2$; if $D(A_1) = D(A_2)$, then \tilde{A}_1 is considered to be equivalent to \tilde{A}_2 , denoted as $\tilde{A}_1 \sim \tilde{A}_2$.

Parameters Estimation Method

This paper refers to some clustering methods of fuzzy numbers [26], and uses IBM SPSS Statistics software to perform cluster analysis on the panel data of a certain place. Panel data has a minimum value, a most probable interval, and a maximum value, and the peak value of the data is relatively flat, which is suitable for constructing the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; \omega)$.

Suppose there are n indicators, and there are p years of data in m regions under the indicators, then the data in the k -th year of the j -th indicator in the i -th region is expressed as x_{ijk} , $i = 1, 2, \dots, m, j = 1, 2, \dots, n, k = 1, 2, \dots, p$.

Perform longitudinal cluster analysis on the data of each region, and cluster the p -year data of the i -th region into s classes according to the Euclidean distance of the data in the group. Among the s classes, there is a class with the largest number of year data, and the minimum value of the data in this class is taken as the left endpoint of the most probable interval, denoted as x_{ijk}^L , the maximum value is taken as the right endpoint of the most probable interval, denoted as x_{ijk}^U , and the number of data in this class is z_{ij} . The matrix form of the generalized trapezoidal fuzzy number is constructed, and its mathematical expression is

$$(\tilde{A}_{ij})_{mn} = ((a_{ij}, b_{ij}, c_{ij}, d_{ij}; \omega_{ij}))_{mn}$$

where

$$a_{ij} = \min_k(x_{ijk}), b_{ij} = x_{ijk}^L, c_{ij} = x_{ijk}^U, d_{ij} = \max_k(x_{ijk}), \omega_{ij} = \frac{z_{ij}}{p}, k = 1, 2, \dots, p$$

Determination of the Indicators in the Evaluation System

The forest ecological carrying capacity system mainly includes resource indicators and damage rate indicators, and the environmental pollution pressure system (mainly includes pollution source indicators such as water pollution, soil pollution, and air pollution). Based on the scientific nature of the established indicators and the availability of data, this paper refers to the existing evaluation index system of forest ecological carrying capacity [6] and theories of sustainable forestry development and sustainable forest management [27, 28], selecting 12 evaluation indicators. Among them, the resource indicators B_1 is the forest land area C_1 , and the forest coverage rate C_2 , forest stock volume C_3 , total stock of living standing trees C_4 , national nature reserve rate C_5 . Damage rate indicators B_2 include the area of fire-affected forest C_6 , and the area of forest pests and rodents C_7 . The environmental pollution pressure system measures the environmental pollution caused by human production and life in the forest ecosystem and is constructed to measure the pollution situation with the environmental pollution source as an evaluation indicator. The environmental pollution indicators B_3 are the discharge of chemical oxygen demand in wastewater C_8 , the discharge of ammonia nitrogen in the wastewater C_9 , the discharge of sulfur dioxide C_{10} , the total amount of sewage treatment C_{11} , and the amount of industrial solid waste generated C_{12} . Obviously, there are interactions between these indicators, such as the area of damage and resource indicators, the amount of environmental pollution, and resource indicators. For benefit-type indicators whose indicator nature is "+", the larger the attribute value, the better, for cost-type indicators whose indicator nature is "-", the smaller the attribute value, the better.

Acquisition and Normalization of Data Matrix

This paper selects the provincial panel data of 5 provinces along the southeastern coast of China for 17 years from 2004 to 2020, namely, Zhejiang, Fujian, Jiangxi, Guangdong, and Guangxi as the evaluation objects. The differences in forest ecological carrying capacity of each province were analyzed. Provide a more scientific basis for the construction of an environment-friendly economic society and sustainable development in the southeastern region. The data comes from the seventh national forest resource inventory (2004-2008), the eighth national forest resource inventory (2009-2013), the ninth national forest resource inventory (2014-2018), and the data platform of the National Bureau of Statistics, individual missing year data are forecasted by the widely recognized ARIMA method in the time series forecasting method. According to the parameter estimation method in Section 2.3, the generalized trapezoidal fuzzy number matrix is generated, and the arrangement is shown in Table 2.

Because the dimensions of the data are different, if the data is directly calculated, it will lose its practical significance. Therefore, it is necessary to standardize the data to make the processed data computable and comparable. Inspired by the idea of the range variation method, the standardized formula of generalized trapezoidal fuzzy number is defined [29]. Let the initial generalized trapezoidal fuzzy number matrix be $(\tilde{A}_{ij})_{mn} = ((a_{ij}, b_{ij}, c_{ij}, d_{ij}, \omega_{ij}))_{mn}$, and the normalized generalized trapezoidal fuzzy number matrix be $(\hat{A}_{ij})_{mn} = ((\hat{a}_{ij}, \hat{b}_{ij}, \hat{c}_{ij}, \hat{d}_{ij}, \hat{\omega}_{ij}))_{mn}$, and there are the following relational expressions:

Table 1. Evaluation Indicator System of Forest Ecological Carrying Capacity and Environmental Pollution Pressure.

Target layer	Primary indicators	Secondary indicators	Unit	Indicator nature
Evaluation of Forest Ecological Carrying Capacity	Resource indicators B_1	Forest land area C_1	hm^2	+
		Forest coverage rate C_2	%	+
		Forest stock volume C_3	10^6m^3	+
		Total stock of living standing trees C_4	10^6m^3	+
		National nature reserve rate C_5	%	+
	Damage rate indicators B_2	Area of fire-affected forest C_6	hm^2	-
		Area of forest pests and rodents C_7	10^4m^2	-
	Environmental pollution indicators B_3	Discharge of chemical oxygen demand in wastewater C_8	$10kt$	-
		Discharge of ammonia nitrogen in the wastewater C_9	$10kt$	-
		Discharge of sulfur dioxide C_{10}	$10kt$	-
		Total amount of sewage treatment C_{11}	$10kt$	-
		Amount of industrial solid waste generated C_{12}	$10kt$	-

Table 2. Generalized Trapezoidal Fuzzy Number Matrix of Forest Ecological Carrying Capacity Indicator in Five Southeast Provinces.

Region	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
Zhejiang	(1887,1958,2114, 5326;0.5294)	(57.4,59.1,59.4, 59.4;0.7059)	(1.72,1.72,2.17, 2.81;0.5882)	(1.94,1.94,2.42, 3.14;0.5882)	(0.9456,1.4367,1.4412, 1.4608;0.4118)	(107.79,107.79,2089.54, 10838.87;0.8235)
Fujian	(38810,54341,60145, 60242;0.7059)	(63.1,66,66.8, 66.8;0.7059)	(4.84,6.08,7.29, 7.29;0.7059)	(5.32,6.67,7.97, 7.97;0.7059)	(1.3270,1.6983,2.0775, 2.0775;0.8235)	(217.44,217.44,3428.86, 13589.8;0.7647)
Jiangxi	(337670,467304,488125, 488125;0.4706)	(58.3,60,61.2, 61.2;0.7059)	(3.95,3.95,4.08, 5.07;0.5882)	(4.5,4.5,4.7, 5.76;0.5882)	(0.4882,0.4882,1.108, 1.5629;0.5294)	(190.2,190.2,2053.66, 12711.18;0.7647)
Guangdong	(34212,55440,73128, 83359;0.8235)	(49.4,49.4,51.3, 53.5;0.5882)	(3.02,3.02,3.57, 4.68;0.5882)	(3.22,3.22,3.78, 5.01;0.5882)	(1.3270,1.6983,2.0775, 2.0775;0.8235)	(287.17,287.17,1475.58, 2886.2;0.9412)
Guangxi	(30004,36557,44843, 44843;0.8235)	(52.7,56.5,60.2, 60.2;0.7059)	(4.69,4.69,5.09, 6.78;0.5882)	(5.11,5.11,5.58, 7.44;0.5882)	(0.4882,0.4882,1.1018, 1.5629;0.5294)	(593.97,593.97,1600.04, 4997.6;0.8235)

Region	C ₇	C ₈	C ₉	C ₁₀	C ₁₁	C ₁₂
Zhejiang	(4.8,4.8,21.14, 51.97;0.8824)	(20.62,48.7,81.83, 81.83;0.7647)	(1.34,1.34,6.29, 11.54;0.7059)	[5.15,53.78,86, 86;0.7059]	(94479.3,94479.3,193751, 330795;0.5294)	(2318,4262.6,4591, 4591;0.6471)
Fujian	(17.65,17.65,32.13, 36.82;0.7059)	(25.16,25.16,39.5, 67.94;0.6471)	(1.56,1.56,5.16, 9.54;0.7059)	(7.88,32.6,46.9, 46.9;0.7059)	(42653.6,80855,139680, 139680;0.7059)	(3361,3361,4956, 8535;0.4706)
Jiangxi	(19.11,19.11,40.56, 54.85;0.9412)	(31.68,31.68,47.4, 101.48;0.6471)	(2.61,2.61,4.59, 9.34;0.7059)	(10.25,45.81,63.4, 63.4;0.7647)	(23983,23983,68069, 108167;0.6471)	(6524,10777,12665, 12665;0.5882)
Guangdong	(27.32,27.32,53.8, 80.66;0.8824)	(63.48,63.48,105.8, 188.45;0.6471)	(4.48,4.48,12.24, 23.09;0.7059)	(11.69,67.83,129.40, 129.40;0.7059)	(158337.4,543394,811273, 811273;0.6471)	(2609,5456,5966, 6944;0.4118)
Guangxi	(22.82,33.85,39.61, 39.61;0.8824)	(30.55,71.12,111.9, 111.9;0.7647)	(2.26,4.74,8.94, 8.94;0.7647)	(8.78,8.78,52.1, 102.40;0.5882)	(39446.8,96160,133791, 151517;0.5294)	(3291,6938,9030, 9030;0.4706)

For benefit indicators, there are

$$\hat{A}_{ij} = \left(\frac{a_{ij}}{d_j^*}, \frac{b_{ij}}{d_j^*}, \frac{c_{ij}}{d_j^*}, \frac{d_{ij}}{d_j^*}; \omega_{ij} \right) \tag{6}$$

For cost indicators, there are

$$\hat{A}_{ij} = \left(\frac{a_j^*}{d_{ij}}, \frac{a_j^*}{c_{ij}}, \frac{a_j^*}{b_{ij}}, \frac{a_j^*}{a_{ij}}; \omega_{ij} \right) \tag{7}$$

where $d_j^* = \max_i(d_{ij}), a_j^* = \min_i(a_{ij})$

After standardizing the data in Table 2 according to the formula (6) and (7), a standard generalized trapezoidal fuzzy number matrix can be obtained, as shown in Table 3.

Association Aggregation of Indicator Information

Quantitative analysis is carried out on the standardized matrix, and the similarity between the attribute and the positive and negative ideal solutions is calculated based on the closeness of the indicator attribute to the ideal attribute [30]. Combined with the idea of association aggregation, the optimal fuzzy measurement model was constructed to sort and evaluate the ecological carrying capacity of the provincial forests. Therefore, the calculation method of relative similarity between \hat{A}_{ij} and positive and negative ideal solution $\tilde{r}_j^+, \tilde{r}_j^-$ in [31]. Let $\tilde{R}_{B_t}^+ = (\tilde{r}_1^+, \tilde{r}_2^+, \dots, \tilde{r}_n^+)$, $\tilde{R}_{B_t}^- = (\tilde{r}_1^-, \tilde{r}_2^-, \dots, \tilde{r}_n^-)$, $t = 1, 2, 3$ be the positive and negative ideal solution vectors, respectively, where

$$\tilde{r}_j^+ = (\max_i \hat{a}_{ij}, \max_i \hat{b}_{ij}, \max_i \hat{c}_{ij}, \max_i \hat{d}_{ij}; \max_i \hat{\omega}_{ij}), j = 1, 2, \dots, n$$

$$\tilde{r}_j^- = (\min_i \hat{a}_{ij}, \min_i \hat{b}_{ij}, \min_i \hat{c}_{ij}, \min_i \hat{d}_{ij}; \min_i \hat{\omega}_{ij}), j = 1, 2, \dots, n.$$

Then the relative similarity formula between \hat{A}_{ij} and the positive and negative ideal solutions $\tilde{r}_j^+, \tilde{r}_j^-$ is [31]:

$$K_{ij} = \frac{Sim(\hat{A}_{ij}, \tilde{r}_j^+)}{Sim(\hat{A}_{ij}, \tilde{r}_j^+) + Sim(\hat{A}_{ij}, \tilde{r}_j^-)} \tag{8}$$

where *Sim* is the similarity between two generalized trapezoidal fuzzy numbers which is defined by formula (2).

Taking the $C_8, C_9, C_{10}, C_{11}, C_{12}$ attributes under the environmental pollution indicator B_3 in Table 3 as an example, calculate the positive and negative ideal solution vectors

$$\tilde{R}_{B_3}^+ = ((0.3035, 0.5220, 0.8196, 1; 0.7647), (0.1499, 0.2919, 1, 1; 0.7647), (0.1098, 0.1098, 0.5866, 1; 0.7647), (0.2217, 0.3523, 1, 1; 0.7059), (0.5049, 0.5049, 0.6897, 1; 0.6471))$$

$$\tilde{R}_{B_3}^- = ((0.1094, 0.1843, 0.2899, 0.3248; 0.6471), (0.058, 0.1095, 0.2827, 0.2991; 0.7059), (0.0398, 0.0398, 0.0759, 0.4405; 0.5882), (0.0296, 0.0296, 0.0441, 0.1515; 0.5294), (0.183, 0.183, 0.2151, 0.3553; 0.4118))$$

According to the similarity formula, carry out the pairwise calculation between \hat{A}_{ij} and the positive and negative ideal solutions $\tilde{r}_j^+, \tilde{r}_j^-$, and obtain the similarity matrix and relative similarity matrix $(S_{ij}^+)_{|B_3| \times |B_3|}, (S_{ij}^-)_{|B_3| \times |B_3|}, (K_{ij})_{|B_3| \times |B_3|}$ of the positive and

negative ideal solutions, as shown below.

Where $B_3 = \{C_8, \dots, C_{12}\}$.

$$(S_{ij}^+)_{|B_3| \times |B_3|} = \begin{pmatrix} 1.164 & 0.857 & 0.929 & 1.256 & 0.584 \\ 0.951 & 0.948 & 1.038 & 1.039 & 0.742 \\ 1.206 & 1.261 & 1.160 & 0.583 & 0.155 \\ 1.753 & 1.633 & 1.248 & 1.447 & 0.743 \\ 1.495 & 1.386 & 0.946 & 1.043 & 0.911 \end{pmatrix}, i, j \in B_3.$$

$$(S_{ij}^-)_{|B_3| \times |B_3|} = \begin{pmatrix} 0.496 & 0.316 & 0.486 & 0.487 & 0.215 \\ 0.355 & 0.373 & 0.564 & 0.353 & 0.321 \\ 0.525 & 0.585 & 0.653 & 0.111 & 0.632 \\ 0.922 & 0.855 & 0.718 & 0.612 & 0.322 \\ 0.730 & 0.674 & 0.498 & 0.356 & 0.444 \end{pmatrix}, i, j \in B_3.$$

$$(K_{ij})_{|B_3| \times |B_3|} = \begin{pmatrix} 0.701 & 0.731 & 0.657 & 0.720 & 0.731 \\ 0.728 & 0.717 & 0.648 & 0.746 & 0.698 \\ 0.697 & 0.683 & 0.640 & 0.841 & 0.646 \\ 0.655 & 0.656 & 0.635 & 0.703 & 0.697 \\ 0.672 & 0.673 & 0.655 & 0.746 & 0.672 \end{pmatrix}, i, j \in B_3.$$

In the same way, the relative similarity matrix under other Primary attributes can be obtained, $B_1 = \{C_1, \dots, C_3\}$, $B_2 = \{C_6, C_7\}$.

$$(K_{ij})_{|B_1| \times |B_1|} = \begin{pmatrix} 0.788 & 0.609 & 0.714 & 0.712 & 0.703 \\ 0.805 & 0.621 & 0.850 & 0.847 & 0.756 \\ 0.990 & 0.611 & 0.770 & 0.772 & 0.674 \\ 0.808 & 0.598 & 0.749 & 0.744 & 0.756 \\ 0.800 & 0.607 & 0.807 & 0.804 & 0.674 \end{pmatrix}, i, j \in B_1.$$

$$(K_{ij})_{|B_2| \times |B_2|} = \begin{pmatrix} 0.749 & 0.796 \\ 0.693 & 0.718 \\ 0.701 & 0.714 \\ 0.686 & 0.707 \\ 0.671 & 0.710 \end{pmatrix}, i, j \in B_2.$$

Most of the hesitant fuzzy aggregation operators are based on the assumption that the importance of elements is independent. However, in practical multi-attribute evaluation problems, there are interrelated and interdependent phenomena among attributes. When there is a certain degree of interdependence and interaction between elements or experts, the aggregation operator based on additive measures is not suitable for decision-making [32-34]. As an effective tool to measure the importance of elements and the relationship between elements, fuzzy measures solve this problem well [35]. To fully reflect the interaction between elements in the set, measure the overall impact of each element combination, and consider

Table 3. Standard Generalized Trapezoidal Fuzzy Number Matrix of Forest Ecological Carrying Capacity Indicator in Five Southeast Provinces.

Region	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
Zhejiang	(0.0039, 0.0039, 0.0043, 0.0039, 0.0109; 0.5294)	(0.8593, 0.8847, 0.8892, 0.8892; 0.7059)	(0.2359, 0.2359, 0.2977, 0.3885; 0.5882)	(0.2434, 0.2434, 0.3036, 0.3940; 0.5882)	(0.4552, 0.6915, 0.6937, 0.7031; 0.4118)	(0.0099, 0.0516, 1, 1; 0.8235)
Fujian	(0.0795, 0.1113, 0.1232, 0.1234; 0.7059)	(0.9446, 0.9880, 1, 1; 0.7059)	(0.6639, 0.8340, 1, 1; 0.7059)	(0.6675, 0.8369, 1, 1; 0.7059)	(0.6388, 0.8175, 1, 1; 0.8235)	(0.0079, 0.0314, 0.4957, 0.4957; 0.7647)
Jiangxi	(0.6918, 0.9573, 1, 1; 0.4706)	(0.8728, 0.8982, 0.9162, 0.9162; 0.7059)	(0.5418, 0.5418, 0.5597, 0.6955; 0.5882)	(0.5646, 0.5646, 0.5897, 0.7227; 0.5882)	(0.235, 0.235, 0.5303, 0.7523; 0.5294)	(0.0085, 0.0525, 0.5667, 0.5667; 0.7647)
Guangdong	(0.0701, 0.1136, 0.1498, 0.1708; 0.8235)	(0.7395, 0.7395, 0.7680, 0.8009; 0.5882)	(0.4143, 0.4143, 0.4897, 0.6420; 0.5882)	(0.404, 0.404, 0.4743, 0.6286; 0.5882)	(0.6388, 0.8175, 1, 1; 0.8235)	(0.0373, 0.073, 0.3754, 0.3754; 0.9412)
Guangxi	(0.0615, 0.0749, 0.0919, 0.0919; 0.8235)	(0.7889, 0.8458, 0.9012, 0.9012; 0.7059)	(0.6433, 0.6433, 0.6982, 0.93; 0.5882)	(0.6412, 0.6412, 0.7001, 0.9335; 0.5882)	(0.235, 0.235, 0.5303, 0.7523; 0.5294)	(0.0216, 0.0674, 0.1815, 0.1815; 0.8235)

Region	C ₇	C ₈	C ₉	C ₁₀	C ₁₁	C ₁₂
Zhejiang	(0.0924, 0.2271, 1, 1; 0.8824)	(0.252, 0.252, 0.4234, 1; 0.7647)	(0.1161, 0.213, 1, 1; 0.7059)	(0.0599, 0.0599, 0.0958, 1; 0.7059)	(0.0725, 0.1238, 0.2538, 0.2538; 0.5294)	(0.5049, 0.5049, 0.5438, 1; 0.6471)
Fujian	(0.1304, 0.1494, 0.272, 0.272; 0.7059)	(0.3035, 0.522, 0.8196, 0.8196; 0.6471)	(0.1405, 0.2597, 0.859, 0.859; 0.7059)	(0.1098, 0.1098, 0.158, 0.6536; 0.7059)	(0.1717, 0.1717, 0.2966, 0.5623; 0.7059)	(0.2716, 0.4677, 0.6897, 0.6897; 0.4706)
Jiangxi	(0.0875, 0.1183, 0.2512, 0.2512; 0.9412)	(0.2032, 0.435, 0.6509, 0.6509; 0.6471)	(0.1435, 0.2919, 0.5134, 0.5134; 0.7059)	(0.0812, 0.0812, 0.1124, 0.5024; 0.7647)	(0.2217, 0.3523, 1, 1; 0.6471)	(0.183, 0.183, 0.2151, 0.3553; 0.5882)
Guangdong	(0.0595, 0.0892, 0.1757, 0.1757; 0.8824)	(0.1094, 0.1949, 0.3248, 0.3248; 0.6471)	(0.058, 0.1095, 0.2991, 0.2991; 0.7059)	(0.0398, 0.0398, 0.0759, 0.4405; 0.7059)	(0.0296, 0.0296, 0.0441, 0.1515; 0.6471)	(0.3338, 0.3886, 0.4249, 0.8885; 0.4118)
Guangxi	(0.1212, 0.1212, 0.1418, 0.2103; 0.8824)	(0.1843, 0.1843, 0.2899, 0.675; 0.7647)	(0.1499, 0.1499, 0.2827, 0.5929; 0.7647)	(0.0503, 0.0988, 0.5866, 0.5866; 0.5882)	(0.1583, 0.1583, 0.2494, 0.608; 0.5294)	(0.2567, 0.2567, 0.3341, 0.7043; 0.4706)

the interaction between attribute indicators, Marichal [35] combined Shapley's method [36] and fuzzy measure. To consider the importance of the combination of indicators in general and to reflect the interaction between them, and to take into account reducing the complexity of the fuzzy measure solution and calculation at the same time, the fuzzy measure is applied to the generalized Shapley value [37], which is expressed as follows:

$$\Phi_s(g_\lambda, N) = \sum_{T \subseteq N \setminus S} \frac{(n-t-s)!t!}{(n-s+1)!} (g_\lambda(T \cup S) - g_\lambda(T)), \forall S \subseteq N \tag{9}$$

where g_λ is the λ -fuzzy measure, and its expression is

$$g_\lambda(A) = \begin{cases} \frac{1}{\lambda} (\prod_{i \in A} (1 + \lambda g_\lambda(i)) - 1), & \lambda \neq 0 \\ \sum_{i \in A} g_\lambda(i), & \lambda = 0 \end{cases} \tag{10}$$

It can be known from formula (10) that for a set N with n elements, the fuzzy measure of any subset in the set N can be obtained only by determining n values. According to the $\mu(N) = 1$ condition of the fuzzy measure, the value of λ can be determined by formula (11), so when the value of $g_\lambda(i)$ is given, the value of λ can be obtained [34].

$$\lambda + 1 = \prod_{i \in N} (1 + \lambda g_\lambda(i)) \tag{11}$$

Combined with the concept of Choquet integral over discrete sets given by Grabisch [38], Meng [37] defined the arithmetic λ -Shapley-Choquet integral operator as follows:

$$C_{\Phi(g_\lambda, N)}(f(x_{(1)}), f(x_{(2)}), \dots, f(x_{(n)})) = \sum_{i=1}^n f(x_{(i)}) (\Phi_{A_{(i)}}(g_\lambda, N) - \Phi_{A_{(i+1)}}(g_\lambda, N)) \tag{12}$$

where (\cdot) represents the permutation of $f(x_{(i)})$, satisfies $0 \leq f(x_{(1)}) \leq f(x_{(2)}) \leq \dots \leq f(x_{(n)})$, $A_{(i)} = \{x_{(i)}, \dots, x_{(n)}\}$ and $A_{(n+1)} = \emptyset$.

Since all schemes are non-inferior, if the attribute weight information is partially known, the optimal fuzzy metric linear programming model [15] on the attribute set is established as follows:

$$\begin{aligned} & \min \sum_{i=1}^m \sum_{j=1}^n K_{ij} \Phi_{c_j}(\mu, C) \\ & \text{s.t.} \begin{cases} \mu(\emptyset) = 0, \mu(C) = 1 \\ \mu(S) \leq \mu(T) \quad \forall S, T \subseteq C, S \subset T \\ \mu(c_j) \in U_{c_j}, \mu(c_j) \geq 0, \forall c_j \in C \end{cases} \end{aligned} \tag{13}$$

where $\Phi_{c_j}(\mu, C)$ is the Shapley value of c_j , and μ is a fuzzy measure on C . The possible interval U_{c_j} of the weight is estimated according to the importance, and the optimal value is found within the interval.

Results and Discussion

Taking the $C_8, C_9, C_{10}, C_{11}, C_{12}$ attributes under the environmental pollution indicator B_3 as an example, the weight value range of the attribute is shown in Table 4, and the optimal fuzzy measurement linear programming model on the attribute set C is established.

Table 4. Attribute importance interval values.

Target layer	Primary indicators	Weight interval	Secondary indicators	Weight interval
Evaluation of Forest Ecological Carrying Capacity	Resource indicators B_1	[0.4, 0.5]	Forest land area C_1	[0.1, 0.3]
			Forest coverage rate C_2	[0.4, 0.5]
			Forest stock volume C_3	[0.1, 0.4]
			Total stock of living standing trees C_4	[0.2, 0.3]
			National nature reserve rate C_5	[0.2, 0.3]
	Damage rate indicators B_2	[0.2, 0.5]	Area of fire-affected forest C_6	[0.7, 0.9]
			Area of forest pests and rodents C_7	[0.3, 0.5]
	Environmental pollution indicators B_3	[0.4, 0.7]	Discharge of chemical oxygen demand in wastewater C_8	[0.1, 0.7]
			Discharge of ammonia nitrogen in the wastewater C_9	[0.1, 0.3]
			Discharge of sulfur dioxide C_{10}	[0.4, 0.6]
			Total amount of sewage treatment C_{11}	[0.1, 0.3]
			Amount of industrial solid waste generated C_{12}	[0.3, 0.5]

$$\min\{0.0042(\mu(C_9, C_{10}, C_{11}, C_{12}) - \mu(C_8)) + 0.0022(\mu(C_8, C_{10}, C_{11}, C_{12}) - \mu(C_9)) + 0.0590(\mu(C_8, C_9, C_{11}, C_{12}) - \mu(C_{10})) - 0.0715(\mu(C_8, C_9, C_{10}, C_{12}) - \mu(C_{11})) + 0.0062(\mu(C_8, C_9, C_{10}, C_{11}) - \mu(C_{12})) + 0.0021(\mu(C_{10}, C_{11}, C_{12}) - \mu(C_8, C_9)) + 0.0211(\mu(C_9, C_{11}, C_{12}) - \mu(C_8, C_{10})) - 0.0225(\mu(C_9, C_{10}, C_{12}) - \mu(C_8, C_{11})) + 0.0035(\mu(C_9, C_{10}, C_{11}) - \mu(C_8, C_{12})) + 0.0204(\mu(C_8, C_{11}, C_{12}) - \mu(C_9, C_{10})) - 0.0231(\mu(C_8, C_{10}, C_{12}) - \mu(C_9, C_{11})) + 0.0028(\mu(C_8, C_{10}, C_{11}) - \mu(C_9, C_{12})) - 0.0042(\mu(C_8, C_9, C_{12}) - \mu(C_{10}, C_{11})) + 0.0217(\mu(C_8, C_9, C_{11}) - \mu(C_{10}, C_{12})) - 0.0218(\mu(C_8, C_9, C_{10}) - \mu(C_{11}, C_{12})) + 3.4698\}$$

$$s.t. \begin{cases} \mu(C_8, C_9, C_{10}, C_{11}, C_{12}) = 1 \\ \mu(S) \leq \mu(T) \quad \forall S, T \subseteq C, S \subset T \\ \mu(C_8) \in [0.12, 0.3] \\ \mu(C_9) \in [0.12, 0.3] \\ \mu(C_{10}) \in [0.25, 0.5] \\ \mu(C_{11}) \in [0.2, 0.4] \\ \mu(C_{12}) \in [0.31, 0.45] \end{cases}$$

Solving the model, we get

$$\begin{aligned} \mu(C_8) &= \mu(C_9) = \mu(C_{11}) \\ &= \mu(C_8, C_9) = \mu(C_8, C_{11}) \\ &= \mu(C_9, C_{11}) = \mu(C_8, C_9, C_{11}) = 0.2, \\ \mu(C_{10}) &= \mu(C_8, C_{10}) = \mu(C_9, C_{10}) = \mu(C_{10}, C_{11}) \\ &= \mu(C_8, C_9, C_{10}) = \mu(C_8, C_{10}, C_{11}) = \mu(C_8, C_{10}, C_{12}) \\ &= \mu(C_9, C_{10}, C_{11}) = \mu(C_9, C_{10}, C_{12}) = \mu(C_{10}, C_{11}, C_{12}) \\ &= \mu(C_8, C_9, C_{10}, C_{11}) = \mu(C_8, C_9, C_{10}, C_{12}) \\ &= \mu(C_8, C_{10}, C_{11}, C_{12}) = \mu(C_9, C_{10}, C_{11}, C_{12}) = 0.5, \\ \mu(C_{12}) &= \mu(C_8, C_{12}) = \mu(C_9, C_{12}) = \mu(C_{10}, C_{12}) = \mu(C_{11}, C_{12}) \\ &= \mu(C_8, C_9, C_{12}) = \mu(C_8, C_{11}, C_{12}) \\ &= \mu(C_9, C_{11}, C_{12}) = \mu(C_8, C_9, C_{11}, C_{12}) = 0.45, \\ \mu(C_8, C_9, C_{10}, C_{11}, C_{12}) &= 1. \end{aligned}$$

The fuzzy measure $\mu(C_j)$ is represented by $g(C_j)$ and substituted into formula (11), the solution has five complex domain roots, and the approximate solution $\lambda_{B_3} = -0.7697$, of B_3 is obtained by using Matlab software, and the λ -fuzzy measure is calculated by substituting it into formula (10).

$$\begin{aligned} g_\lambda(C_8) &= g_\lambda(C_9) = g_\lambda(C_{11}) = 0.2, \\ g_\lambda(C_{12}) &= 0.45, g_\lambda(C_{10}) = 0.5, g_\lambda(C_{10}, C_{12}) = 0.7768, \\ g_\lambda(C_8, C_9) &= g_\lambda(C_8, C_{11}) = g_\lambda(C_9, C_{11}) = 0.3692, \\ g_\lambda(C_8, C_{10}) &= g_\lambda(C_9, C_{10}) = g_\lambda(C_{10}, C_{11}) = 0.6230, \\ g_\lambda(C_8, C_{12}) &= g_\lambda(C_9, C_{12}) = g_\lambda(C_{11}, C_{12}) = 0.5807, \\ g_\lambda(C_8, C_9, C_{11}) &= 0.5124, \\ g_\lambda(C_8, C_9, C_{10}) &= g_\lambda(C_8, C_{10}, C_{11}) = g_\lambda(C_9, C_{10}, C_{11}) = 0.7271, \\ g_\lambda(C_8, C_{10}, C_{12}) &= g_\lambda(C_9, C_{10}, C_{12}) = g_\lambda(C_{10}, C_{11}, C_{12}) = 0.8572, \\ g_\lambda(C_8, C_9, C_{12}) &= g_\lambda(C_8, C_{11}, C_{12}) = g_\lambda(C_9, C_{11}, C_{12}) = 0.6913, \\ g_\lambda(C_8, C_9, C_{10}, C_{11}) &= 0.8152, g_\lambda(C_8, C_9, C_{11}, C_{12}) = 0.7849, \\ g_\lambda(C_8, C_9, C_{10}, C_{12}) &= g_\lambda(C_8, C_{10}, C_{11}, C_{12}) = g_\lambda(C_9, C_{10}, C_{11}, C_{12}) = 0.9253, \\ g_\lambda(C_8, C_9, C_{10}, C_{11}, C_{12}) &= 1. \end{aligned}$$

According to formula (10) (11), we have

$$\begin{aligned} \Phi_{C_8, C_9, C_{10}}(g_\lambda, C) &= \Phi_{C_9, C_{10}, C_{11}}(g_\lambda, C) = 0.5639, \\ \Phi_{C_9, C_{10}, C_{12}}(g_\lambda, C) &= \Phi_{C_{10}, C_{11}, C_{12}}(g_\lambda, C) = 0.7378, \\ \Phi_{C_{10}, C_{12}}(g_\lambda, C) &= 0.4975, \Phi_{C_8, C_9, C_{11}}(g_\lambda, C) = 0.4905, \\ \Phi_{C_8, C_{10}, C_{11}}(g_\lambda, C) &= 0.5405, \Phi_{C_8, C_{10}, C_{12}}(g_\lambda, C) = 0.7144, \\ \Phi_{C_8, C_9, C_{10}, C_{11}}(g_\lambda, C) &= 0.6826, \Phi_{C_8, C_9, C_{11}, C_{12}}(g_\lambda, C) = 0.6425, \\ \Phi_{C_8, C_9, C_{12}}(g_\lambda, C) &= \Phi_{C_8, C_{11}, C_{12}}(g_\lambda, C) = \Phi_{C_9, C_{11}, C_{12}}(g_\lambda, C) = 0.5245, \\ \Phi_{C_8, C_9, C_{10}, C_{12}}(g_\lambda, C) &= \Phi_{C_8, C_{10}, C_{11}, C_{12}}(g_\lambda, C) = \Phi_{C_9, C_{10}, C_{11}, C_{12}}(g_\lambda, C) = 0.8627, \\ \Phi_{C_8, C_9, C_{10}, C_{11}, C_{12}}(g_\lambda, C) &= 1. \end{aligned}$$

Table 5. Fuzzy measures of secondary indicator attributes.

S	$\mu(C_S)$	S	$\mu(C_S)$	S	$\mu(C_S)$
{1}	0.2	{2, 4}	0.45	{1, 4, 5}	0.2
{2}	0.45	{2, 5}	0.45	{2, 3, 4}	0.45
{3}	0.19	{3, 4}	0.2	{2, 3, 5}	0.45
{4}	0.2	{3, 5}	0.2	{2, 4, 5}	0.45
{5}	0.2	{4, 5}	0.2	{3, 4, 5}	0.2
{1, 2}	0.45	{1, 2, 3}	0.45	{1, 2, 3, 4}	0.45
{1, 3}	0.2	{1, 2, 4}	0.45	{1, 2, 3, 5}	0.45
{1, 4}	0.2	{1, 2, 5}	0.45	{1, 2, 4, 5}	0.45
{1, 5}	0.2	{1, 3, 4}	0.2	{1, 3, 4, 5}	0.2
{2, 3}	0.45	{1, 3, 5}	0.2	{2, 3, 4, 5}	0.45
{6}	0.7	{7}	0.35	{1, 2, 3, 4, 5} {6, 7}	1

$$\begin{aligned} \Phi_{\emptyset}(g_{\lambda}, C) &= 0, \Phi_{c_{10}}(g_{\lambda}, C) = 0.3365, \Phi_{c_{12}}(g_{\lambda}, C) = 0.2969, \\ \Phi_{c_8}(g_{\lambda}, C) &= \Phi_{c_9}(g_{\lambda}, C) = \Phi_{c_{11}}(g_{\lambda}, C) = 0.1222, \\ \Phi_{c_8, c_9}(g_{\lambda}, C) &= \Phi_{c_8, c_{11}}(g_{\lambda}, C) = \Phi_{c_9, c_{11}}(g_{\lambda}, C) = 0.2028, \\ \Phi_{c_8, c_{10}}(g_{\lambda}, C) &= \Phi_{c_9, c_{10}}(g_{\lambda}, C) = \Phi_{c_{10}, c_{11}}(g_{\lambda}, C) = 0.3721, \\ \Phi_{c_8, c_{12}}(g_{\lambda}, C) &= \Phi_{c_9, c_{12}}(g_{\lambda}, C) = \Phi_{c_{11}, c_{12}}(g_{\lambda}, C) = 0.3419, \end{aligned}$$

Similarly, the fuzzy measurement values of other secondary indicator attributes can be obtained, as shown in Table 5. According to formula (11), there are five complex domain roots in the solution, and the approximate solution of B_1, B_2 is obtained as $\lambda_{B_1} = \lambda_{B_2} = -0.4766$ by using Matlab, and the λ -fuzzy measure is calculated by substituting into formula (10) to obtain Table 6. From formula (9), the generalized Shapley value of the secondary indicator attribute can be calculated, as shown in Table 7.

According to the ranking indicator method of generalized trapezoidal fuzzy numbers, the centroid ranking value of the secondary indicator attribute is calculated, as shown in Table 8. Rearrange the evaluation values from smallest to largest. Use the arithmetic λ -Shapley-Choquet integral operator (Equation (12)) to obtain the comprehensive value of the B_i -th attribute of the i -th area, as shown in Table 9. The centroid ranking values corresponding to the comprehensive evaluation values are listed in Table 10.

After calculating the attribute value of the primary indicator, the fuzzy measurement value of the primary indicator attribute can be obtained, that is $\mu(B_{\{1\}}) = 0.4, \mu(B_{\{2\}}) = 0.25, \mu(B_{\{3\}}) = 0.5, \mu(B_{\{1,2\}}) = 0.4, \mu(B_{\{1,3\}}) = 0.5, \mu(B_{\{2,3\}}) = 0.5,$ and $\mu(\emptyset) = 0, \mu(B_1, B_2, B_3) = 1$. According

to formula (11), the solution has five complex domain roots, the approximate solution $\lambda = -0.1180$ is obtained by using Matlab, and the λ -fuzzy measure is calculated by substituting into formula (10) to obtain: $g_{\lambda}(B_{\{1\}}) = 0.4, g_{\lambda}(B_{\{2\}}) = 0.25, g_{\lambda}(B_{\{3\}}) = 0.5, g_{\lambda}(B_{\{1,2\}}) = 0.6382, g_{\lambda}(B_{\{1,3\}}) = 0.8764, g_{\lambda}(B_{\{2,3\}}) = 0.7352,$ and $g_{\lambda}(\emptyset) = 0, g_{\lambda}(B_1, B_2, B_3) = 1$. The generalized Shapley value of the secondary index attribute can be calculated by formula (9): $\Phi_{\{1\}}(g_{\lambda}, B) = 0.3490, \Phi_{\{2\}}(g_{\lambda}, B) = 0.2034, \Phi_{\{3\}}(g_{\lambda}, B) = 0.4475, \Phi_{\{1,2\}}(g_{\lambda}, B) = 0.5691, \Phi_{\{1,3\}}(g_{\lambda}, B) = 0.8132, \Phi_{\{2,3\}}(g_{\lambda}, B) = 0.6676,$ and $\Phi_{\emptyset}(g_{\lambda}, B) = 0, \Phi_{B_1, B_2, B_3}(g_{\lambda}, B) = 1$.

Using the arithmetic λ -Shapley-Choquet integral operator to get the comprehensive evaluation value of each region:

$$\begin{aligned} \tilde{Z}_{zhejiang} &= (0.1556, 0.2009, 0.6622, 0.7593; 0.4118), \\ \tilde{Z}_{fujian} &= (0.3652, 0.4479, 0.6096, 0.6319; 0.4706), \\ \tilde{Z}_{jiangxi} &= (0.2999, 0.3441, 0.5278, 0.5983; 0.4706), \\ \tilde{Z}_{guangdong} &= (0.2332, 0.2703, 0.3722, 0.4800; 0.4118), \\ \tilde{Z}_{guangxi} &= (0.2343, 0.2497, 0.3635, 0.5081; 0.4706). \end{aligned}$$

Calculate the centroid sorting of the comprehensive evaluation value of each region, and get $D(\tilde{Z}_{zhejiang}) = 0.1456, D(\tilde{Z}_{fujian}) = 0.1853, D(\tilde{Z}_{jiangxi}) = 0.1597, D(\tilde{Z}_{guangdong}) = 0.0998, D(\tilde{Z}_{guangxi}) = 0.1137$. Therefore $\tilde{Z}_{fujian} \succ \tilde{Z}_{jiangxi} \succ \tilde{Z}_{zhejiang} \succ \tilde{Z}_{guangxi} \succ \tilde{Z}_{guangdong}$.

The calculation shows that Fujian Province has the highest forest ecological carrying capacity index, and its forest ecological carrying capacity is the best; followed by Jiangxi Province, Zhejiang Province,

Table 6. λ -Fuzzy measures of secondary indicator attributes.

S	$g_{\lambda}(C_S)$	S	$g_{\lambda}(C_S)$	S	$g_{\lambda}(C_S)$
{1}	0.2	{2, 4}	0.5828	{1, 4, 5}	0.5149
{2}	0.45	{2, 5}	0.5828	{2, 3, 4}	0.6901
{3}	0.19	{3, 4}	0.3616	{2, 3, 5}	0.6901
{4}	0.2	{3, 5}	0.3616	{2, 4, 5}	0.6958
{5}	0.2	{4, 5}	0.3701	{3, 4, 5}	0.5076
{1, 2}	0.5828	{1, 2, 3}	0.6901	{1, 2, 3, 4}	0.7871
{1, 3}	0.3616	{1, 2, 4}	0.6958	{1, 2, 3, 5}	0.7871
{1, 4}	0.3701	{1, 2, 5}	0.6958	{1, 2, 4, 5}	0.7919
{1, 5}	0.3701	{1, 3, 4}	0.5076	{1, 3, 4, 5}	0.6318
{2, 3}	0.5762	{1, 3, 5}	0.5076	{2, 3, 4, 5}	0.7871
{6}	0.7	{7}	0.35	{1, 2, 3, 4, 5} {6, 7}	1

Table 7. Generalized Shapley values of secondary indicator attributes.

S	$\Phi_S(g_\lambda, C)$	S	$\Phi_S(g_\lambda, C)$	S	$\Phi_S(g_\lambda, C)$
{1}	0.1614	{2, 4}	0.3761	{1, 4, 5}	0.4435
{2}	0.3616	{2, 5}	0.3761	{2, 3, 4}	0.6357
{3}	0.1673	{3, 4}	0.2175	{2, 3, 5}	0.6357
{4}	0.1614	{3, 5}	0.2175	{2, 4, 5}	0.6429
{5}	0.1614	{4, 5}	0.2232	{3, 4, 5}	0.4364
{1, 2}	0.3761	{1, 2, 3}	0.6404	{1, 2, 3, 4}	0.7936
{1, 3}	0.2175	{1, 2, 4}	0.6429	{1, 2, 3, 5}	0.7936
{1, 4}	0.2404	{1, 2, 5}	0.6429	{1, 2, 4, 5}	0.8010
{1, 5}	0.2232	{1, 3, 4}	0.4364	{1, 3, 4, 5}	0.5909
{2, 3}	0.3708	{1, 3, 5}	0.4364	{2, 3, 4, 5}	0.7936
{6}	0.6750	{7}	0.3250	{1, 2, 3, 4, 5} {6, 7}	1

Table 8. Indicator values of generalized trapezoidal fuzzy number centroid ranking of secondary indicator.

Region	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
Zhejiang	0.0025	0.3908	0.1212	0.1236	0.1493	0.3512	0.4154	0.2338	0.3361	0.1065	0.0736	0.2387
Fujian	0.0525	0.4548	0.4578	0.4582	0.5298	0.1629	0.1183	0.3040	0.3024	0.1005	0.1431	0.1876
Jiangxi	0.2705	0.4569	0.2090	0.2250	0.1745	0.1877	0.1342	0.2330	0.1980	0.0806	0.3367	0.0867
Guangdong	0.0732	0.3276	0.1984	0.1917	0.5298	0.1658	0.0876	0.1185	0.1083	0.0568	0.0194	0.1192
Guangxi	0.0498	0.4488	0.2734	0.2752	0.1745	0.0735	0.0853	0.1605	0.1493	0.1595	0.1075	0.1125

Guangxi Province, and Guangdong Province. One of the reasons is that Fujian Province is superior to other provinces in terms of forest stock volume C_3 , total stock volume of standing trees C_4 , rate of national nature reserves C_5 , and COD discharge in wastewater C_8 . From Table 8, it is manifested that the centroid ranking index value of C_3, C_4, C_5, C_8 in Fujian Province is higher than the corresponding attribute values of other provinces. It can be seen that in terms of resource indicators such as forest stock volume, total standing tree volume, and the rate of national nature reserves, Fujian Province both occupy a greater advantage, indicating that in the forest ecological carrying rate index system, resource indicators (such as forest stock volume) play an important role, followed by environmental pollution on forests. Fig. 1 clearly shows that Fujian's resource indicator B_1 is far superior to the other four provinces. It should be noted that the situation of Zhejiang Province is very special from the perspective of the damage rate index, with the corresponding ranking value of 0.3458, which is obviously different from other provinces. This is one of the reasons why Zhejiang Province did not rank in the top two in the comprehensive ranking. Therefore,

while focusing on resource-based development, the impact of fire damage and pests and rodents on forests should not be ignored as well.

The generalized Shapley weight value Φ_{B_3} of environmental pollution index B_3 accounts for 45%, which is higher than 35% of resource index B_1 and 20% of damage rate index B_2 , indicating that in the evaluation process of forest ecological carrying capacity

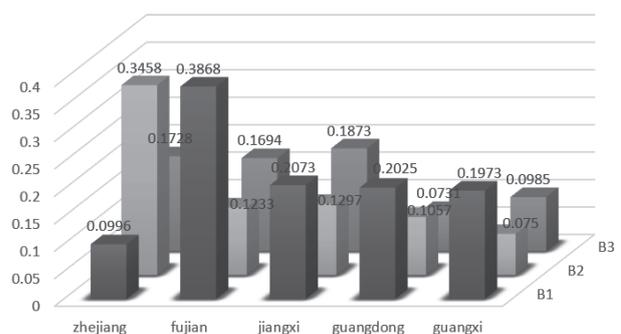


Fig. 1. The ranking value of the attribute centroid of the primary indicator.

Table 9. Attribute values of Primary indicators.

Region	B_1	B_2	B_3
Zhejiang	(0.3038, 0.3225, 0.3583, 0.4109; 0.4118)	(0.0362, 0.1060, 0.9500, 0.9500; 0.8235)	(0.1744, 0.2102, 0.5358, 0.8976; 0.5294)
Fujian	(0.6355, 0.7329, 0.8145, 0.8146; 0.7059)	(0.0841, 0.1036, 0.3311, 0.3311; 0.7059)	(0.2405, 0.3884, 0.6138, 0.7150; 0.4706)
Jiangxi	(0.5460, 0.5666, 0.6448, 0.7674; 0.4706)	(0.0575, 0.0910, 0.3412, 0.3412; 0.7647)	(0.1658, 0.2738, 0.5717, 0.6428; 0.5882)
Guangdong	(0.3959, 0.4337, 0.5136, 0.6076; 0.5882)	(0.0493, 0.0795, 0.2318, 0.2318; 0.8824)	(0.1360, 0.1765, 0.2747, 0.4726; 0.4118)
Guangxi	(0.4008, 0.4127, 0.5460, 0.6891; 0.5294)	(0.0529, 0.0815, 0.1595, 0.1818; 0.8235)	(0.1701, 0.1723, 0.3004, 0.6330; 0.4706)

Table 10. The centroid ranking values of the first-level index attributes.

Region	$D(B_1)$	$D(B_2)$	$D(B_3)$
Zhejiang	0.0996	0.3458	0.1728
Fujian	0.3868	0.1233	0.1694
Jiangxi	0.2073	0.1297	0.1873
Guangdong	0.2025	0.1057	0.0731
Guangxi	0.1973	0.0750	0.0985

environmental pollution indicators have a large impact on the results. Therefore, only relying on the advantages of a single aspect to promote the development of forest carrying capacity is limited, and we should also pay attention to the impact of environmental pollution on forest ecology. The improvement of forest carrying capacity requires not only improving the self-regulating

function and ecological function of the forest system itself, more need to reduce environmental pollution.

The generalized Shapley weight values and their combined weights of the primary indicator attributes are shown as radar charts in Fig. 2. It can be observed that a single environmental pollution indicator $\Phi_{B_3} = 0.4475$ is greater than $\Phi_{B_1} = 0.3490$, $\Phi_{B_2} = 0.2034$, and the combined weight $\Phi_{B_1, B_3} = 0.8132$ of the resource indicator B_1 and the environmental pollution indicator B_3 is higher than the other combined weights $\Phi_{\{2,3\}}(g_\lambda, B) = 0.6676$, $\Phi_{B_1, B_2} = 0.5691$.

In addition, the lower the indicator after standardized treatment, the greater the pressure of environmental pollution and the more serious the environmental pollution. According to Table 8, although the bottom three provinces performed well in various resource indicators, their numerical values in environmental pollution indicators such as the discharge of chemical oxygen demand in wastewater and the total amount of sewage treatment were relatively low, this is also one of the reasons for its low comprehensive evaluation value, forests bear a heavy burden of environmental pollution while maintaining self-renewal. To improve the forest carrying capacity, we should implement environmental protection policies and actively develop an environment-friendly economy to ensure the high-quality development of forest resources in the southeast, to achieve the sustainable development goal of economic and ecological civilization construction in the southeast.

Conclusions

In the evaluation of forest ecological carrying capacity, according to the characteristics of the collected panel data, a generalized trapezoidal fuzzy number is fitted to the data for information expression, which can fully express the initial information of the data. Considering that there may be interactions between attribute indicators, the combination of

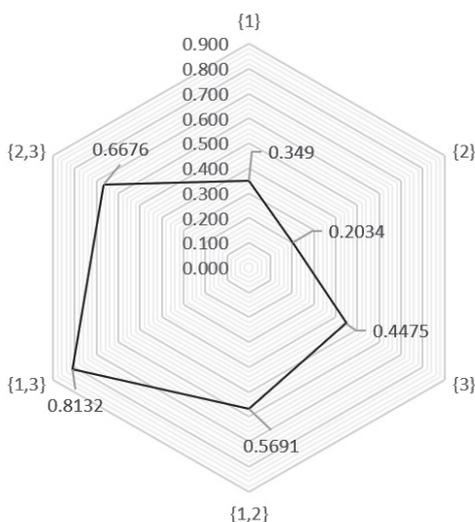


Fig. 2. Combined weight radar chart.

different attribute indicators has different degrees of importance, and the assumption that attribute indicators are independent may not hold in practical applications, therefore, the application scope of traditional multi-attribute decision-making based on additive measures is limited. In combination with the idea of game theory, we use the Shapley value with the λ -fuzzy measures, which can determine the attribute weight according to the contribution degree when considering the correlation between attribute indicators. The optimal fuzzy measure linear programming model is established, and the generalized Shapley value of each layer attribute indicator is determined based on the similarity. The arithmetic λ -Shapley-Choquet integral is applied for information aggregation to obtain the comprehensive evaluation value, to evaluate the forest ecological carrying capacity of the five provinces in the past 17 years. It is worth mentioning that the method given in this article is a useful supplement to the provincial forest ecological carrying capacity evaluation method without considering the interaction of attribute indicators.

Acknowledgments

This work was supported by the National Social Science Foundation (No. 22BGL303): Research on dynamic game strategy and long-term win-win mechanism of cooperative management on collective forest land.

Competing Interests

The authors declare no competing interests.

References

- CHUMAKOV A.N. Our common future. Value Inquiry Book Series, **276**, 359, **2014**.
- TANG X., GUAN X.L., LU S.S., QIN F., LIU X., ZHANG D.H. Examining the spatiotemporal change of forest resource carrying capacity of the Yangtze River Economic Belt in China. Environmental Science and Pollution Research, **27** (17), 21213, **2020**.
- LIAO W.M., QIN K.Q., QIU H.L., GAO X.P. Study on dynamic relationship between forest ecological carrying capacity and environmental pollution pressure – taking seven provinces in the Middle East Regions for examples. Forestry Economics, **40** (3), 46, **2018**.
- LIAO W.M., QIN K.Q., TONG T., PENG T.Z. Coupling relationship between urbanization and forest ecological carrying capacity on the Yangtze River Economic Belt. Journal of Zhejiang A & F University, **36** (2), 349, **2019**.
- LI Y., HUANG C.J., WANG K., WANG Y.T., ZHANG D.H. Forest ecological carrying capacity evaluation and gravity center transfer analysis in anhui province. Resources and Environment in the Yangtze Basin, **30** (01), 87, **2021**.
- JIANG J., YANG C.Y., CHEN C.G., LU Y.L., CHEN H.L., FAN J., ZHANG Y.P. Construction of evaluation index system of forest ecological carrying capacity – taking Guangdong Province as an example. Forestry and Environmental Science, **37** (04), 146, **2021**.
- MI Y., WU P.X., ZHAI X.J., CI S., DONG Q., ZHANG M.F., ZHAO G.P. Study on dynamic relationship between forest ecological carrying capacity and environmental pollution pressure in Shaanxi Province. South China Agriculture, **14** (21), 166, **2020**.
- CUI H.T., CUI Y.D., ZENG W.C., WANG F.Y., XIONG N. Dynamic relationship between forest ecological carrying capacity and environmental pollution pressure in western China. Southwest China Journal of Agricultural Sciences, **34** (11), 2510, **2021**.
- SONG L.S., WANG H., YANG C., SONG W.M., WANG S.Y., WANG S. Forest environmental carrying capacity based on deep learning. Computational intelligence and neuroscience, **2022**, 7547645, **2022**. DOI: 10.1155/2022/7547645
- MOUKHTAR I., IBRAHIM A.A., EI S.T.A., MOSTAFA S. Assessment of the environmental carrying capacity for protected areas: a study of petrified forest and Hassanah Dome, the Great Cairo. Environmental Research, Engineering & Management, **76** (4), 106, **2020**.
- ZHU L.Y., ZHANG D.H. Forest ecological carrying capacity evaluation on Changbai County in Jilin Province. Ecological Science, **35** (05), 176, **2016**.
- LIAO X., FANG C.L., SHU T.H. Multifaceted land use change and varied responses of ecological carrying capacity: A case study of Chongqing, China. Applied Geography, **148**, **2022**.
- WU C., JIANG A.D., ZHENG W.L. Study on the Measures for Optimizing China's Provincial Territorial Space Based on the Perspective of Resource and Environmental Carrying Capacity in the New Situation. Sustainability, **14** (21), **2022**.
- CHI Y.Y., BAI G.Q., DONG H. A new multicriteria decision-making method for the selection of Sponge City schemes with Shapley-Choquet aggregation operators. Mathematical Problems in Engineering, **2021**, **2021**. DOI:10.1155/2021/6615709
- MENG F.Y., CHEN X.H., ZHANG Q. Generalized hesitant fuzzy generalized Shapley-Choquet integral operators and their application in decision making. International Journal of Fuzzy Systems, **16**, (3), 400, **2014**.
- JU Y.B., YANG S.H., LIU X.Y. Some new dual hesitant fuzzy aggregation operators based on Choquet integral and their applications to multiple attribute decision making. Journal of Intelligent and Fuzzy Systems, **27** (6), 2857, **2014**.
- WAN S.P., YAN J., ZOU W.C., DONG J.Y. Generalized Shapley Choquet integral operator based method for interactive interval-valued hesitant fuzzy uncertain linguistic multi-criteria group decision making. IEEE Access, **8**, 202194, **2020**.
- KAKATI P., BORKOTOKEY S. Generalized interval-valued intuitionistic fuzzy Hamacher generalized Shapley Choquet integral operators for multicriteria decision making. Iranian Journal of Fuzzy Systems, **17** (1), 121, **2020**.
- LIN M.W., HUANG C., XU Z.S., CHEN R.Q. Evaluating IoT platforms using integrated probabilistic linguistic MCDM method. IEEE Internet of Things Journal, **7** (11), 11195, **2020**.

20. LIN M.W., HUANG C., CHEN R.Q., FUJITA H., WANG X. Directional correlation coefficient measures for Pythagorean fuzzy sets: their applications to medical diagnosis and cluster analysis. *Complex & Intelligent Systems*, **7** (2), 1025, **2021**.
21. HUANG C., LIN M.W., XU Z.S. Pythagorean fuzzy MULTIMOORA method based on distance measure and score function: its application in multicriteria decision making process. *Knowledge Information Systems*, **62** (11), 4373, **2020**.
22. MANJOT K., AMIT K. An algorithm for solving fuzzy maximal flow problems using generalized triangular fuzzy numbers. *International journal of hybrid intelligent systems*, **8** (1), 15, **2011**.
23. LIU P.D., JIN F. A weighted aggregation operators multi-attribute group decision-making method based on interval-valued trapezoidal fuzzy numbers. *Applied Mathematical Modelling*, **38** (6), 1053, **2011**.
24. CHEN S.J., CHEN S.M. Fuzzy risk analysis based on similarity measures of generalized fuzzy numbers. *Journal of Zhengzhou University*, **11** (1), 45, **2013**.
25. WANG L., YI P.T., LI W.W., LIU J. Generalized Trapezoidal Fuzzy Number Density Weighted Operator and Its Application. *Operations Research and Management Science*, **28** (12), 106, **2019**.
26. LIN M.W., WANG H.B., XU Z.S., YAO Z.Q., HUANG J.L. Clustering algorithms based on correlation coefficients for probabilistic linguistic term sets. *International Journal of Intelligent Systems*, **33** (12), 2402, **2018**.
27. OMAR M. International legal protection of forests in the context of sustainable development. *IOP Conference Series, Earth and Environmental Science*, **779**, 012131, **2021**.
28. ADAMOWICZ W. Economic indicators of sustainable forest management: theory versus practice. *Journal of Forest Economics*, **9** (1), 27, **2003**.
29. WU Y.K., SONG R.S., CHEN B. Group decision making for information security risk assess method based on Trapezoidal Fuzzy Number. *Journal of Nanjing Normal University(Engineering and Technology Edition)*, (1), 51, **2011**.
30. SHIH H.S., SHYUR H.J., LEE E.S. An extension of TOPSIS for group decision making. *Mathematical & Computer Modelling*, **45** (7-8), 801, **2007**.
31. BEKEIEN S., VASILIAUSKAS A.V., HOSKOVA M.Š., VASILIENE V.V. Comprehensive assessment of distance learning modules by fuzzy AHP-TOPSIS method. *Mathematics*, **9** (4), 1, **2021**.
32. ZHANG W.Y., CHEN J., ZHANG G.F., LIN L., ZHANG G.Y. Optimization model of multi-attribute ship form project based on Choquet fuzzy integral operator. *Journal of Dalian University of Technology*, (4), 380, **2015**.
33. WU Y.F., ZHANG Y.F., FANG X., LIAN P.T., LV X. Ecological restoration effect evaluation of artificial reefs with choquet integral algorithm. *Marine Environmental Science*, **41** (3), 467, **2022**.
34. SUGENO M. Theory of fuzzy integrals and its applications. Doctoral Thesis, Tokyo Institute of Technology, **1974**.
35. MARICHAL J.L. The influence of pseudo-Boolean function with application to game theory and multicriteria decision making. *Discrete Applied Mathematics*, **107** (1), 139, **2007**.
36. SHAPLEY L.S. A value for n-person game. *Classics in game theory*, 69-79, **1997**.
37. MENG F.Y., ZHANG Q., CHENG H. Approaches to multiple-criteria group decision making based on interval-valued intuitionistic fuzzy Choquet integral with respect to the generalized λ -Shapley index. *Knowledge-Based Systems*, **37** (1), 237, **2013**.
38. GRABISCH M. The application of fuzzy integrals in multicriteria decision making. *European Journal of Operational Research*, **89** (3), 445, **1996**.