Original Research

Research on Carbon Emission Reduction and Promotion Strategies for Home Appliance **Enterprises Considering Carbon Cap** and Trade Mechanism

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Abstract

The carbon cap and trade mechanism is an important policy for reducing carbon emissions. This paper investigates the selection of carbon emission reduction and promotional strategies in the production and sales processes of home appliances under the carbon cap and trade mechanism. First, we find that cooperative emission reduction and promotional decisions enhance the operational performance of home appliance enterprises. Cooperative emission reduction benefits the profits of home appliance manufacturers, and cooperative promotions benefit the profits of home appliance dealers. Second, regardless of how carbon emission reduction costs vary, the level of carbon emission reduction positively correlates with consumer low-carbon preferences and carbon trading prices. Conversely, under different investments in carbon emission reduction costs, significant differences arise in the pricing of home appliances, the level of marketing efforts, and market demand volumes. Finally, compared to price, current consumers are more concerned about home appliances' carbon emission reduction levels. At the same time, home appliance manufacturers must make reasonable investments in carbon emission reduction and effectively respond to the impact of changes in carbon trading prices.

Keywords: Carbon cap and trade, decision, firm performance, promotion

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Introduction

With the continuous deepening of the global industrialization process, carbon emissions have increased along with economic growth; such growth will continue until a peak is reached, a process known as carbon peaking [1, 2]. Governments at all levels in China have introduced corresponding carbon emission reduction policies to reduce carbon emissions and achieve 'carbon peaking' earlier. One such policy, carbon cap-and-trade, is widely considered one of the most effective ways to reduce carbon emissions [3]. Carbon cap-and-trade is a mechanism for buying and selling carbon credits; the government allocates current carbon quotas to businesses for free based on their historical carbon emissions. If a company's carbon emissions are lower than the allocated quota, it can sell the surplus carbon credits; if higher, it can buy additional credits from the market [4]. When a company's carbon quota is insufficient, it can reduce its carbon emission level or purchase additional quotas. Government environmental policies force companies to engage in energy conservation and low-carbon production [5]. Compared to price, consumers have a high demand for green products [6, 7]. Therefore, government price subsidies for green appliances can stimulate consumers' desire to purchase them, promoting increased sales. Consumer attention to environmental labels is an initial crucial stage in the decision-making process of purchasing ecofriendly products [8]. As such, low-carbon consumers hope businesses can provide information about the energy consumption of electronic products on household appliances and regularly release environmental statements, making promotional decisions more transparent and responsible for appliance companies. Senior corporate leaders increasingly understand that climate change and carbon emission regulations affect their business decisions [9]. Consequently, these companies actively attach energy efficiency labels to their eco-friendly products and regularly publish annual reports and corporate social responsibility reports. In the appliance industry, companies like Gree Electric Appliances, Midea Group, and Haier Smart Home use carbon labels to make their environmental practices more transparent to increase consumer confidence and the market share of appliances with carbon labels [10-12]. Countries including China, the European Union, and Australia are all implementing carbon cap-andtrade mechanisms [13-15]. The carbon cap-and-trade mechanism in China primarily applies to six major industries: steel, petrochemicals, power generation, cement, aviation, and papermaking. The home appliance industry is not included; however, the production, manufacturing, use, and recycling process of home appliances is closely linked to power, energy, steel, and chemicals—sectors that are significant participants in carbon trading. According to the 'National Carbon Emission Trading Market Construction Scheme (Power Generation Industry)' (hereinafter referred to as 'the Scheme'), the carbon constraints and trading policy use the power industry as a breakthrough point to develop the national carbon market. For other industries, the principle of expanding the carbon market is 'to include an industry once it is mature'. As the national carbon emission reduction policies advance deeply, the home appliance industry will likely be incorporated into the carbon cap-and-trade list. This paper considers the home appliance industry in advance and analyzes the optimal carbon emission reduction and promotional decisions for home appliance companies under the carbon capand-trade mechanism. This analysis can provide theoretical references for home appliance companies to choose appropriate operational decisions and for the government to make optimal decisions in formulating future carbon emission reduction policies for the home appliance industry.

This paper aims to address the following three issues: (1) How will the early incorporation of the carbon capand-trade mechanism into the home appliance industry affect the operation of home appliance companies? (2) Which decision-making scheme is optimal for different decision scenarios? (3) When decentralized decision-making by home appliance manufacturers and dealers has lower economic performance than cooperative decision-making, how can contract coordination improve corporate performance?

The main contributions of this paper include the following: (1) This study incorporates the carbon cap-and-trade mechanism into the home appliance industry in advance, providing a theoretical basis for the government and home appliance companies to formulate scientific carbon emission reduction policies. (2) Considering the different scenarios of carbon emission reduction and promotion among companies, this paper identifies the marginal conditions for home appliance companies to choose emission reduction strategies and presents decision-making schemes to promote the sales of green home appliances. (3) Current research on home appliance supply chains mainly focuses on developing green home appliances, overlooking the importance of promotion. According to Huh S et al. [16], products can only achieve good economic and social benefits when sold; therefore, it is necessary to incorporate carbon emission reduction and promotional decisions into the green home appliance supply chain.

The remainder of this paper is organized as follows: The relevant literature is reviewed in Section 2. The problem and presents the model assumptions in Section 3. Section 4 constructs and solves the differential game model of carbon emission reduction and promotion strategies for home appliance enterprises considering carbon caps and trade mechanisms. Section 5 compares the equilibrium results of the model and discusses the impact of the consumer low-carbon preference coefficient and carbon trading prices on the decision outcomes. Numerical examples are given in Section 6 to illustrate some results of the paper. Section 7 concludes

the paper and discusses the possible future research directions.

Literature Review

The carbon cap-and-trade mechanism refers to the process whereby the government allocates a certain carbon quota to businesses. If a company does not use its carbon quota during production, it can sell the remaining quota in the carbon trading market; conversely, if an additional quota is needed, the company can purchase more [17, 18]. As an effective means to encourage enterprises to reduce carbon emissions, this mechanism is increasingly applied in industries such as steel, petrochemicals, and electricity [19]. Decisionmaking issues regarding supply chain operations under the carbon cap-and-trade mechanism have also received attention from scholars. For example, Yang et al. [20] studied the impact of compliance and non-compliance behaviors of closed-loop supply chain participants under the carbon emission cap-and-trade mechanism. Zhang et al. [21] extended the market demand under carbon trading policies to uncertainty, exploring optimal decision-making by companies under stochastic demand. Some scholars have analyzed current policies on carbon trading and emission reduction in practice. Their content mainly extends further based on industrial agglomeration, ultimately relating to performance [22, 23]. For instance, under the conditions of carbon cap-and-trade, Zhou [24] studied the impact of resource allocation on overall corporate performance. Furthermore, Li et al. [25] took the coal supply chain as the research object and introduced four policies (emission cap, carbon tax, carbon trading, and carbon offset) as constraints, establishing a comprehensive decision-making model for the sustainable design of the coal supply chain. They discovered that lowcarbon green technology, low-carbon production costs, green energy utilization, and energy efficiency interact and are the most fundamental manifestations of the decarbonization of the coal supply chain. Enterprises must construct proactive environmental strategies [26] to achieve a win-win situation for economic benefits and environmental protection, minimizing the impact of environmental regulations on corporate performance management. For example, Yao et al. [27] analyzed the role of binding energy-saving and emission reduction targets in coordinating energy-saving, emission reduction, and economic growth, finding that emission reduction targets negatively affected the production performance of industrial enterprises.

In addition to research on carbon emission reduction, scholars have also examined retailers' marketing effort strategies. Hrabec et al. [28] argued that optimal marketing is equivalent to its deterministic equivalent. Xue et al. [29] proposed that green marketing alone has no sensible impact on product greenness; however, with ecological labeling policies, green marketing may

motivate retailers to invest in higher marketing efforts, increasing economic profits and product greenness. Li et al. [30] and Li et al. [31] studied the impact of marketing effort levels on enterprises' green product contracting forms and supply chain contract choices. They found that when the level of marketing efforts is high or low, the improvement of product greenness can benefit enterprises; however, this benefit is inconsistent when marketing efforts are moderate.

The existing literature demonstrates an increasing recognition that carbon emission reduction under the carbon cap-and-trade mechanism is a dynamic process; however, few studies have addressed the impact of random disturbances on carbon emission reduction within the supply chain system. Furthermore, the studies above overlooked promotional strategies for home appliances under the carbon cap-and-trade mechanism. Different promotional methods can lead to varying emission reduction benefits [32]. Therefore, exploring how home appliance companies choose between emission reduction and promotional methods to improve emission reduction effects and member profits is necessary. In practice, Gree Air Conditioner's 'Zero Carbon Source' technology can save 80% of energy; if popularized, this technology could prevent the global temperature from rising by 0.5°C [33]. Inspired by corporate practices and theoretical research, this paper establishes carbon emission reduction and promotion decision models for home appliance enterprises under different decision scenarios. These scenarios are based on research and development regarding carbon emission reduction technology and distributor marketing efforts in the context of carbon quota trading. This study derives equilibrium solutions and compares and analyzes their trends and optimal corporate profits with changes in consumer low-carbon preferences and carbon trading prices. Finally, we verify the conclusions with numerical examples.

Material and Methods

Description of the Problem

This paper primarily examines a two-tier supply chain composed of an individual home appliance manufacturer and its distributor, focusing on the optimal strategies for collaborative emission reduction and promotional activities within the production and marketing processes of green home appliances. As shown in Fig. 1. This study devises emission reduction and promotional decision-making models for home appliance enterprises under three scenarios: (1) Centralized decision-making, where the home appliance manufacturer collaborates with its distributor, integrating their respective interests as a unified entity, herein denoted as *MC* for ease of reference. (2) Decentralized decision-making, where both the home appliance manufacturer and distributor pursue

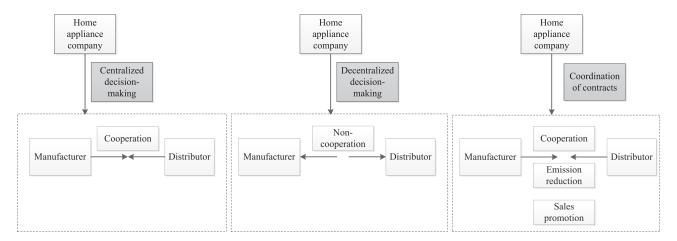


Fig 1. Operational Decision-Making Frameworks for Home Appliance Enterprises Under Varying Decision Scenarios.

the maximization of their own interests, making decisions independently, denoted as NC. (3) Contractual coordination, which aims to reconcile the outcome disparities between the aforementioned decision-making scenarios. This approach starts with adjustments to the wholesale price, denoted by ω , to incentivize collaboration between the home appliance manufacturer and distributor in both emission reduction and promotional efforts, represented respectively by DT and ST. To facilitate description, we correlate the functional relationships under scenarios MC, NC, DT, and ST to the superscripts MC, RC, DT, and ST, respectively. The superscript "*" indicates the optimal equilibrium decision outcomes for these four models.

Model Assumptions

For the purpose of investigating the aforementioned issue, the primary assumptions are as follows:

Assumption 1: There exists a Stackelberg game between the home appliance manufacturer and the distributor, where the distributor acts as the market leader and the manufacturer is the market follower. There is symmetric information between the two parties, and both have a risk preference that is neutral [34].

Assumption 2: The cost per unit for manufacturing a home appliance is denoted by c, which includes raw materials, labor wages, depreciation, and energy consumption, among other costs. In this scenario, the wholesale price of an individual home appliance is represented by ω , and the retail price is denoted by p. Where $p>\omega>0$

Assumption 3: The market demand for home appliances is influenced not only by the pricing and carbon emission reduction levels of the products but also by the marketing efforts of distributors. When distributors promote low-carbon home appliances to consumers through discounts, recommendations, and counter displays, they can effectively guide consumers towards low-carbon consumption habits and encourage a shift towards environmentally friendly practices among

consumers who were previously not eco-conscious [35]. Following the research by Chen [36], the market demand function is characterized as: D = a - bp + kv + re, where "a" signifies the potential consumption volume, with a>0; "p" denoting the retail price of the product; "b" represents the consumer sensitivity coefficient to the retail price, with b>0; "v" indicates the level of marketing efforts by the distributor; "e" signifies the carbon emission reduction level of the home appliance. "k" and "r" are the sensitivity coefficients of consumers to the marketing effort "v" and the carbon emission reduction level "e" of the home appliance, respectively, with k, r, e, and v all greater than v.

Assumption 4: To enhance the carbon emission reduction level of their products, home appliance manufacturers need to invest in carbon emission reduction costs. The higher the level of carbon emission reduction, the greater the investment in carbon emission reduction costs [37, 38], i.e., $g'(e) = h_m e^2/2$. Where h_m represents the manufacturer's carbon emission reduction cost coefficient; similarly, the expression for the marketing effort cost investment by the home appliance distributor is $g'(v) = h_r v^2/2$, where h_r denotes the distributor's marketing effort cost coefficient.

Assumption 5: The government allocates a carbon emission cap of c_g to the manufacturer, with p_e as the carbon trading price and e_0 as the initial level of carbon emissions. Consequently, the total carbon emissions of the manufacturer are expressed as $E = (e_0 - e)(a - bp + kv + re)$. At this point, the cost for the manufacturer to purchase carbon emissions on the carbon trading market is denoted as, $C_e = p_e[(e_0 - e)(a - bp + kv + re) - C_o]$ [39].

Results and Discussion

Centralized Decision Making (MC Model)

Centralized decision-making emphasizes the profit maximization of the decision-maker as a whole, which is represented by the upper corner mark MC [40]. At this time, the appliance manufacturer and distributor collaborate to determine the optimal retail price p^{MC} , carbon emission level e^{MC} , and marketing effort v^{MC} for the appliance products. In this case, the objective function of the decision is:

$$\max_{p,e,v} \pi^{MC} = D(p - c_m) - p_e[(e_0 - e)D - C_g] - h_m e^2 / 2 - h_r v^2 / 2$$
(1)

Proposition 1: The optimal equilibrium strategy for enterprises under centralized decision-making is as follows:

$$e^{MC^*} = -\frac{h_r \left(a - b \left(c_m + e_0 p_e \right) \right) \left(b p_e + r \right)}{h_m k^2 + b^2 h_r p_e^2 + h_r r^2 - 2b h_r \left(h_m - p_e r \right)}$$
(2)

$$v^{MC^*} = \frac{h_m k \left(-a + b \left(c_m + e_0 p_e\right)\right)}{h_m k^2 + b^2 h_r p_e^2 + h_r r^2 - 2b h_r \left(h_m - p_e r\right)}$$
(3)

$$p^{MC^*} = \frac{\left(c_m + e_0 p_e\right) \left(-b h_m h_r + h_m k^2 + b h_r p_e r + h_r r^2\right) + a h_r \left(-h_m + p_e \left(b p_e + r\right)\right)}{h_m k^2 + b^2 h_r p_e^2 + h_r r^2 - 2b h_r \left(h_m - p_e r\right)}$$

(4)

(6)

$$D^{MC*} = \frac{bh_m h_r (bc_m - a + be_0 p_e)}{h_m k^2 + b^2 h_r p_e^2 + h_r r^2 - 2bh_r (h_m - p_e r)}$$
(5)

$$\pi^{MC^*} = \frac{(((-(e_0p_e + c_m)^2h_m + 2p_e^3C_g)b^2 + ((2(ae_0 - 2C_g)p_e + 2ac_m)h_m + 4p_e^2rC_g)b + 2p_er^2C_g - a^2h_m)h_r + 2Cgh_mk^2p_e)}{h_mk^2 + b^2h_pp_e^2 + h_rr^2 - 2bh_r(h_m - p_er)}$$

Proof: According to equation (1), the Hessian matrix with respect to p, e and v is:

$$H_{1} = \begin{bmatrix} \frac{\partial^{2} \pi^{MC}}{\partial p^{2}} & \frac{\partial^{2} \pi^{MC}}{\partial p \partial e} & \frac{\partial^{2} \pi^{MC}}{\partial p \partial v} \\ \frac{\partial^{2} \pi^{MC}}{\partial e \partial p} & \frac{\partial^{2} \pi^{MC}}{\partial e \partial e^{2}} & \frac{\partial^{2} \pi^{MC}}{\partial e \partial v} \\ \frac{\partial^{2} \pi^{MC}}{\partial v \partial p} & \frac{\partial^{2} \pi^{MC}}{\partial v \partial e} & \frac{\partial^{2} \pi^{MC}}{\partial v^{2}} \end{bmatrix} = \begin{bmatrix} -2b & r - bp_{e} & k \\ r - bp_{e} & 2p_{e}r - h_{m} & kp_{e} \\ k & kp_{e} & -h_{r} \end{bmatrix}$$

$$v^{RC*} = \frac{d(m_{r}(pp_{e} - 2n_{m} + p_{e}') + n_{m}k) + b(c_{m} + c_{0}p_{e})(h_{r}(r - 2n_{m} + p_{e}') + h_{r}k)}{b(-4bh_{m}h_{r} + 2h_{m}k^{2} + b^{2}h_{r}p_{e}^{2} + 2bh_{r}p_{e}r + h_{r}r^{2})}$$

$$v^{RC*} = \frac{h_{m}k(-a + b(c_{m} + e_{0}p_{e}))}{-4bh_{m}h_{r} + 2h_{m}k^{2} + b^{2}h_{r}p_{e}^{2} + 2bh_{r}p_{e}r + h_{r}r^{2}}$$

$$(7) \qquad (((-(e_{0}p_{e} + c_{m})^{2}h_{m} + 2p_{e}^{3}C_{e})b^{2} + (((2ae_{0} - 8C_{e})p_{e} + 2abh_{e})p_{e}r + h_{r}r^{2})$$

The matrix indicates when Hessian that third-order principal minor $\left|H\right|\!=\!-2b(2\,p_{\scriptscriptstyle e}r-h_{\scriptscriptstyle m})-(r-bp_{\scriptscriptstyle e})^2>0$, specifically when $h_m > (bp_e + r)^2 / (2b)$, the determinant of the Hessian matrix is negatively definite, meaning π^{MC} is a jointly concave function with respect to p, e and v. Consequently, the MC model possesses a unique optimal solution. By solving for $\partial \pi^{MC} / \partial p = 0$, $\partial \pi^{MC} / \partial e = 0$, and $\partial \pi^{MC} / \partial v = 0$ simultaneously,

we can derive the optimal retail price p^{MC^*} , carbon emission level e^{MC^*} , and marketing effort v^{MC^*} for the appliance product. Substituting the results for p^{MC^*} , e^{MC^*} , and v^{MC^*} into the market demand function yields the optimal market demand for the appliance under MC decision-making as D^{MC^*} . Furthermore, this allows for the calculation of the optimal total profit π^{MC^*} for the appliance company.

Proof complete

Decentralized Decision-Making (RC Model)

Decentralized decision-making emphasizes the maximization of the respective interests of the decisionmakers, which is represented by the superscript RC. At this point, the order of the game of the enterprise is as follows: Initially, the distributor decides on the degree of marketing effort v^{RC} and the retail pricing p^{RC} for the home appliance products; thereafter, the manufacturer establishes the optimal carbon emission level e^{RC} and wholesale pricing ω^{RC} for the home appliance products. The decision objective function is:

$$\max_{v,p} \pi_r = (p - \omega)D - h_r v^2 / 2$$
 (8)

$$\max_{e,\omega} \pi_m = (\omega - c_m)D - p_e[(e_0 - e)D - C_g] - h_m e^2 / 2$$
(9)

Proposition 2: The optimal equilibrium strategy for enterprises under decentralized decision-making is as

$$e^{RC^*} = -\frac{h_r \left(a - b \left(c_m + e_0 p_e \right) \right) \left(b p_e + r \right)}{-4b h_m h_r + 2h_m k^2 + b^2 h_r p_e^2 + 2b h_r p_e r + h_r r^2}$$
(10)

$$\omega^{RC^*} = \frac{a\left(bh_r\left(bp_e^2 - 2h_m + p_e r\right) + h_m k^2\right) + b\left(c_m + e_0 p_e\right)\left(h_r\left(r^2 - 2bh_m + bp_e r\right) + h_m k^2\right)}{b\left(-4bh_m h_r + 2h_m k^2 + b^2 h_r p_e^2 + 2bh_r p_e r + h_r r^2\right)}$$
(11)

$$v^{RC^*} = \frac{h_m k \left(-a + b \left(c_m + e_0 p_e\right)\right)}{-4bh_m h_r + 2h_m k^2 + b^2 h_r p_e^2 + 2bh_r p_e r + h_r r^2}$$
(12)

$$\pi_{m}^{RC*} = \frac{\left(\left(\left(-\left(e_{0}p_{e} + c_{m}\right)^{2} h_{m} + 2 p_{e}^{3} C_{g}\right) b^{2} + \left(\left(\left(2a e_{0} - 8 C_{g}\right) p_{e} + 2a c_{m}\right) h_{m}\right)}{\left(2\left(-4b h_{m} h_{r} + 2 h_{m} k^{2} + b^{2} h_{r} p_{e}^{2} + 2b h_{r} p_{e} r + h_{r} r^{2}\right)\right)}$$

$$(13)$$

$$p^{RC^*} = \frac{a(h_r b(-3h_m + bp_e^2 + p_e r) + h_m k^2) + b(c_m + e_0 p_e)(h_r (r^2 - bh_m + bp_e r) + h_m k^2)}{b(-4bh_m h_r + 2h_m k^2 + b^2 h_r p_e^2 + 2bh_r p_e r + h_r r^2)}$$
(14)

$$D^{RC*} = \frac{bh_{m}h_{r}(bc_{m} - a + be_{0}p_{e})}{-4bh_{m}h_{r} + 2h_{m}k^{2} + b^{2}h_{r}p_{e}^{2} + 2bh_{r}p_{e}r + h_{r}r^{2}} (15)$$

$$\pi_{r}^{RC*} = \frac{h_{m}^{2}h_{r}(-k^{2} + 2bh_{r})(bc_{m} - a + be_{0}p_{e})^{2}}{2(h_{r}b^{2}p_{e}^{2} + 2h_{r}bp_{e}r - 4h_{m}h_{r}b + 2h_{m}k^{2} + h_{r}r^{2})} (16)$$

$$\frac{\left(h_{m}^{2}h_{r}(2bh_{r} - k^{2})(a - b(c_{m} + e_{0}p_{e}))^{2} - (h_{r}(-4bh_{m} + b^{2}p_{e}^{2} + 2bp_{e}r + r^{2}) + 2h_{m}k^{2})\right)}{\left(a^{2}h_{m}h_{r} - 2abh_{m}h_{r}(c_{m} + e_{0}p_{e}) + b^{2}h_{r}(c_{m}^{2}h_{m} + 2c_{m}e_{0}h_{m}p_{e} + e_{0}^{2}h_{m}p_{e}^{2} - 2C_{g}p_{e}^{3})\right)}{\left(2(h_{r}b^{2}p_{e}^{2} + 2h_{r}bp_{e}r - 4h_{m}h_{r}b + 2h_{m}k^{2} + h_{r}r^{2})^{2}\right)} (17)$$

Proof: We use backward induction to solve. Firstly, compute the first-order partial derivatives of π_r with respect to p and v. Set $\partial \pi_r/\partial p = 0$ and $\partial \pi_r/\partial v = 0$, and solve to obtain the results:

$$p^{RC} = \frac{-\omega k^2 + ah_r + eh_r r + bh_r \omega}{-k^2 + 2bhr},$$
$$v^{RC} = \frac{k(a + er - b\omega)}{-k^2 + 2bhr}$$

Secondly, by substituting the expressions for p^{RC} and v^{RC} into the manufacturer's profit function, we can derive the Hessian matrix of π_m with respect to ω and e as follows:

$$H = \begin{bmatrix} \frac{\partial^{2} \pi_{m}}{\partial e^{2}} & \frac{\partial^{2} \pi_{m}}{\partial e \partial \omega} \\ \frac{\partial^{2} \pi_{m}}{\partial \omega \partial e} & \frac{\partial^{2} \pi_{m}}{\partial \omega^{2}} \end{bmatrix} = \begin{bmatrix} \frac{2brp_{e}h_{r}}{2bh_{r} - k^{2}} - hm & \frac{r - bp_{e}}{2} \\ \frac{r - bp_{e}}{2} & \frac{-2b^{2}h_{r}}{2bh_{r} - k^{2}} \end{bmatrix}$$

$$(18)$$

It can be seen from Equation (18) that when it satisfies the condition, $|H| = -\frac{2b^2h_r(h_mk^2 - 2bh_mh_r + 2bh_rp_er)}{(2bhr - k^2)^2} - \frac{(r - bp_e)^2}{4} > 0$ The determinant of the Hessian matrix is negative definite when $-\frac{2b^2h_r(h_mk^2 - 2bh_mh_r + 2bh_rp_er)}{(2bhr - k^2)^2} > \frac{(r - bp_e)^2}{4}.$

Therefore, the profit function π_m of the home appliance manufacturer is a concave joint function with respect to ω and e. Consequently, the RC model possesses a unique optimal solution. By combining equations $\partial \pi_m/\partial e=0$ and $\partial \pi_m/\partial \omega=0$, we can get ω^{RC^*} and e^{RC^*} . Similarly, we can conclude that p^{RC^*} , v^{RC^*} , p^{RC^*} , r^{RC^*} , r^{RC^*} and r^{RC^*} . Proof complete.

To ensure that the calculation results are meaningful and the optimal decision-making equilibrium results and profits of the manufacturer and the dealer are both positive, the equilibrium results in the above results need to meet the following conditions: $b(e_0p_e+c_m)>a \qquad k<\sqrt{2bh_r} \qquad , \\ h_r(bp_e+r)^2>2h_m(2h_rb-k^2)$

Stackelberg Game with the Introduction of Coordination of Contracts

From the comparison of equilibrium outcomes between Proposition 1 and Proposition 2, it is evident that the optimal equilibrium strategies under decentralized decision-making are uniformly lower than those under centralized decision-making. This indicates the presence of a double marginalization effect. Based on this, the article introduces cooperative promotion and cooperative emission reduction decisions to achieve contractual coordination.

Firstly, we investigated the cooperative promotional decision-making scheme (DT model). This section adopts cooperative promotional decision-making to achieve joint coordination of carbon emission levels, marketing effort intensity, and retail prices, thereby enabling companies to achieve the same total profit under decentralized decision-making as they would under centralized decision-making. In this scenario, first, the manufacturer offers a lower wholesale price to the dealer to encourage the appliance dealer to sell products at the retail price and marketing effort consistent with centralized decision-making, while the appliance manufacturer produces low-carbon appliances at the carbon emission level consistent with centralized decision-making. Secondly, the dealer transfers a fixed payment S to the manufacturer to compensate for the loss of benefits incurred by the manufacturer in implementing the cooperative promotional decision.

In this case, the objective function of the decision is:

$$\max_{\omega^{DT}, S} \pi_{m} = (\omega - c_{m})D - p_{e}[(e_{0} - e)D - Cg] - h_{m}e^{2} / 2 + S$$
s.t.
$$\begin{cases} (\omega - c_{m})D - p_{e}[(e_{0} - e)D - Cg] - h_{m}e^{2} / 2 + S \ge \pi_{m}^{RC^{*}} \\ (p - \omega)D - h_{r}v^{2} / 2 - S \ge \pi_{r}^{RC^{*}} \\ p^{DT^{*}} = p^{MC^{*}}, e^{DT^{*}} = e^{MC^{*}}, v^{DT^{*}} = v^{MC^{*}}, D^{DT^{*}} = D^{MC^{*}} \end{cases}$$
(19)

Proposition 3: The optimal equilibrium strategy for enterprises under cooperative promotion decisions is as follows:

$$\omega^{DT^*} = \frac{(h_m k^2 + h_r r^2)(c_m + e_0 p_e) + h_r r p_e (a + b c_m + b e_0 p_e) + b h_r (a p_e^2 - 2 c_m h_m - 2 e_0 h_m p_e)}{h_r b^2 p_e^2 + 2 h_r b p_e r - 2 h_m h_r b + h_m k^2 + h_r r^2}$$
(20)

The optimal values of the total profit of the manufacturer, the distributor, and the enterprise are:

$$\pi_{m}^{DT^{*}} = \frac{h_{r}^{2}h_{m}(bp_{e}+r)^{2}[a-b(c_{m}+e_{0}p_{e})]^{2} - 2C_{g}p_{e}(h_{r}b^{2}p_{e}^{2}+2h_{r}bp_{e}r-2h_{m}h_{r}b+h_{m}k^{2}+h_{r}r^{2})^{2}}{-2(h_{r}b^{2}p_{e}^{2}+2h_{r}bp_{e}r-2h_{m}h_{r}b+h_{m}k^{2}+h_{r}r^{2})^{2}} + S$$
(21)

$$\pi_{r}^{DT*} = \frac{h_{m}^{2} h_{r} (2bh_{r} - k^{2}) (a - b(c_{m} + e_{0}p_{e}))^{2}}{2(h_{m}k^{2} + b^{2}h_{r}p_{e}^{2} + h_{r}r^{2} - 2bh_{r}(h_{m} - p_{e}r))^{2}} - S$$
(22)

$$\pi^{DT^*} = \frac{\left[h_r(bp_e + r)^2 + h_m(k^2 - 2bh_r)\right]\left[2C_g p_e \left[h_r(bp_e + r)^2 + h_m(k^2 - 2h_r b)\right] + h_m h_r(bc_m - a + be_0 p_e)^2\right]}{-2(h_r b^2 p_e^2 + 2h_r bp_e r - 2h_m h_r b + h_m k^2 + h_r r^2)^2}$$
(23)

To obtain at least the retained profit under RC decision, the value range of transfer payment S can be obtained as $[S, \bar{S}]$, where:

$$S = \frac{h_m^3 h_r (k^2 - 2bh_r)^2 (bc_m - a + be_0 p_e)^2}{2(h_r b^2 p_e^2 + 2h_r bp_e r - 2h_m h_r b + h_m k^2 + h_r r^2)^2 (h_r b^2 p_e^2 + 2h_r bp_e r - 4h_m h_r b + 2h_m k^2 + h_r r^2)}$$
(24)

$$\bar{S} = \frac{h_m^3 h_r (k^2 - 2bh_r)^2 (bc_m - a + be_0 p_e)^2 (2h_r b^2 p_e^2 + 4h_r bp_e r - 6h_m h_r b + 3h_m k^2 + 2h_r r^2)}{2(h_r b^2 p_e^2 + 2h_r bp_e r - 2h_m h_r b + h_m k^2 + h_r r^2)^2 (h_r b^2 p_e^2 + 2h_r bp_e r - 4h_m h_r b + 2h_m k^2 + h_r r^2)^2}$$
(25)

Proof: In the objective function of equation (19), the first two constraint conditions are participation constraints for the manufacturer and the dealer. This signifies that both parties are willing to engage in the contract only if they can secure at least the reservation profit obtained under the *RC* decision-making scenario. The second constraint condition is an incentive compatibility constraint. By employing the backward induction method to solve the model, the results from Equations (20), (21), (22), (23), (24) and (25) can be derived.

Proof complete.

Secondly, we investigated the cooperative emission reduction decision-making scheme (ST model). This section utilizes the principle of revenue-sharing contracts to adopt cooperative emission reduction decisions aimed at jointly coordinating carbon emission levels, marketing effort intensity, and retail prices. The goal is to align the total profits of companies under decentralized decision-making with those achieved under centralized decision-making. In this scenario, the manufacturer initially offers a lower wholesale price to the dealer to encourage the appliance dealership to sell products at the retail price and marketing efforts under MC decision-making. Meanwhile, the household appliance manufacturer produces low-carbon home appliances at the carbon emission level under MC

decision-making. Then, based on the carbon emission level, the manufacturer and dealer distribute the sales revenue. In this setup, the household appliance manufacturer receives a revenue-sharing proportion of λ , and consequently, the dealer receives a revenue-sharing proportion of $1 - \lambda$, where λ is the revenue-sharing factor.

In this case, the objective function of the decision is:

$$\max_{\omega^{ST}, \lambda, e} \pi_m = (\lambda p + \omega^{ST} - c_m) D - p_e [(e_0 - e) D - C_g] - h_m e^{2/2}$$
(26)

$$\max \pi_r = [(1 - \lambda)p - \omega^{ST}]D - h_r v^2 / 2$$
 (27)

Proposition 4: The optimal equilibrium strategy for enterprises under cooperative emission reduction decisions is as follows:

$$\omega^{ST*} = \frac{(1-\lambda)[(c_m + e_0 p_e)[h_m k^2 + h_r r^2 + bh_r (rp_e - 2h_m)] + ah_r p_e (r + bp_e)]}{h_r b^2 p_e^2 + 2h_r bp_e r - 2h_m h_r b + h_m k^2 + h_r r^2}$$
(28)

The optimal values of the total profit of the manufacturer, the distributor, and the enterprise are:

$$\pi_{m}^{ST^*} = \frac{2C_{g} p_{e} [h_{r} (bp_{e} + r)^{2} + h_{m} (k^{2} - 2h_{r} b)]^{2} + h_{r}^{2} h_{m} [2bh_{m} \lambda - (r + bp_{e})^{2}] (bc_{m} - a + be_{0} p_{e})^{2}}{2(h_{r} b^{2} p_{e}^{2} + 2h_{r} bp_{e} r - 2h_{m} h_{r} b + h_{m} k^{2} + h_{r} r^{2})^{2}}$$
(29)

$$\pi_r^{ST*} = \frac{-h_m^2 h_r (bc_m - a + be_0 p_e)^2 (k^2 - 2bhr + 2bhr\lambda)}{2(h_r b^2 p_e^2 + 2h_r bp_e r - 2h_m h_r b + h_m k^2 + h_r r^2)^2}$$
(30)

$$\pi^{ST^*} = \frac{\left[h_r(bp_e + r)^2 + h_m(k^2 - 2h_rb)\right]\left[2C_g p_e \left[h_r(bp_e + r)^2 + h_m(k^2 - 2h_rb)\right] - h_m h_r(bc_m - a + be_0 p_e)^2\right]}{2(h_rb^2 p_e^2 + 2h_rbp_e r - 2h_m h_rb + h_m k^2 + h_r r^2)^2}$$
(31)

According to the previous description, the premise for the implementation of the cooperative emission reduction decision is that $\pi_m^{ST*} \geq \pi_m^{RC*}$, $\pi_r^{ST*} \geq \pi_r^{RC*}$.

From this, it can be derived that the range of values for the revenue-sharing factor λ in the collaboration between the manufacturer and the dealer for emission reduction is $[\lambda, \bar{\lambda}]$, where:

$$\lambda = \frac{4h_{m}bh_{r}k^{2} - 4h_{m}b^{2}h_{r}^{2} - h_{m}k^{4}}{2b^{3}h_{r}^{2}p_{e}^{2} + 4b^{2}h_{r}^{2}p_{e}r - 8h_{m}b^{2}h_{r}^{2} + 2bh_{r}^{2}r^{2} + 4h_{m}bh_{r}k^{2}}$$
(32)

$$\dot{\lambda} = \frac{h_m (k^2 - 2bh_r)^2 (2h_r b^2 p_e^2 + 4h_r bp_e r - 6h_m h_r b + 3h_m k^2 + 2h_r r^2)}{2bh_r (h_r b^2 p_e^2 + 2h_r bp_e r - 4h_m h_r b + 2h_m k^2 + h_r r^2) 2}$$
(33)

Proof: To achieve the level of MC decision under cooperative emission reduction decision, the equilibrium result of the ST model needs to satisfy: $p^{ST^*} = p^{MC^*}$, $e^{ST^*} = e^{MC^*}$, $v^{ST^*} = v^{MC^*}$, $D^{ST^*} = D^{MC^*}$. On this basis, the first-order partial derivatives of p^{ST} and v^{ST} in Equation (27) are obtained, set them equal to 0, and the combined equation can obtain the equilibrium result.

Proof complete.

Result Comparison

Corollary 1: In centralized decision-making, as consumers' low-carbon preferences increase, the equilibrium outcomes of household appliance products and the total profit of the supply chain also increase. When the carbon emission reduction investment cost coefficient h_m is low, the marketing effort of dealers, market demand, and total profit of enterprises all increase with the rise in carbon trading prices, but the retail price of green household appliances will decrease. When the carbon emission reduction investment cost coefficient h_m is high, the results are just the opposite.

Corollary 1 shows that to meet consumers' demand for green household appliances, manufacturers strive to improve the carbon emission reduction levels of their products. For dealers, an increase in the retail price of household appliances and marketing efforts helps boost market demand and the total profit of the supply chain. When the carbon emission reduction investment cost coefficient h_m is low, higher carbon trading prices lead to increased production costs for household appliances. To reduce the cost of carbon trading, manufacturers will invest more in carbon emission costs to produce more appliances with carbon labels, thereby promoting an increase in market demand for household appliances. Due to the reduction in carbon trading costs, an increase in carbon emission reduction costs does not raise the total expenses of the supply chain. Therefore, the carbon emission reduction investment cost coefficient h_{\perp} and the sales price will still remain at a lower level. When the carbon emission reduction investment cost coefficient h_m is high, although investment in carbon emission reduction costs can improve carbon emission reduction levels, an excessively high h_m will lead to an increase in the retail prices of household appliances, consequently resulting in reduced market demand. At this point, the total profit of the supply chain will also decrease.

Proof: Under the MC model, the equilibrium outcomes for household appliances and firm profits can be determined by calculating their first-order partial derivatives with respect to the consumer low-carbon preference coefficient r and the carbon trading price p_e . This process yields the results of Corollary 1. Proof complete.

Corollary 2: Irrespective of the changes in the carbon emission reduction investment cost coefficient h_{m} , when the consumer low-carbon preference coefficient rincreases, there is a corresponding increase in the carbon emission reduction level, marketing effort, market demand, and corporate profits for household appliances. At this juncture, while an increase in the carbon trading price elevates the carbon emission reduction level, it simultaneously reduces the manufacturer's profits. When the carbon emission reduction investment cost coefficient h is relatively low, as the consumer lowcarbon preference coefficient r rises, both the wholesale and retail prices of household appliances also increase. In this scenario, the carbon trading price is positively correlated with the degree of marketing effort, market demand, and dealer profits, but negatively correlated with the retail price of household appliances. Conversely, when the carbon emission reduction investment cost coefficient h_{m} is relatively high, the outcomes are precisely the opposite.

Corollary 2 indicates that under the dual impact of increased carbon emission reduction cost investments by appliance manufacturers and a rising consumer low-carbon preference coefficient, the carbon emission reduction level, market demand, and dealer profits for household appliances will all increase. When the carbon emission reduction investment cost coefficient h_m is low, as the carbon trading price rises, manufacturers will reduce their carbon purchases and instead produce more appliances with carbon labels to enhance market demand. Considering consumer interests, manufacturers will lower wholesale prices, and consequently, retail prices will decrease. For dealers, the primary source of profit comes from increased product market demand, thus leading to an intensification of marketing efforts by dealers. When the carbon emission reduction investment cost coefficient h_m is high, it increases the production costs of household appliances, resulting in reduced manufacturer profits. To compensate for the losses, manufacturers will raise the wholesale prices of household appliances, which will in turn increase retail prices, ultimately leading to a decrease in both market demand for household appliances and dealer profits.

Proof: Under the RC model, the equilibrium outcomes for household appliances and firm profits can be determined by calculating their first-order partial derivatives with respect to the consumer low-carbon preference coefficient r and the carbon trading price p_e . This process yields the results of Corollary 2. Proof complete.

Corollary 3: A comparative analysis of the equilibrium outcomes for household appliances under varying circumstances indicates that:

(1)
$$v^{DT*} = v^{ST*} = v^{MC*} > v^{RC*}, \quad D^{DT*} = D^{ST*} = D^{MC*} > D^{RC*},$$

$$e^{DT*} = e^{ST*} = e^{MC*} > e^{RC*};$$
(2) When $h_r < h_m k^2 / (2bh_m - r^2)$ or $h_m < h_r r^2 / (2bh_r - k^2)$,

$$\omega^{RC^*} > \omega^{DT^*} > \omega^{ST^*}, p^{RC^*} < p^{MC^*} = p^{DT^*} = p^{ST^*},$$
When $h_m > h_r r(r + bp_e) / (bh_r - k^2)$ or $h_r > k^2 h_m / (bh_m - r^2 - rbp_e)$, $\omega^{DT^*} < \omega^{ST^*} < \omega^{RC^*}$, $p^{RC^*} > p^{MC^*} = p^{DT^*} = p^{ST^*}$

Proof: Let $\Delta\omega_1$ and $\Delta\omega_2$ respectively denote the difference in the optimal wholesale prices for household appliances under decentralized decision-making and contract coordination strategies, with the results as follows:

$$\Delta \omega_{i} = \omega^{DT^*} - \omega^{ST^*}$$

$$= \frac{\lambda[(c_m + e_0 p_e)(h_m k^2 + h_r r^2 - 2bh_m h_r) + bh_r p_e^2(e_0 r + a) + h_r p_e r(a + bc_m)]}{h_r b^2 p_e^2 + 2h_r bp_e r - 2h_m h_r b + h_m k^2 + h_r r^2}$$

$$\Delta \omega_{\gamma} = \omega^{DT^*} - \omega^{RC^*}$$

$$= \frac{h_m(k^2 - 2bh_r)(bc_m - a + be_0p_e)[h_mk^2 + h_rr^2 + bp_er - 2bh_m)]}{b[h_rb^2p_e^2 + 2h_rb(p_er - h_m) + h_rr^2 + h_mk^2][h_rb(bp_e^2 + 2p_er - 4h_m) + 2h_mk^2 + h_rr^2]}$$
 So, when $h_r < \frac{h_mk^2}{2bh_m - r^2}$, or $h_m < \frac{h_rr^2}{2bh_r - k^2}$, it can be inferred that $\omega^{DT*} > \omega^{ST*}$. When $h_r < \frac{h_mk^2}{2bh_m - r^2}$, or $h_m < \frac{h_rr^2}{2bh_r - k^2}$, it can be inferred that $\omega^{DT*} < \omega^{ST*}$. When $h_m < \frac{h_rr^2}{2bh_r - k^2}$, or $h_r < \frac{h_mk^2}{2bh_m - r^2 - brp_e}$, it can be inferred that $\omega^{DT*} > \omega^{RC*}$. When $h_m < \frac{h_rr(r + bp_e)}{2bh_r - k^2}$, or $h_r < \frac{h_mk^2}{2bh_m - r^2 - brp_e}$, it can be inferred that $\omega^{DT*} > \omega^{RC*}$. When $h_m < \frac{h_rr(r + bp_e)}{2bh_r - k^2}$, or $h_r < \frac{h_mk^2}{2bh_r - r^2 - brp_e}$, it can be

inferred that $\omega^{DT^*} < \omega^{RC^*}$.

By integrating the results from Corollary 3, part (2), when $h_r < \frac{h_m k^2}{2hh - r^2}$, or $h_m < \frac{h_r r^2}{2hh - k^2}$, we have that $\omega^{RC^*} > \omega^{DT^*} > \omega^{ST^*}$ $p^{RC^*} < p^{MC^*}.$ When $h_m > \frac{h_r r(r + bp_e)}{bh - k^2}$ or, $h_r > \frac{k^2 h_m}{bh_m - r^2 - rbp_a}$, we have that $\omega^{DT^*} < \omega^{ST^*} < \omega^{RC^*}, \quad p^{RC^*} > p^{MC^*}.$ Additionally, given the framework of contractual harmony, $p^{DT^*} = p^{ST^*} = p^{MC^*}$, $e^{DT^*} = e^{ST^*} = e^{MC^*}, v^{DT^*} = v^{ST^*} = v^{MC^*}, D^{DT^*} = D^{ST^*} = D^{MC^*}.$ Thus, the proof of Corollary 3 is established.

Proof complete.

Corollary 4: A comparative analysis of the profits between home appliance manufacturers and distributors, as well as the total supply chain profit, reveals that:

(1)
$$\pi^{DT^*} = \pi^{ST^*} = \pi^{MC^*} > \pi^{RC^*};$$

(2) When
$$S > \frac{bh_m^2 h_r^2 \lambda (bc_m - a + be_0 p_e)^2}{(h_r b^2 p_e^2 + 2h_r bp_e r - 2h_m h_r b + h_m k^2 + h_r r^2)^2}$$

$$\pi_m^{DT*} > \pi_m^{ST*} > \pi_m^{RC*}, \pi_r^{ST*} > \pi_r^{DT*} > \pi_r^{RC*},$$
When
$$S < \frac{bh_m^2 h_r^2 \lambda (bc_m - a + be_0 p_e)^2}{(h_r b^2 p_e^2 + 2h_r bp_e r - 2h_m h_r b + h_m k^2 + h_r r^2)^2},$$

$$\pi_m^{ST*} > \pi_m^{DT*} > \pi_m^{RC*}, \pi_r^{DT*} > \pi_r^{ST*} > \pi_r^{RC*}.$$

Proof: Let $\Delta \pi_m$, $\Delta \pi_n$ and $\Delta \pi$ respectively represent the differences in profits for home appliance manufacturers, distributors, and the total supply chain under various scenarios, with the results as follows:

$$\begin{split} \Delta\pi &= \pi^{DT^*} - \pi^{ST^*} = 0 ; \\ \Delta\pi_m &= S - \frac{bh_m^2 h_r^2 \lambda (bc_m - a + be_0 p_e)^2}{(h_r b^2 p_e^2 + 2h_r bp_e r - 2h_m h_r b + h_m k^2 + h_r r^2)^2} \end{split}$$

$$\Delta \pi_r = \frac{bh_m^2 h_r^2 \lambda (bc_m - a + be_0 p_e)^2}{(h_r b^2 p_e^2 + 2h_r bp_e r - 2h_m h_r b + h_m k^2 + h_r r^2)^2} - S$$

From $\Delta \pi = \pi^{DT^*} - \pi^{ST^*} = 0$, we have that $\pi^{DT^*} = \pi^{ST^*}$. By integrating the results from Corollary 4, Part (1), it can be inferred that $\pi^{DT^*} = \pi^{ST^*} = \pi^{MC^*} > \pi^{RC^*}$. From $\Delta \pi_m$ suggest that when $S > \frac{bh_m^2 h_r^2 \lambda (bc_m - a + be_0 p_e)^2}{(h_r b^2 p_e^2 + 2h_r bp_e r - 2h_m h_r b + h_m k^2 + h_r r^2)^2}$, we have that

 $\pi_{_{m}}^{\ DT^{*}} > \pi_{_{m}}^{\ ST^{*}}, \text{ conversely, } \pi_{_{m}}^{\ DT^{*}} < \pi_{_{m}}^{\ ST^{*}}. \text{ From } \pi_{_{m}}^{\ DT^{*}} > \pi_{_{m}}^{\ RC^{*}} \text{ and } \pi_{_{m}}^{\ ST^{*}} > \pi_{_{m}}^{\ RC^{*}}, \text{ we can therefore deduce that } \pi_{_{m}}^{\ DT^{*}} > \pi_{_{m}}^{\ ST^{*}} > \pi_{_{m}}^{\ RC^{*}}, \text{ conversely, } \pi_{_{m}}^{\ ST^{*}} > \pi_{_{m}}^{\ DT^{*}} > \pi_{_{m}}^{\ RC^{*}}.$ From $\Delta\pi$ suggest that when $S < \frac{bh_{_{m}}^{\ 2}h_{_{r}}^{\ 2}\lambda(bc_{_{m}} - a + be_{_{0}}p_{_{e}})^{2}}{(h_{_{r}}b^{2}p_{_{e}}^{\ 2} + 2h_{_{r}}bp_{_{e}}r - 2h_{_{m}}h_{_{r}}b + h_{_{m}}k^{2} + h_{_{r}}r^{2})^{2}}.$

$$S < \frac{bh_m^2 h_r^2 \lambda (bc_m - a + be_0 p_e)^2}{(h_r b^2 p_e^2 + 2h_r bp_e r - 2h_m h_r b + h_m k^2 + h_r r^2)^2}$$

have that $\pi_r^{DT^*} > \pi_r^{ST^*}$, conversely, $\pi_r^{DT^*} < \pi_r^{ST^*}$. From $\pi_r^{DT^*} > \pi_r^{RC^*}$ and $\pi_r^{ST^*} > \pi_r^{RC^*}$, we have that $\pi_r^{DT^*} > \pi_r^{ST^*} > \pi_r^{ST^*}$ $\pi_r^{RC^*}$, conversely, $\pi_r^{ST^*} > \pi_r^{DT^*} > \pi_r^{RC^*}$.

Proof complete

Upon analyzing the findings from corollaries 3 and 4, it becomes evident that both collaborative marketing and collaborative emission abatement decisions can harmonize the holistic performance metrics of domestic appliance companies, aligning them with the outcomes associated with the MC decision-making model. In scenarios characterized by substantive fixed payment fee s, the inclination towards a collaborative marketing strategy amplifies the advantages for the producer. Contrastingly, when the fixed payment fee s is modest, the impetus for a collaborative marketing strategy shifts favorable outcomes towards the distributor. Within the framework of a collaborative emission abatement decision, both manufacturers and distributors possess the capacity to adapt their strategic choices dynamically

by fine-tuning the revenue-distribution factor λ , thereby enhancing their position in business negotiations.

Numerical Simulation Analysis

In China, home appliance manufacturers such as Haier Smart Home, Midea Group, Gree Electric Appliances, Sichuan Changhong, and Hisense Visual Technology are all committed to corporate social responsibility. They continuously publish corporate social responsibility reports and strive to produce green home appliances with carbon labels to promote carbon emission reduction. Offline traditional chain retail enterprises like Gome Electrical Appliances and Suning, as well as online e-commerce retail platforms represented by JD.com, are adopting various marketing strategies to seize a dominant position in the home appliance industry. Therefore, this section uses home appliance manufacturers and dealers as a case study for numerical analysis of household appliances. operating conditions of home appliance manufacturers and dealers (e-commerce platforms) are shown in Table 1:

Assuming that a home appliance dealer A places an order for home appliances with manufacturer B, the selling price of the home appliances primarily depends on the operating costs of both the manufacturer and the dealer. It refers to the research of Xu et al. [41] and Cui et al. [42] and the operation situation of home appliance manufacturers. We set $p=900,\ \omega=750,\ c_m=350,\ b=0.8,\ a=1000,\ r=0.76,\ k=0.65,\ p_e=100,\ C_g=5,\ e_0=1,\ h_m=2000000,\ and\ h_r=1000000.$

Equilibrium Decision Analysis

The above parameters were brought into the above model to obtain the simulation results as shown in Table 2.

Table 2 shows that compared to the centralized decision-making, decentralized decision-making, and contract coordination strategies. The results show that the wholesale price of home appliances satisfy: $\omega^{RC^*} > \omega^{DT^*} < \omega^{ST^*}$; the retail prices satisfy: $p^{RC^*} > p^{DT^*} = p^{ST^*} = p^{MC^*}$; while the carbon emission levels, marketing effort, market demand, and total corporate profits all satisfy: $e^{RC^*} < e^{DT^*} = e^{ST^*} = e^{MC^*}$, $v^{RC^*} < v^{DT^*} = v^{ST^*} = v^{MC^*}$, v^{RC^*

Table 1. Operating	Status of Home	Appliance Ma	mufacturers I	(2023 year)
Table 1. Operating	2 Status of Home	Appliance ivia	muraciureis i	2023 year J.

Chinese electric appliance enterprises	Annual revenue (Thousand RMB)	Operating costs (Thousand RMB)	Gross profit rate (%)
Midea Group	113,890,764	79,112,626	30.54
Gree Electric Appliances	122403396	77,430,333	34.32
Sichuan Changhong Electric	62,109,982	56,491,627	9.05
Hisense Group	3,536,7010	2,853,1180	19.33
Haier Wise House	10,724,1360	7,416,6140	30.84
Suning Appliance	40,844,302	35,703,175	12.59
Gome Electrical Appliances	26,104,102	22,289,001	13.94
JD.com Group	400,927,013	35,071,028	16.85

Table 2. Equilibrium Outcomes and Profit Comparison under MC, RC, DT, and ST Decisions.

Variable of decision	Centralized Decision Making	Decentralized Decision-Making	Coordination of Contracts $S = 79683.958, \lambda = 0.68$	
	MC	RC	DT	ST
ω false	_	845.959	433.512	325.134
p false	841.835	1048.019	841.835	841.835
e false	0.165	0.082	0.165	0.165
v false	0.003	0.001	0.003	0.003
D false	326.659	161.648	326.659	326.659
π_m false	_	65159.105	77465.383	88481.4777
π_r false	_	32662.586	53698.120	42682.0253
π false	131163.503	97821.691	131163.503	131163.503

Manufacturer's profits satisfy: $\pi_m^{RC^*} > \pi_m^{DT^*} > \pi_m^{MC^*}$; Dealer's profits satisfy: $\pi_r^{RC^*} > \pi_r^{ST^*} > \pi_r^{RC^*}$. From this, we can conclude that: (1) When manufacturers and dealers collaborate in the operation of carbon-labeled home appliances, the operational performance of the home appliance products is higher than when they operate independently; (2) Contract coordination not only promotes an increase in profits for both manufacturers and dealers but also optimizes the operational performance of home appliance products; (3) Looking at the manufacturer's profits, the ST decision is optimal, while from the retailer's perspective, the DT decision is most favorable.

Sensitivity Analysis of Various Parameters

The following sections employ graphical representations to further understand the impact of the consumer low-carbon preference coefficient r and the consumer low-carbon preference p_e on equilibrium outcomes and the profits of home appliance companies. The sensitivity analysis of various parameters is shown in the figures below.

Fig. 2 analyzes the impact of the consumer low-carbon preference coefficient r on the equilibrium outcomes, as detailed below:

Fig. 2 shows that: (1) The price of home appliances (wholesale ω and retail price p), the level of carbon emission reduction e, the degree of marketing effort v,

and market demand D all increase with the increase in consumer low-carbon preference coefficient r, while ω^{DT^*} and ω^{ST^*} decrease with the increase in consumer low-carbon preference coefficient r, indicating that during the process of contract coordination, manufacturers have adopted decisions on cooperative promotion and carbon emission reduction with lower cost pricing. (2) Looking at the trend of changes in all equilibrium outcomes, the consumer low-carbon preference coefficient r has the greatest impact on the marketing effort of home appliance products, indicating that cooperative promotion is beneficial in enhancing consumer low-carbon preferences.

Fig. 3 analyzes the impact of the consumer low-carbon preference coefficient r on corporate profits, as detailed below:

Fig. 3 shows that: (1) The profits of home appliance dealers (π_p) , manufacturers (π_m) , and the total corporate profit (π) all increase with an increase in the consumer low-carbon preference coefficient r, indicating that consumer low-carbon preferences are beneficial for enhancing corporate profits. (2) Looking at the trend in profit changes for home appliance products, the consumer low-carbon preference coefficient r has the greatest impact on profits under decisions involving cooperative promotion and cooperative emission reduction. Although the sharing of sales revenue between dealers and manufacturers somewhat reduces the dealers' profits, it still allows dealers to maintain

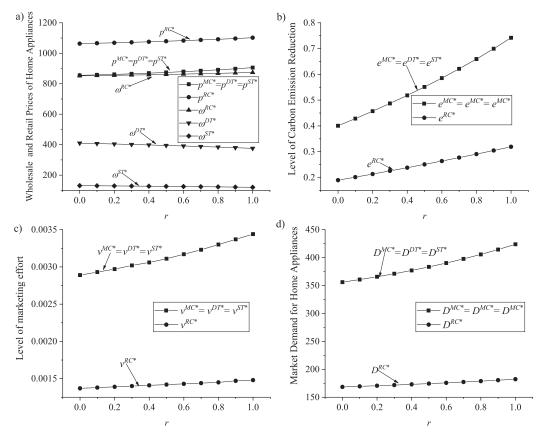


Fig 2. Impact of changes in consumer low-carbon preference coefficient r on equilibrium outcomes.

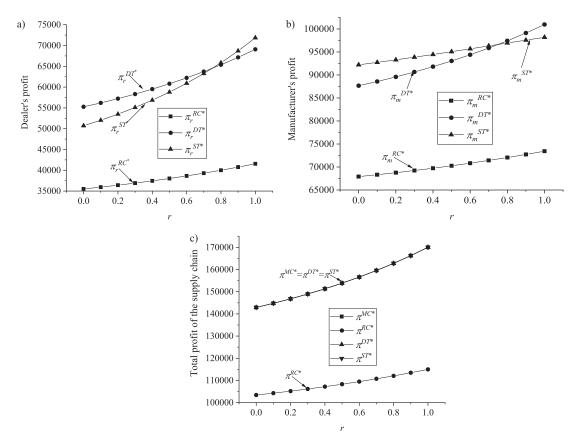


Fig. 3. Impact of changes in consumer low-carbon preference coefficient r on corporate profit.

a higher level of profit growth. (3) Comparing the profits of home appliance companies, the profits for both dealers and manufacturers under *DT* and *ST* decisions are higher than those under *NC* decisions and equal to the profit values under *MC* decisions. This indicates that decisions on cooperative promotion and cooperative carbon emission reduction can facilitate contract coordination, helping manufacturers and dealers achieve optimal profit levels.

Fig. 4 analyzes the impact of changes in carbon trading prices on equilibrium outcomes, as detailed below:

Fig. 4 shows that: (1) When the carbon reduction investment is high $(h_m=200000)$, both the prices of home appliances (wholesale and retail) and the level of carbon reduction increase with the increase in carbon trading price p_e , while the degree of marketing effort and market demand decrease with the increase in carbon trading price. (2) Conversely, when the carbon reduction investment is low $(h_m = 50000)$, the prices of home appliances (wholesale and retail) decrease with the increase in carbon trading price p_a , whereas the level of carbon reduction, degree of marketing effort, and market demand increase with the rise in carbon trading price. (3) Examining the trends in the equilibrium outcomes for home appliances, when the carbon reduction investment is high $(h_m = 200000)$, the carbon trading price has the most significant impact on the level of carbon reduction; when the carbon reduction

investment is low ($h_m = 50000$), the carbon trading price predominantly influences the level of carbon reduction e, wholesale price ω , and market demand D^{MC^*} . This indicates that an increase in the government's carbon trading price p_e facilitates carbon reduction. However, regardless of the changes in the manufacturer's carbon reduction investment, the costs ultimately affect the prices of home appliances, thereby influencing market demand. Therefore, to maximize their respective interests, it is essential for the government, enterprises, and consumers to negotiate and find an equilibrium point that balances the interests of all parties.

Fig. 5 analyzes the impact of changes in carbon trading prices on corporate profits, as detailed below:

Fig. 5 shows that when the carbon reduction investment is high $(h_m = 200000)$, the total profits of home appliance dealers, manufacturers, and the supply chain decrease with the increase in carbon trading price p_e . Conversely, when the carbon reduction investment is low $(h_m = 50000)$, the profits of home appliance dealers and the total profits of the supply chain under cooperative decision scenarios increase with the rise in carbon trading price p_e , while the profits of home appliance manufacturers and the total profits of the supply chain under decentralized decision scenarios decrease with the increase in carbon trading price p_e . This indicates that although a high carbon reduction investment beneficially enhances the carbon reduction level of home appliances to a certain extent, it does

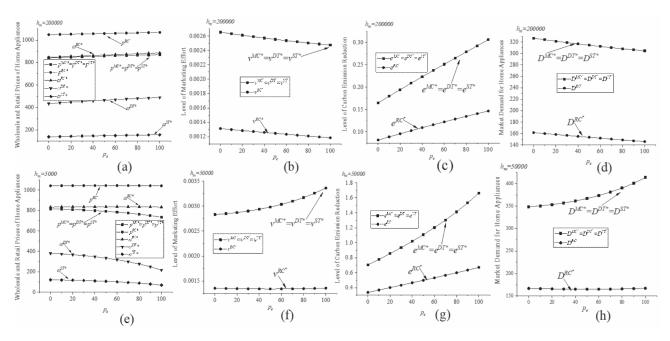


Fig. 4. Impact of Carbon Trading Prices on Equilibrium Outcomes.

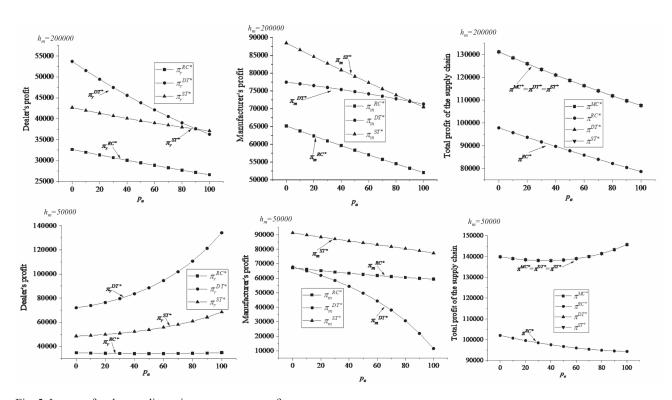


Fig. 5. Impact of carbon trading price on corporate profits.

not contribute to profit increases for businesses. On the other hand, a low carbon reduction investment maintains profit margins for businesses to some degree, but for home appliance manufacturers, their profits decrease with the increase in carbon trading price p_e . Therefore, for home appliance enterprises to maximize their interests, it is crucial to make reasonable carbon reduction investments and effectively counteract the profit losses brought by dynamic changes in carbon

trading prices. However, regardless of the variations in manufacturers' carbon reduction investments, the profits under cooperative scenarios are greater than those under non-cooperative scenarios. This suggests that home appliance enterprises seeking to optimize profits should avoid acting independently and instead strive to strengthen cooperation among upstream and downstream enterprises as well as within the enterprise itself.

Conclusions

This paper primarily analyzes a two-stage green supply chain comprising a single home appliance manufacturer and a distributor. We examine optimal decision-making issues regarding cooperative emission reduction and promotion among home appliance centralized enterprises under decision-making, decentralized decision-making, and contractual coordination. The paper further analyzes how consumer low-carbon preferences, carbon trading prices, and investment costs related to carbon emission reduction affect equilibrium outcomes and corporate profits. The research findings indicate the following:

- (1) When home appliance manufacturers and distributors cooperate for carbon-labeled home appliances, the operational performance of the home appliance products is higher than when operating independently.
- (2) Decisions on cooperative promotion and emission reduction can enhance the profits of both manufacturers and distributors and optimize the operational performance of home appliance products. Comparing equilibrium outcomes reveals that decisions on cooperative emission reduction are optimal from the perspective of the home appliance manufacturer's profits, while decisions on cooperative promotion are optimal from the retailer's profit viewpoint.
- (3) The optimal pricing of home appliance products, the level of carbon emission reduction, the extent of marketing efforts, and market demand volume are influenced by several factors. These factors include consumer low-carbon preferences, government-set carbon trading prices, the manufacturer's carbon emission reduction costs, and the distributor's marketing effort cost investments. Regardless of how the manufacturer's carbon emission reduction costs vary, the level of carbon emission reduction positively relates to consumer lowcarbon preferences and carbon trading prices. When the manufacturer's investment in carbon emission reduction costs is high, the government-set carbon trading price positively relates to the pricing of home appliance products. Conversely, the extent of marketing efforts and market demand volume are negatively related. The results are the opposite when the manufacturer's investment in carbon emission reduction costs is low.
- (4) Reducing carbon emissions in home appliance products is more important than price for consumers. For home appliance manufacturers, maximizing their interests requires adopting reasonable carbon emission reduction investments and effectively dealing with the profit losses brought about by changes in carbon trading prices. Regardless of changes in consumer lowcarbon preferences and government-set carbon trading prices, corporate profits under cooperative scenarios are greater than those under non-cooperative scenarios. Therefore, home appliance manufacturers distributors should strive to strengthen cooperation to maximize profits.

This study derived the following management insights: On the one hand, manufacturers and distributors should dynamically adjust their respective carbon emission reduction or promotional efforts in response to environmental changes. Manufacturers should produce green home appliances, distributors should actively promote and advertise to stimulate consumption and boost sales, generating more profits. On the other hand, the greater the impact of carbon emission reduction levels for products and promotional effort levels on market demand, the more motivated manufacturers are to invest in distributors. Likewise, more motivated distributors invest in promotional efforts to promote the consumption of green home appliance products. Furthermore, establishing a close contractual mechanism between distributors is essential for substantially enhancing distributor carbon emission reduction, distributor promotional efforts, and overall supply chain performance. This paper also has limitations and shortcomings that require further exploration and improvement. First, this paper only analyzed the optimal decision-making problem for a single manufacturer and distributor. Future research could expand to price competition among multiple distributors and manufacturers. Second, further research on online and offline differentiated pricing could be conducted. Lastly, designing hybrid contracts to coordinate dual-channel supply chains for green home appliance products could be considered.

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Conflict of Interest

The authors declare no conflict of interest.

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