Original Research

Ideal Point Interval Recognition Model for Dynamic Risk Assessment of Water Inrush in Karst Tunnel and Its Application

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Received: 1 September 2023 Accepted: 1 October 2023

Abstract

Water inrush has become one of the main engineering hazards in tunnel and underground engineering construction. A new ideal point interval recognition model for risk assessment of water inrush was proposed to accurately predict and effectively prevent the hazard. Given the complexity and uncertainty of the geological conditions of tunnel engineering, a continuous interval of a small range was used to assign the evaluation index instead of a fixed value. The positive and negative ideal points and the ideal distance measure function were improved. The fusion method of multi-index ideal distance measure interval and the risk classification standard based on ideal closeness degree was presented. The integrated weighting method combining the analytic hierarchy process (AHP) and frequency statistic method was introduced to determine the weight of the evaluation index. The AHP was improved based on the proposed 1~5 scale and triangular fuzzy theory. Considering the dynamic risk change of water inrush, a dynamic risk assessment method was established to realize the process control of the hazard including the preliminary assessment and secondary assessment. The risk-pregnant environment factors were selected to evaluate the preliminary risk before tunnel construction. In the construction of the tunnels, the environmental factors were modified and the riskcausing factors were introduced to evaluate the secondary risk. The proposed method was used to dynamically evaluate the risk of water inrush in the river-crossing section of the Yuelongmen Tunnel from Chengdu to Lanzhou Railway. The evaluation results were in good agreement with the actual

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situation. The method has better grade discrimination and risk identification and has a certain guiding significance for risk prevention and control of tunnel and underground engineering geological hazards.

Keywords: Karst tunnel, water inrush, dynamic evaluation, ideal interval recognition model, engineering application

Introduction

With the focus of China's infrastructure construction such as roads, railways, water conservancy, and hydropower gradually shifting to the western karst mountain area, a large number of high-risk karst tunnels have emerged. Once encountering karst caves, faults, karst pipelines, and other bad geology during tunnel excavation, it is very easy to induce geological hazards such as water and mud inrush, collapse, and rock burst [1, 2]. According to statistics, water and mud inrush have become one of the most frequent and harmful geological hazards in tunnel construction, seriously affecting tunnel and underground engineering [3, 4]. Therefore, it is very necessary to carry out research on risk assessment, prediction, and early warning of water inrush in karst tunnels.

In recent decades, scholars at home and abroad have conducted a few researches regarding the risk management and risk assessment of water and mud inrush hazards [5-7]. In terms of risk assessment model, the attribute mathematical theory [7, 8], fuzzy mathematical theory [9, 10], analytic hierarchy analysis [11], cloud model [12, 13], set pair analysis [14], grey theory [15], extension theory [16], random forest model [17], were used to establish a risk assessment model of tunnel water inrush. However, the existing risk assessment models have the following questions in the application process. First, it is difficult to characterize the uncertainty and complexity of the geological conditions. Second, the applicability of each model is different. Therefore, Li et al. [18] proposed an attribute interval evaluation model based on the attribute mathematical theory. Li et al [19] subdivided the identity, opposition, and difference into the identity, good and bad opposition, and good and bad difference, and introduced the fuzzy mathematical theory to determine the connection function to improve the set pair analysis method. Based on the fuzzy mathematical theory, Wang et al. [20] used an interval number to present the evaluation index values, membership degrees, and weight vector, and carried out a relative superiority analysis of the interval matrix. Wang et al. [21] presented a non-linear attribute measure function based on a normal distribution function and adopted an interval to quantify the evaluation index. Yang and Zhang [22] improved the linear measurement functions in the attribute mathematical theory based on the trigonometric function. Yuan et al. [23] used the center triangle whitening weight function and upper and lower limit whitening weight function to solve the crossing properties of the grey clustering and presented

a modified grey clustering model. Ye et al. [24] proposed a highly coupled fractal analysis model for tunnel excavation by coupling the porous media fractal theory with multi-field effects to assess the risk of water inrush.

As a common multi-target decision method, the ideal point model can realize the comprehensive assessment of multiple factors and multiple objects at the same time. Due to the simple principle and calculation, it was widely applied in the field of rockburst prediction [25], rock mass quality classification [26], risk evaluation of water inrush [27], and failure risk of prestressed anchor cable [28]. Therefore, the ideal point method was introduced for risk recognition of water inrush in karst tunnels. A new ideal point interval recognition model was proposed. The weight of the evaluation index was determined based on the improved AHP and frequency statistic method. The risk-pregnant factors and riskcausing factors were selected as the evaluation index of water inrush. A dynamic risk assessment method and the early warning criteria for tunnel water inrush were established.

Material and Methods

Traditional Ideal Point Model

The basic principle of the ideal point model is to regard the evaluation object as a pint in the *m*-dimensional space, construct the positive and negative ideal points based on the prior information, and then use the constructed objective function to find the feasible solution that is closest to the positive ideal point and farthest from the negative ideal point. The method has the advantages of simple principle, easy calculation, and high resolution, and is widely used in multi-objective optimization decision-making problems.

(1) Evaluation index decision-making matrix construction

Assuming that an evaluation object contains n evaluation index I_j (j=1, 2,..., n), and the index I_j is regarded as the jth objective function of the decision-making for the evaluation object. The objective function vector is defined as:

$$F(x) = [f_1(x), f_2(x), ..., f_n(x)]$$
 (1)

The weights corresponding to the n objective functions are denoted as:

$$W = [w_1, w_2, ..., w_n]$$
 (2)

Where
$$0 < w_j < 1$$
, $\sum_{j=1}^{n} w_j = 1$.

The ideal value of the object to be evaluated under the objective function $f_j(x)$ is defined as μ_j . The judgment matrix can be constructed as follows:

$$R = \{\mu_1, \mu_2, ..., \mu_n\} \left[w_1, w_2, ..., w_n\right]^T$$
(3)

(2) Selecting a positive ideal point and negative ideal point

The evaluation indices can be divided into 2 categories: very large type and very small type. For very large indices, the larger the value, the more dangerous it is. For very small indices, the smaller the value, the more dangerous it is. It is assumed that each evaluation index I_j (j = 1, 2, ..., n) can be divided into K risk levels, as shown in Table 1.

When the evaluation index belongs to the very large type, and the $a_{jk} < b_{jk}$, $a_{j1} < a_{j2} < ... < a_{jK}$ and $b_{j1} < b_{j2} < ... < b_{jK}$ are satisfied, the definitions of positive ideal point and negative ideal point are as follows:

$$\begin{cases}
f_{jk}^*(+) = b_{jk} \\
f_{jk}^*(-) = a_{jk}
\end{cases}$$
(4)

When the evaluation index belongs to the very small type, and the $a_{jk} < b_{jk}$, $a_{j1} > a_{j2} > ... > a_{jK}$, and $b_{j1} > b_{j2} > ... > b_{jK}$ are satisfied, the definitions of positive ideal point and negative ideal point are as follows:

$$\begin{cases} f_{jk}^*(+) = a_{jk} \\ f_{jk}^*(-) = b_{jk} \end{cases}$$

$$(5)$$

Where $f_{jk}^*(+)$ and $f_{jk}^*(-)$ are the positive ideal point and negative ideal point of the evaluation index I_j belonging to the risk level C_k respectively. a_{jk} and b_{jk} are the upper limit and lower limit of the evaluation index I_j belonging to the risk level C_k respectively.

(3) Constructing ideal point functions

The distance between the measured value of the evaluation index and the ideal point is defined as the ideal point function. When the distance between the index and the positive ideal point is smaller,

Table 1. Classification criteria of evaluation index.

Evaluation		Risk level					
index (I_j)	$C_{_1}$	C_2	•••	C_{K}			
I_1	$a_{11} \sim b_{11}$	$a_{12} \sim b_{12}$	•••	$a_{\scriptscriptstyle 1K} \sim b_{\scriptscriptstyle 1K}$			
I_2	$a_{21} \sim b_{21}$	$a_{22} \sim b_{22}$	•••	$a_{2K} \sim b_{2K}$			
I_n	$a_{n1} \sim b_{n1}$	$a_{n2} \sim b_{n2}$	•••	$a_{nK} \sim b_{nK}$			

and the distance between the index and the negative ideal point is larger, the risk level is considered higher. The functional expression is as follows:

$$||f(x)-f^*(+)|| \to \min, ||f(x)-f^*(-)|| \to \max_{(6)}$$

The Minkowski distance is generally selected as the ideal point function, and the distance D between the evaluation object and the ideal point in the n-dimensional space is calculated as

$$\begin{cases}
D_{1k} = \left\{ \sum_{j=1}^{n} w_{j} \left[\frac{f_{j}(x_{j}) - f_{jk}^{*}(+)}{f_{j}^{*U} - f_{j}^{*L}} \right]^{P} \right\}^{1/P} \\
D_{2k} = \left\{ \sum_{j=1}^{n} w_{j} \left[\frac{f_{j}(x_{j}) - f_{jk}^{*}(-)}{f_{j}^{*U} - f_{j}^{*L}} \right]^{P} \right\}^{1/P}
\end{cases} (7)$$

Where D_{1k} and D_{2k} are the multi-index distance measure values between the object to be evaluated and the positive ideal point, and the negative ideal point, respectively. w_j is the weight of the evaluation index I_j . f_j^{*U} and f_j^{*L} are the upper limit and lower limit of the index I_j respectively. x_j is the actual value of the index I_j . P is the Minkowski distance function coefficient, which is usually taken as P = 2.

(4) Ideal point closeness

The ideal point closeness is used to describe the degree of the object to be evaluated belonging to the risk level $C_k(k = 1, 2, ..., K)$. Its calculation formula is as follows:

$$T = D_2 / (D_1 + D_2) \tag{9}$$

Where $0 \le T \le 1$. The larger the closeness T, the smaller the distance between the object and the positive ideal point, and the larger the distance value between the object and the negative ideal point.

Improved Ideal Point Interval Model

The traditional ideal point model has the following shortcoming when making a target decision: (1) A fixed value is often adapted to quantify the evaluation index. Due to the complexity and uncertainty of the geological conditions, the measured value of the evaluation index is assigned by a small range of intervals. (2) The unreasonable selection of the ideal points leads to the confusion of the risk level. As an example, it is assumed that the object to be evaluated is a point in one-dimensional space, and its location coordinate is 60, as shown in Fig. 1. The distance between the object and the positive and negative ideal points and the closeness degree are calculated, as shown in Table 2.

Fig. 1. 1-D location map of the evaluation object and the ideal points.

As can be seen from Table 1., the risk of the object is Level IV. The location point of the object belongs to Level III. Therefore, the ideal points and ideal point functions are improved.

(1) For a very large index, the maximum value is selected as the positive ideal point and the minimum value as the negative ideal point. For a very small index, the opposite is true. The specific formula is as follows:

When the evaluation index belongs to a very large type:

$$\begin{cases} f_j^*(+) = \max f_j(x) \\ f_j^*(-) = \min f_j(x) \end{cases}$$
 (10)

When the evaluation index belongs to a very small type:

$$\begin{cases} f_j^*(+) = \min f_j(x) \\ f_j^*(-) = \max f_j(x) \end{cases}$$
(11)

Where $f_j(x)$ represents the actual value of the *i*th evaluation index.

(2) According to the selected positive and negative ideal points, the ideal distance measure functions D are constructed based on Eq.(7).

$$\begin{cases}
D_{1j} = \left[\frac{f_j - f_j^*(+)}{f_j^{*U} - f_j^{*L}} \right]^2 \\
D_{2j} = \left[\frac{f_j - f_j^*(-)}{f_j^{*U} - f_j^{*L}} \right]^2
\end{cases} \qquad \underline{f}_j \leq f_j \leq \overline{f}_j$$
(12)

(3) To accurately characterize the complexity and uncertainty of the geological conditions along the tunnel, the value of the evaluation index I_j s extended to a continuous small range of mathematical interval

 $[\underline{f_j}(x), \overline{f_j}(x)]$. However, the ideal point functions can only realize the superposition of single values of multiple evaluation indices, and cannot be directly used for the superposition of interval values of multiple evaluation indices.

Therefore, for any $f_j(x) \in [\underline{f}_j(x), \overline{f}_j(x)]$, the distance between the actual measured value of the evaluation index and the ideal points can be calculated by Eq.(12). A continuous distance interval $[\underline{\mu}_j, \overline{\mu}_j]$ will be obtained.

For any $\mu_j \in \left[\underline{\mu}_j, \overline{\mu}_j\right]$, if Eq.(7) is used for the superposition of the distances between multiple evaluation indices and the ideal points, both D_{1k} and D_{2k} have countless values and are discontinuous. Therefore, it is necessary to first deal with the single-index distance interval $\left[\underline{\mu}_j, \overline{\mu}_j\right]$. The specific formula is as follows:

$$u'_{j} = \alpha_{1} \underline{\mu}_{j} + \alpha_{2} \overline{\mu}_{j}$$

$$\alpha_{1} + \alpha_{2} = 1$$

$$\alpha_{1} > 0, \alpha_{2} > 0$$
(13)

Where μ_j' is the weighted average value of the ideal point distance interval $\left[\underline{\mu}_j, \overline{\mu}_j\right]$. α_1 and α_2 are the weighting coefficient of the lower limit $\underline{\mu}_j$ and upper limit $\overline{\mu}_j$ of the ideal point distance respectively, which are determined by the experts according to the specific situation.

(4) The weighted summation of the distances between the multiple evaluation indices and the ideal points in the *n*-dimensional space is calculated as follows:

$$\begin{cases}
D_{1k} = \left\{ \sum_{j=1}^{n} w_{j} \mu_{1jk} \right\}^{1/2} \\
D_{2k} = \left\{ \sum_{j=1}^{n} w_{j} \mu_{2jk} \right\}^{1/2}
\end{cases}$$
(14)

(5) Eq.(6) is used to calculate the closeness degree between the object to be evaluated and the ideal point. However, the obtained closeness degree is a single value, which cannot effectively characterize the risk level of the object. Therefore, according to its variation range, the closeness degree is divided into four risk levels on average: Level IV, Level III, Level II, and Level I, as shown in Table 3.

Table 2. Calculation results of the given example.

Risk level	Distance to the positive ideal point	Distance to the negative ideal point	Closeness degree
Level I	0.35	0.6	0.368
Level II	0.1	0.35	0.222
Level III	0.15	0.1	0.6
Level IV	0.4	0.15	0.727

Table 3. Grading criteria of water inrush based on degrees keeping close to the ideal point.

Risk level	Level I	Level II	Level III	Level IV
T	0~0.25	0.25~0.5	0.5~0.75	0.75~1.0

Dynamic Risk Assessment Method of Water Inrush in Karst Tunnels

It is well known that the risk of water inrush in the process of tunnel construction is changing. To realize the dynamic identification of the risk, a two-stage dynamic risk assessment method for water inrush in karst tunnels is proposed: the preliminary assessment and the secondary assessment. The preliminary assessment is carried out in the survey and design stage, which can provide a reference for the design. The secondary assessment is carried out in the construction stage, which can provide a reference for the tunnel excavation.

Dynamic Evaluation Index System for Water Inrush

Based on a large number of tunnel water inrush case statistics, the influencing factors of water inrush can be divided into 3 categories: geological and hydrological factors, climatic and environmental factors, and construction and design factors. The geological and hydrological factors include the formation lithology, geological structure, groundwater, topography and

geomorphology, rock formation occurrence, and unconformity structural plane. The climatic and environmental factors mainly include temperature and rainfall. The construction and design factors include the excavation method, support measures, advance geological forecast, and monitoring and measurement.

Based on the statistical analysis of the important influencing factors of water inrush, combined with the existing research [18], the dynamic risk assessment index system of water inrush in karst tunnels is established. For the preliminary assessment, the hazard-pregnant environment factors including formation lithology I_1 , unfavorable geology I_2 , groundwater level I_3 , topography and geomorphology I_4 , dip angle of rock formation I_5 , unconformity structural plane I_6 are selected as the evaluation indices. The construction and support I_{τ} , and advance geological forecast I_{g} based on the hazardpregnant environment indices are added as evaluation indices of the secondary evaluation. The risk of water inrush from low to high is divided into Level IV (Low risk), Level III (Medium risk), Level II (High risk), and Level I (Very high risk), as shown in Fig. 2.

It is difficult to quantitatively describe the formation lithology I_1 , unfavorable geology I_2 , unconformity

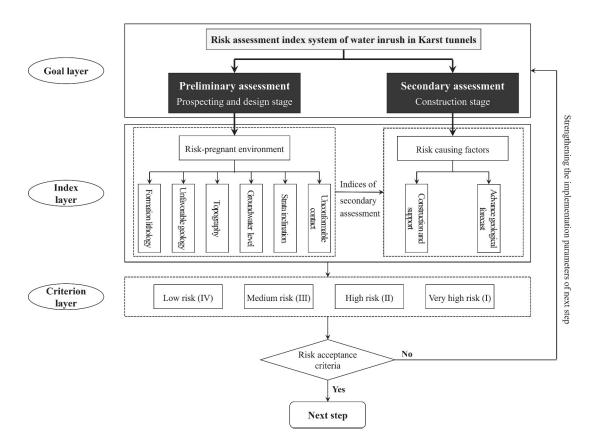


Fig. 2. Two-stage dynamic risk assessment method of water inrush in the karst tunnels.

structural plane I_6 , construction and support I_7 , and advance geological forecast I_8 . Therefore, according to the qualitative grading standard, the expert scoring method is used to quantify these indices. The negative terrain area ratio is used to quantify the topography and geomorphology I_4 . The dip angle of rock formation $25^\circ\sim65^\circ$ is the most favorable for karst development, but the index does not meet the definition of a very large index and very small index. To satisfy the feasibility calculation of the ideal interval evaluation method, the index I_5 is corrected. The grading standard of the evaluation indices is shown in Table 4.

Early Warning and Risk Acceptance Criteria

The dynamic assessment results can only reflect the possibility of water inrush, but can not reflect the harm degree. Therefore, the water inflow is introduced to establish a four-color early warning method for water inrush, as shown in Table 5.

To effectively avoid the occurrence of water inrush, the risk acceptance criteria are formulated. That is, the acceptance line is introduced to divide the risk into acceptable area and unacceptable area. When the risk level of the assessment is in the unacceptable area, the support parameters, excavation methods, and monitoring

and measurement can be dynamically adjusted to reduce the risk to an acceptable area, as shown in Table 6.

Index Weighting Method

The reasonable weighting method of the evaluation indices is very important for the risk evaluation results of water inrush. Therefore, an integrated method based on subjective weight and objective weight is used to determine the weighting of the evaluation indices.

$$w_i = \beta w_{is} + (1 - \beta) w_{io} \tag{15}$$

Where w_{is} is the subjective weight of the index I_i , which is determined by the improved analytic hierarchy process method. w_{io} is the objective weight of the index I_i , which is determined by the improved analytic hierarchy process method. β and 1- β are the distribution coefficients of subjective weight and objective weight, and their specific values are determined by the experts according to the field situation.

(1) The objective weight

Through the frequency statistics of the influencing factors of the water inrush examples, Li et al [9] obtained the objective weights of the evaluation indices $I_1 \sim I_5$. That is, (formation lithology I_1 , unfavorable geology I_2)

Table 4. Indices and criteria for risk assessment of water inrush in karst tunnels.

T 1		Risk level		
Index	Level I	Level II	Level III	Level IV
I_{1}	Thick to medium-thick strong- soluble rock, such as pure limestone, ancient siliceous cemented dolomite, carbonaceous and asphaltene limestone.	Thick to medium-thick medium-soluble rock, such as marble rock, dolomite, and argillaceous limestone.	Thin weak-soluble rock, such as marble rock, dolomite, argillaceous limestone	Non-soluble rock, such as sandstone, shale, etc.
	[90, 100]	[80, 90)	[60, 80)	[0, 60)
I_2	There are large water-bearing and water- conducting structures near the tunnels	There are medium water-bearing and water-conducting structures near the tunnels	There are small water- bearing and water- conducting structures near the tunnels	There are no water- bearing or water- conducting structures
	[90, 100]	[80, 90)	[60, 80)	[0, 60)
$I_3(h)$	<i>h</i> ≥60 m	30 m≤ <i>h</i> <60 m	10 m <h<30 m<="" td=""><td><i>h</i>≤10 m</td></h<30>	<i>h</i> ≤10 m
I_4	Large negative terrain with a strong catchment capacity	Medium-sized negative terrain with medium catchment capacity	Small negative terrain with low catchment capacity	No negative terrain
	[60%, 100%]	[40%, 60%)	[20%, 40%)	[0, 20%)
$I_{5}(\varphi)$	25°<φ≤45°	10°<φ≤25°	5°<φ≤10°	0°<φ≤5°
I_6	Strongly conducive to karst development	Moderately conducive to karst development	Weakly conducive to karst development	No conducive to karst development
6	[90, 100]	[80, 90)	[60, 80)	[0, 60)
7	Very unreasonable	Unreasonable	Basically reasonable	Reasonable
I_7	[0, 60)	[60, 80)	[80, 90)	[90, 100]
ī	Very inaccurate	Inaccurate	Basically accurate	Accurate
I_8	[0, 60)	[60, 80)	[80, 90)	[90, 100]

Four-colo	Four-color warning		Water inflow (m ³ ·d ⁻¹)				
>10	0000	3000~10000	500~3000	< 500			
	Level I	Red	Orange	Orange	Yellow		
Risk level	Level II	Orange	Orange	Yellow	Yellow		
KISK IEVEI	Level III	Orange	Yellow	Yellow	Blue		
	Level IV	Yellow	Yellow	Blue	Blue		

Table 5. Four-color warning method for water inrush.

groundwater level I_3 , topography and geomorphology I_4 , dip angle of rock formation I_5) = (0.188, 0.388, 0.259, 0.109, 0.056). However, the unconformity structural plane I_6 is not considered. Its objective weight is determined according to the reference [11], that is $w_{6s} = 0.180$. The index weights of the preliminary assessment can be obtained by normalizing the above weights:

$$W_s = (0.159, 0.329, 0.219, 0.092, 0.047, 0.153)$$
 (16)

For the secondary assessment, the risk-causing factors including construction I_7 and support, advance geological forecast I_8 , are introduced. Their objective weights are determined according to the frequency statistics in the reference [11]. That is, $w_{7s} = 0.048$ and $w_{8s} = 0.192$. The objective weight vector of the secondary evaluation indices is as follows:

$$W_s = (0.128, 0.266, 0.177, 0.074, 0.038, 0.123, 0.039, 0.155)$$
 (17)

(2) The subjective weight

The analytic hierarchy process (AHP) method is used to calculate the subjective weight of the evaluation indices. The AHP can give full play to the experts' experience and knowledge but it also has two limitations: One is that the index weights are easily affected by subjectivity and risk preference. The other is that there are fuzziness and uncertainty in the relative importance between the evaluation indices, and using the single scale to quantify the importance can easily lead to information loss. Therefore, the triangular fuzzy number theory (TFN) is introduced to improve the AHP, and the specific steps are as follows:

a. The triangular fuzzy number $M_{ij} = (r^l_{ij}, r^m_{ij}, r^u_{ij})$ is used to characterize the relative importance between the evaluation indices I_i and I_j , where r^l , r^m , and r^u represent the lower limit value, the most likely value, and the upper limit value, respectively.

The relative importance between the evaluation indices is quantified based on $1\sim9$ scales method proposed by the Saaty [29]. However, the $1\sim9$ scales method is easy to cause scale confusion and unqualified consistency checking. Therefore, a new $1\sim5$ scales method is proposed to determine the values of r^{I} , r^{m} , and r^{u} , as shown in Table 7.

b. A n-order judgment complementary matrix can be constructed from the triangular fuzzy number M_{ij} , denoted as $M = (M_{ij})_{n \times n}$. The matrix needs to satisfy:

$$M_{ii} \otimes M_{ii} = 1$$
, that is $M_{ii} = (M_{ii})^{-1}$ (18)

c. The triangular fuzzy matrix M is defuzzified. Taking $M_{ij} = (r^l_{ij}, r^m_{ij}, r^u_{ij})$ as an example, its calculation formula is as follows:

$$r_{ij} = \frac{r_{ij}^{l} + 4r_{ij}^{m} + r_{ij}^{u}}{6} \tag{19}$$

$$R = (r_{ij})_{n \times n} \tag{20}$$

d. The AHP is used to solve the subjective weight vector Wo of the matrix R according to the reference [30]. And consistency checking is carried out.

$$W_o = (w_{o1}, w_{o2}, ..., w_{on})$$
 (21)

Results and Discussion

Engineering Background of Yuelongmen Tunnel

The Yuelongmen Tunnel is one of the key control projects of the Chengdu-Lanzhou Railway. The tunnel is repaired by two separate lines with a left line length of 19974.3 m and a right line length of 20044.0 m. The maximum buried depth is about 1445.5 m. The tunnel area is located in the central Longmen Mountain Fault zone and passes through the Gaochuanping active fault, Gaochuanping overturned syncline, and Qianfoshan fault. The geological conditions are very complex, and show typical "four extreme and three high" characteristics, namely " extremely strong terrain cutting, extremely complex and active structure condition, extremely weak and broken lithology condition, extremely significant Wenchuan earthquake effect, high geostress, high earthquake intensity, and high geological hazard risk" [31]. Therefore, engineering geological problems such as active fault, high geostress, large deformation of soft rock, and karst in the tunnel area are significant.

Toble 6	Diele	accentance	critaria	for	water inrush.	
Table o.	KISK	acceptance	criteria	IOT	water inrush.	

W1	Acceptance	Treatment measures			
Warning level	criteria	Preliminary assessment	Secondary assessment		
Red	Non-acceptable	Special design for support and excavation	Stop work. Expert demonstration and strengthening monitoring		
Orange	Unacceptable	Strengthening support and construction design	Stop work. Required measures need to be taken and strengthening monitoring		
Yellow	Acceptable	-	Strengthening monitoring		
Blue	Negligible	-	Construction		

Table 7. The 1~5 scales method.

Scale	Linguistic scale for importance
1	I_i and I_j are equally importance
2	I_i is slightly more important than I_j
3	I_i is obviously more important than I_j
4	I_i is strongly more important than I_j
5	I_i is extremely more important than I_j

The section D1K93+440~D2K96+250 of Yuelongmen Tunnel is in the deep circulation zone of groundwater. The lithology is dominated by soluble rocks such as dolomitic limestone, and limestone. The karst is moderately developed. There are several rivers in the tunnel area. The section YD2K94+605~+701 passes through the Suishui River, and the minimum buried depth is about 50m, as shown in Fig. 3. The proposed dynamic risk assessment method based on the new fuzzy interval recognition model is used to evaluate

the water inrush risk of the river-crossing section YD2K94+605~+701 in the right line of the tunnel. According to the engineering geological survey, special hydrological survey, and other data of the river-crossing section from the Yuelongmen Tunnel, the measured interval values of the preliminary assessment indices are determined, and the interval values of the secondary assessment indices are modified in combination with geological conditions revealed by the on-site excavation, as shown in Table 8.

Integrated Weighting Determination

The proposed TFN-AHP method is used to construct the triangular fuzzy judgement matrix for the dynamic risk assessment of water inrush in karst tunnels. The matrix is defuzzified and the weighting calculation is carried out. The subjective index weights of the preliminary assessment and secondary assessment are obtained. Moreover, the constructed judgement matrix satisfies the consistency checking, as detailed in Table 9.

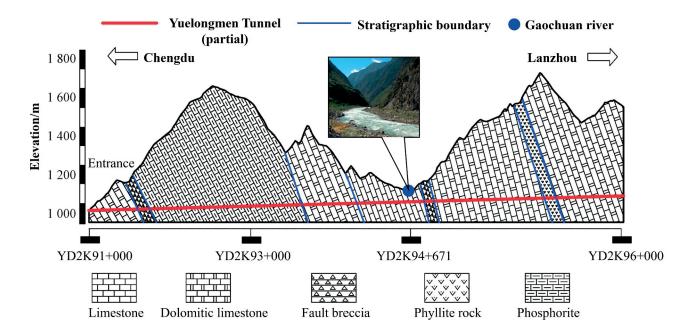


Fig. 3. Engineering geological profile of Yuelongmen Tunnel.

Table 8. Measured interval value of dynamic evaluation index in the river-crossing section of Yuelongmen Tunnel.

I_8	ı	ı	Basically accurate	[65,70]
I_7	ı	-	Basically reasonable	[65,70]
I_{ϵ}	Para-unconformity contact with the underlying Zongchanggou Group (C1zn) of the Lower Carboniferous System	[85,90]	The fissures are developed	[80,85]
$I_{s}/^{\circ}$	75°-80°	[75,80]	Correction according to exposed geological condition	[80,85]
$I_4^{\prime 0/0}$	The Gaochuan River on the surface is developed and has a strong catchment capacity.	[70,75]	As the same as above	[70,75]
I_3/m	The minimum buried depth of this section is 51 m, and the hydraulic connectivity between groundwater and the Gaochuan River is good.	[63,67]	As the same as above	[63,67]
I_2	The Gaochuanping overturned syncline and secondary reverse fault are developed, which are good water-bearing and water-conducting structures	[85,90]	The geological structures are developed, and their water conductivity is good	[85,90]
I_1	The lithology is mainly mediumthick layered limestone, which is soluble rock	[56,95]	The lithology is mainly hard limestone, and its dissolution is not obvious.	[80,85]
Index	Preliminary assessment		Secondary	

Note: The groundwater level I_i represents the elevation difference between the groundwater level and the tunnel floor [9].

Table 9. Triangular fuzzy judgment matrix for subjective weights analysis.

$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$									
$(1,1,1) \qquad (1/3,1/3,1/2) \qquad (1/3,1/3,1/2) \qquad (2,2,3)$ $(2,3,3) \qquad (1,1,1) \qquad (1,1,1) \qquad (3,4,4)$ $(2,3,3) \qquad (1,1,1) \qquad (1,1,1) \qquad (3,4,4)$ $(1/3,1/2,1/2) \qquad (1/4,1/4,1/3) \qquad (1/4,1/4,1/3) \qquad (1,1,1)$ $(1/3,1/2,1/2) \qquad (1/5,1/5,1/4) \qquad (1/5,1/5,1/4) \qquad (1/3,1/2,1/2)$ $(1/3,1/2,1/2) \qquad (1/4,1/4,1/3) \qquad (1/4,1/4,1/3) \qquad (1,1,1)$ $(1/3,1/2,1/2) \qquad (1/3,1/2) \qquad (1/3,1/2) \qquad (2,2,3)$ $(1/4,1/3,1/2) \qquad (1/5,1/4) \qquad (1/5,1/5,1/4) \qquad (1/3,1/3,1/2)$ $Preliminary assessment$ $Me = [M_{ij}]_{6:6}$ $Wo = (0.151, 0.31, 0.38, 0.053, 0.088)$ $\lambda_{max} = 6.009, CI = 0.002, CR = 0.001. CI \text{ and } CR \text{ are less than } 0.1, \text{ which satisfies the consistency checking.}$	Index	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8
$(2,3,3) \qquad (1,1,1) \qquad (1,1,1) \qquad (3,4,4)$ $(2,3,3) \qquad (1,1,1) \qquad (1,1,1) \qquad (3,4,4)$ $(1/3,1/2,1/2) \qquad (1/4,1/4,1/3) \qquad (1/4,1/4,1/3) \qquad (1,1,1)$ $(1/3,1/2,1/2) \qquad (1/5,1/5,1/4) \qquad (1/5,1/5,1/4) \qquad (1/3,1/2,1/2)$ $(1/3,1/2,1/2) \qquad (1/4,1/4,1/3) \qquad (1/4,1/4,1/3) \qquad (1,1,1)$ $(1/3,1/2,1/2) \qquad (1/3,1/3,1/2) \qquad (1/3,1/3,1/2) \qquad (2,2,3)$ $(1/4,1/3,1/2) \qquad (1/5,1/5,1/4) \qquad (1/5,1/5,1/4) \qquad (1/3,1/3,1/2)$ $Preliminary assessment$ $Me = [M_{ij}]_{6:6}$ $Me = (0.151, 0.31, 0.088, 0.053, 0.088)$ $\lambda_{max} = 6.009, CI = 0.002, CR = 0.001. CI \text{ and } CR \text{ are less than } 0.1, \text{ which satisfies the consistency checking.}$	I_1	(1,1,1)	(1/3,1/3,1/2)	(1/3,1/3,1/2)	(2,2,3)	(2,3,4)	(2,2,3)	(1,1,1)	(2,3,4)
(1/3, 1/2, 1/2) (1/4, 1/4, 1/3) (1/4, 1/4, 1/3) (1/4, 1/4, 1/3) (1/4, 1/4, 1/3) (1/3, 1/2, 1/2) (1/5, 1/5, 1/4) (1/5, 1/5, 1/4) (1/5, 1/5, 1/4) (1/3, 1/2, 1/2) (1/3, 1/2, 1/2) (1/4, 1/4, 1/3) (1/4, 1/4, 1/3) (1/3, 1/3, 1/2) (1/3, 1/3, 1/2) (1/3, 1/3, 1/2) (1/3, 1/3, 1/2) $M = [M_{ij}]_{6\cdot,6}$ $\lambda_{max} = 6.009, CI = 0.002, CR = 0.001. CI \text{ and } CR \text{ are less than } 0.1, \text{ which satisfies the consistency checking.}$	I_2	(2,3,3)	(1,1,1)	(1,1,1)	(3,4,4)	(4,5,5)	(3,4,4)	(2,3,3)	(4,5,5)
$(1/3, 1/2, 1/2) \qquad (1/4, 1/4, 1/3) \qquad (1/4, 1/4, 1/3) \qquad (1,1,1)$ $(1/3, 1/3, 1/2) \qquad (1/5, 1/5, 1/4) \qquad (1/5, 1/5, 1/4) \qquad (1/3, 1/2, 1/2)$ $(1/3, 1/2, 1/2) \qquad (1/4, 1/4, 1/3) \qquad (1/4, 1/4, 1/3) \qquad (1,1,1)$ $(1/3, 1/2, 1/2) \qquad (1/3, 1/3, 1/2) \qquad (1/3, 1/3, 1/2) \qquad (2,2,3)$ $(1/4, 1/3, 1/2) \qquad (1/5, 1/5, 1/4) \qquad (1/5, 1/5, 1/4) \qquad (1/3, 1/3, 1/2)$ $Preliminary assessment$ $M = [M]_{jocoo}$ $Mo = (0.151, 0.31, 0.38, 0.053, 0.088)$ $\lambda_{max} = 6.009, CI = 0.002, CR = 0.001. CI \text{ and } CR \text{ are less than } 0.1, \text{ which satisfies the consistency checking.}$	I_3	(2,3,3)	(1,1,1)	(1,1,1)	(3,4,4)	(4,5,5)	(3,4,4)	(2,3,3)	(4,5,5)
$(1/3, 1/3, 1/2) \qquad (1/5, 1/5, 1/4) \qquad (1/5, 1/5, 1/4) \qquad (1/3, 1/2, 1/2)$ $(1/3, 1/2, 1/2) \qquad (1/4, 1/4, 1/3) \qquad (1/4, 1/4, 1/3) \qquad (1,1,1)$ $(1/4, 1/3, 1/2) \qquad (1/3, 1/3, 1/2) \qquad (1/5, 1/5, 1/4) \qquad (1/5, 1/5, 1/4)$ $(1/4, 1/3, 1/2) \qquad Preliminary assessment$ $M = [M]_{6^{\circ,6}}$ $Mo = (0.151, 0.31, 0.38, 0.053, 0.088)$ $\lambda_{max} = 6.009, CI = 0.002, CR = 0.001. CI \text{ and } CR \text{ are less than } 0.1, \text{ which satisfies the consistency checking.}$	I_4	(1/3, 1/2,1/2)	(1/4, 1/4, 1/3)	(1/4, 1/4,1/3)	(1,1,1)	(2,2,3)	(1,1,1)	(1/3,1/2,1/2)	(2,3,3)
(1/3, 1/2, 1/2) (1/4, 1/4, 1/3) (1/4, 1/4, 1/3) (1,1,1) (1/3, 1/2, 1/2) (1/3, 1/3, 1/2) (2,2,3) (1/4, 1/3, 1/2) (1/5, 1/5, 1/4) (1/5, 1/5, 1/4) (1/5, 1/5, 1/4) (1/3, 1/3, 1/2) Preliminary assessment $M = [M_{ij}]_{6:6}$ $Wo = (0.151, 0.31, 0.31, 0.088, 0.053, 0.088)$ $\lambda_{max} = 6.009, CI = 0.002, CR = 0.001. CI \text{ and } CR \text{ are less than } 0.1, \text{ which satisfies the consistency checking.}$	I_5	(1/3, 1/3,1/2)	(1/5, 1/5,1/4)	(1/5, 1/5, 1/4)	(1/3, 1/2,1/2)	(1,1,1)	(1/3, 1/2,1/2)	(1/3, 1/3, 1/2)	(1,1,1)
(1,1,1) (1/3,1/2) (1/3,1/2) (2,2,3) (1/4,1/3,1/2) (1/5,1/4) (1/5,1/4) (1/5,1/4) (1/3,1/2) Preliminary assessment $M = [M_{ij}]_{6\times 6}$ $Mo = (0.151, 0.31, 0.38, 0.053, 0.088)$ $\lambda_{max} = 6.009, CI = 0.002, CR = 0.001. CI \text{ and } CR \text{ are less than } 0.1, \text{ which satisfies the consistency checking.}$	I_6	(1/3, 1/2,1/2)	(1/4, 1/4, 1/3)	(1/4, 1/4,1/3)	(1,1,1)	(2,2,3)	(1,1,1)	(1/3,1/2,1/2)	(2,3,3)
(1/4,1/3,1/2)	I_7	(1,1,1)	(1/3, 1/3, 1/2)	(1/3, 1/3,1/2)	(2,2,3)	(2,3,3)	(2,2,3)	(1,1,1)	(2,3,4)
Preliminary assessment $M = [M_{ij}]_{6.6}$ $Wo = (0.151, 0.31, 0.31, 0.088, 0.053, 0.088)$ $\lambda_{max} = 6.009, CI = 0.002, CR = 0.001. CI \text{ and } CR \text{ are less than } 0.1, \text{ which satisfies the consistency checking.}$	I_8	(1/4,1/3,1/2)	(1/5, 1/5, 1/4)	(1/5, 1/5, 1/4)	(1/3, 1/3, 1/2)	(1,1,1)	(1/3, 1/3, 1/2)	(1/4,1/3,1/2)	(1,1,1)
$M = [M_{ij}]_{6:6}$ $Wo = (0.151, 0.31, 0.31, 0.088, 0.053, 0.088)$ $\lambda_{max} = 6.009, CI = 0.002, CR = 0.001. CI \text{ and } CR \text{ are less than } 0.1, \text{ which satisfies the consistency checking.}$			Preliminary	assessment			Secondary	assessment	
	Subjective weight	$\lambda_{max}=6.009,~CI=0$	M = [1] $Wo = (0.151, 0.31, 0.31)$ 1.002, $CR = 0.001$. CR acconsistency	M _j loss 1, 0.088, 0.053, 0.088) and CR are less than 0.1, checking.	which satisfies the	$Wo = \lambda_{max} = 8.046, CI = 0.046$	M = [(0.125, 0.26, 0.26, 0.07, 0.007, CR = 0.005, CI at consistency	M _i] ₈₋₈ 3, 0.044, 0.073, 0.124, C and <i>CR</i> are less than 0.1, y checking.	0.041) which satisfies the

The integrated weight values of the evaluation indices are calculated according to the determined objective weights and subjective weights. The distribution coefficient α is selected as 0.5. The integrated index weights of the preliminary assessment and secondary assessment can be obtained as follows:

$$W_1 = [0.155, 0.32, 0.265, 0.09, 0.05, 0.121]$$

 $W_2 = [0.127, 0.263, 0.219, 0.073, 0.041, 0.098, 0.082, 0.098]$

Positive Ideal Point and Negative Ideal Point

According to the definition of very large index and very small index, the formation lithology I_1 , unfavorable geology I_2 , groundwater level I_3 , topography and geomorphology I_4 , dip angle of rock formation I_5 , unconformity structural plane I_6 belong to very large index, while the construction and support I_7 , and advance geological forecast I_8 belong to very small index. Then, according to the upper and lower limits of the value range of each index, the positive ideal point matrix $F^*(+)$ and the negative point matrix $F^*(-)$ of the dynamic risk assessment of water inrush are determined:

(1) Preliminary assessment

$$\begin{cases}
F^*(+) = [100,100,120,100,45,100] \\
F^*(-) = [0,0,0,0,0,0]
\end{cases}$$
(22)

(2) Secondary assessment

$$\begin{cases}
F^*(+) = [100,100,120,100,45,100,100,100] \\
F^*(-) = [0,0,0,0,0,0,0]
\end{cases}$$
(23)

Risk Grading Recognition

The measured interval values of the evaluation indices in Table 8 are substituted into the Eq. (9)-(14), the distance between the object to be evaluated and the ideal point and the closeness degree of the ideal point are obtained, as shown in Table 10.

According to the interpretation results of advance geological forecast and targeted advanced drilling, it is presumed that the water inflow of the tunnel face is about 7000 m³/d. Based on the warning release criteria



Fig. 4. Water inrush situation of the river-crossing section in the Yuelongmen Tunnel [31].

in Table 5, the early warning level is orange and the risk is unacceptable. Therefore, it is necessary to stop work and take some measures to control water inrush.

Excavation Verification

After the excavation of the river-crossing section in the Yuelongmen Tunnel, the water inrush occurs on the arch roof, as shown in Fig. 4. Since the large water-bearing and water-conducting structure is developed in the river-crossing section, the water inrush is fissure-type and the water inflow is about 130 m³/h. The practicability and feasibility of the proposed method in the dynamic risk assessment and control of water inrush is verified.

Conclusions

- (1) A new ideal interval recognition model is proposed based on the ideal point method. Considering the complexity and uncertainty of the geological conditions along the tunnel, a small range of continuous interval is used to quantify the evaluation index. The positive and negative ideal points and ideal point functions are improved. And the risk grading criteria based on the ideal point closeness is proposed.
- (2) An integrated weighting method of the evaluation index based on AHP and frequency statistic method is proposed. To avoid the scale confusion of the relative

Table 10. Analysis of dynamic risk assessment results of water inrush in the river-crossing section of Yuelongmen Tunnel.

	Pr	eliminary assessme	nt	Sec	ondary assessment	
Mileage	Positive ideal point distance	Negative ideal point distance	Closeness degree	Positive ideal point distance	Negative ideal point distance	Closeness degree
D1K93+440 ~D2K96+250	0.204	0.552	0.731	0.478	0.665	0.660
Risk grading	Le	evel I (Very high ris	k)	Le	evel II (High risk)	

importance between the evaluation indices, a 1-5 scales method is put forward. The triangular fuzzy number theory is introduced to improve the subjectivity and risk preference of the AHP.

- (3) To realize the process control of water inrush, a dynamic assessment method is presented, namely the preliminary assessment in the survey and design stage and the secondary assessment in the construction stage. The early warning release criteria of water inrush are put forward combining the risk level and water inflow. And the risk acceptance criteria are developed.
- (4) The proposed dynamic risk assessment method of water inrush based on the ideal interval recognition model is applied to the river-crossing section D1K93+440~D2K96+250 in the Yuelongmen Tunnel. The evaluation results are in good agreement with the actual situation, which verifies the practicability and feasibility of the method. The proposed method has the advantages of a clear risk level and dynamic risk recognition.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (No. 52078090), Natural Science Foundation of Chongqing (cstc2021jcyj-msxmX0133, cstc2020jcyj-msxmX0679), Science and Technology Research Project of Chongqing Education Commission (KJQN202300714), the Fund of State Key Laboratory of Mountain Bridge and Tunnel Engineering (No. SKLBT-19-006; No. SKLBT- YF2106).

Conflict of Interest

The authors declare no conflict of interest.

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