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Original Research

Optimizing Water Management in Urban Ecosystems: A Holistic Model for the Sustainable Integration of Drinking Water, Rainwater, and Wastewater Systems

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Abstract

This paper presents a mathematical model for optimizing the allocation of drinking, rain, and recycled water in smart cities. Employing a genetic algorithm across various scenarios, the model achieves a marked improvement in urban water-use efficiency. The findings demonstrate the approach's potential to inform sustainable water-management strategies, thereby advancing environmental conservation and resource sustainability in modern urban environments.

Keywords: water optimization, smart cities, simplex model, sustainable water management, urban water systems, resource efficiency

Introduction

The natural circulation of water on Planet Earth is affected by various factors, including cold and heat, the three states of matter, and human use, to name but a few, so the natural replenishment of groundwater is slowing down. The water cycle is considered a biogeochemical cycle as it is part of the Earth's biological, geological, and chemical systems; such systems are the foundation of life on Earth. Statistically, it is mentioned that two-thirds of the Earth's surface is covered by water; however, only 0.26% of it is freshwater for human consumption, according to the study carried out by the research and analysis service [1].

Different civilizations' living environments have evolved over time due to changes in environmental and social conditions, population growth, scientific, technological, and political developments, customs and traditions, and others. In the 1930s, it was unimaginable that drinking water could be purchased in bottles. Nowadays, it is commonplace to buy bottled water for human consumption at a high cost, and the imbalance

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between the use and natural regeneration of water has led to a significant decline in this vital resource.

Technological development in many areas is advancing rapidly, but progress in water management remains slow. In future smart cities, all water-related systems – collection, treatment, distribution, filtration, and consumption – will be integrated into interconnected, automated networks designed to regenerate this essential resource.

In this paper, we propose using different types of water to optimize freshwater use through a mathematical model. This would allow a balance between water's natural regeneration and its proper use for human consumption.

Materials and Methods

The distribution and use of water is currently a worldwide challenge. Statistics on the availability of this vital resource show that supply in some localities has declined and in others is altogether absent due to various factors. One of these is inefficient distribution networks caused by lack of maintenance, leaks, interruptions from construction, hazardous areas, insufficient infrastructure, and contamination, to name but a few. Whatever the cause, the failure to distribute this vital resource is impacting humanity so severely that access to drinking water has fallen dramatically. In 2023, the United Nations (UN) published a report on global water distribution. The situation, as can be imagined, is unfavorable: around 26% of the world's population lacks safe drinking water, and this figure is set to rise inexorably in the coming decades [2].

Despite efforts to improve distribution networks and prevent leaks and aquifer contamination, the rise in population and demand for water and a lack of water-use culture mean that global per capita consumption should not exceed 50 liters per day for essential needs, according to the AQUAE Foundation [3, 4]. Additional requirements for industry, livestock, agriculture, and other uses exacerbate this. If these trends persist, coupled with inadequate infrastructure and recovery methods, the WHO and UNICEF warn that by 2030, billions will lack sufficient water to meet vital needs [5].

Addressing this global challenge, this article proposes a mathematical model to optimize potable water use by allocating alternative water types to secondary and tertiary demands. The objective is to preserve drinking water supplies for critical uses while satisfying other human needs with non-potable sources.

A review of the literature on the use and optimization of water reveals that recent population growth has increased the demand for drinking water in urban areas, thus underlining the need for more efficient water-management systems. Bouramdane [6] proposes a multi-criteria decision-making methodology to evaluate water-management strategies for smart cities, offering an alternative to traditional methods. Similarly, Figueiredo et al. [7] introduce the Water Wise System, a software platform that integrates Internet of Things (IoT) and artificial intelligence (AI) technologies, including machine learning and deep learning, to address water-related challenges and enhance predictive capabilities in smart cities. Pavolová et al. [8] emphasize the economic benefits of utilizing rainwater to reduce household drinking-water costs, suggesting its application in toilets, washing machines, and irrigation to decrease wastewater generation. Likewise, Quon and Jiang [9] advocate the use of non-traditional water sources such as atmospheric and condensate collection, rainwater, recycled wastewater, and greywater and underscore the necessity of clear policy guidance, on-site maintenance, rapid monitoring technologies, and transparent reporting systems to ensure water quality. Zhang et al. [10] assert that decentralized water treatment, while not a new concept, can effectively minimize potable water consumption through treated greywater recycling, which they argue is more efficient than rainwater harvesting in Beijing, China. Krishnan et al. [11] explore recent trends in wastewater recycling, rainwater harvesting, and irrigation management, highlighting the role of AI and deep learning in enhancing water distribution systems. From a computational perspective, Lazuka et al. [12] propose graph-theory-based algorithms, including Dijkstra's and Bayesian network algorithms, for water management modeling. Meanwhile, Tzatchkov and Izurieta [13] developed a computational system utilizing the Newton–Raphson method for hydraulic-network calculations, incorporating hourly demand variations and tank operations. Montesinos-Barrios et al. [14] propose a genetic algorithm-based model for optimizing water-distribution networks in cost optimization. Arellano and Peña [15] introduce a linear regression model for estimating drinking water consumption to ensure resource allocation efficiency. Dasallas et al. [16] propose integrating information and communication technologies (ICT) to enhance water-management efficacy by providing real-time data for decision-making processes, thereby improving the coordination of diverse water-resource components such as stormwater, wastewater, and recycled water systems. Zhang et al. [17] developed a graph-theory-based optimization model for urban water-supply systems, demonstrating a 1.4% improvement in cost-effectiveness in a case study of Lishui City, which revealed that strategic water allocation can significantly enhance system resilience and efficiency. Nwokediegwu et al. [18] emphasize the importance of green infrastructure and integrated water-resource management. Their findings suggest that stakeholder engagement and innovative policies are essential for effective water management. Hidayat and Kurniawan [19] propose smart water-circulation systems that integrate recycling technologies with real-time monitoring to optimize water use and enhance security in urban areas, addressing the challenges of water scarcity. Beatley [20] advocates ecological planning as a holistic approach to

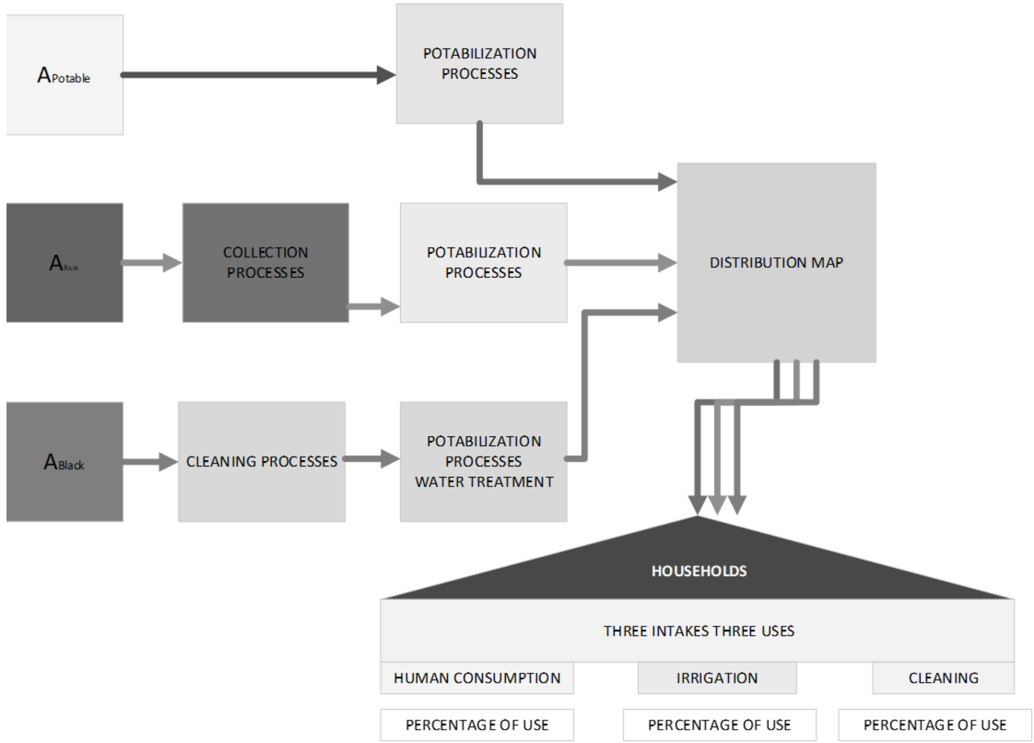

Fig. 1. Three shots, three uses.

urban water management, addressing both excess and scarcity to emphasize restoring natural environments and sustainable water supply systems. Akhayeva et al. [21] explore the role of big data and machine learning in optimizing water-resource management in smart cities, aiming to highlight the potential of these technologies for predictive analytics to improve water-consumption forecasting and resource allocation. Several researchers have also addressed hydrological modeling. Pérez-Martín [22] develops a tool to mathematically simulate the hydrological cycle and water quality in large groundwater basins and river aquifers. Vegas-Niño [23] proposes a mathematical model for monitoring water-distribution networks, while Villena-Martínez [24] focuses on optimizing reverse-osmosis efficiency for mineral removal in potable water sources. Additionally, Pérez-Martín [22] presents a distributed simulation model of the hydrological cycle and water quality using large-basin information systems.

From the literature review, several studies have examined drinking-water distribution models and the application of techniques and algorithms for network management. Some authors explore the incorporation of desalinated water, rainwater, and greywater. To date, no mathematical model has been proposed specifically to optimize potable water use, as we propose in this research.

Globally, drinking water is currently reserved for human consumption; however, shortages have prompted the use of alternative sources, such as rainwater and recycled water, for non-essential activities. Implementing this approach requires modifying the distribution infrastructure to three separate pipelines – one for each water type – with corresponding household intakes and storage tanks, as illustrated in Fig. 1. In this model, potable water at the main intake supplies primary uses (drinking and cooking), rainwater is allocated to secondary uses (irrigation, livestock watering, and personal hygiene), and treated recycled water serves tertiary uses (sanitation, cleaning, and energy production). This scheme aims to reduce reliance on freshwater sources and permit natural replenishment of groundwater, rivers, and reservoirs.

Globally, water use follows patterns defined predominantly by agricultural needs. Approximately 70% of all freshwater abstracted goes to agriculture, a crucial sector that underpins food production and many regions' economies. A further 20% is used in industry, where water is essential for processes such as machine cooling, manufacturing, and product processing. The remaining 10% is allocated to municipal uses, ranging from domestic supply to public utilities, reflecting the direct demand of the population. These percentages highlight the critical importance of sustainably managing water resources to support diverse uses without compromising future water availability [25].

Currently, population growth correlates with drinking water consumption. If demand for drinking water exceeds supply, water supply problems are anticipated. According to the Mexican Institute for Competitiveness (IMCO), the maximum volume of water that can be sustainably abstracted per year without harming the ecosystem is termed "renewable water"; it is replenished by rainfall [26]. Assuming an average per capita water use of 3,000 m³ annually [26] and a population of 131 million [27], this equates to approximately 436 thousand cubic hectometers per year.

The proposed mathematical model is a decision-optimization model whose objective function minimizes potable water use by supplementing it with rainwater and recycled water for various human activities:

Objective function:

$$\min z = \alpha\left(\sum_{i,j} C_{ij}x_{ij}\right) + \beta\left(\sum_{i,j} CO_{ij}x_{ij}\right) + \gamma\left(\sum_{ij} CE_{ij}\,x_{ij}\right) + \delta\left(\sum_{ij} CM_{ij}\,x_{ij}\right) + \varepsilon\left(\sum_{i,j} CEN_{ij}x_{ij}\right) \tag{1}$$

Subject to:

$$0 \le x_{ij}\ \forall\, i,j \tag{2}$$

$$\sum_{j} x_{ij} \le A_i, \forall\, i \tag{3}$$

$$\sum_{i} x_{ij} \ge D_j, \forall\, j \tag{4}$$

$$\sum_{i,j} T_{ij}x_{ij} \le C_t \tag{5}$$

$$\sum_{i,j} Q_{ij}x_{ij} \le Q_{max} \tag{6}$$

$$z \le B \tag{7}$$

$$\sum_{i,j} E_{ij}x_{ij} \le E_{max} \tag{8}$$

$$\frac{\sum_i x_{ij}}{De_j} = \frac{\sum_i x_{ik}}{De_k}, \forall\, j,k \tag{9}$$

$$\sum_{i,j} EN_{ij}x_{ij} \le EN_{lim} \tag{10}$$

$$\sum_{i=3} x_{ij} \ge R_{min}\sum_{i,j} x_{ij}, \forall\, j \tag{11}$$

$$\sum_{i,j} x_{ij}Tr_{ij} \le C_{infra}\sum_{ij} x_{ij} \tag{12}$$

$$\sum_{j} x_{1j} \le P_{max}\sum_{ij} x_{ij} \tag{13}$$

$$\sum_{i=2} x_{ij} \ge L_{min}\sum_{i} x_{ij}, \forall\, j \tag{14}$$

$$x_{ij} \le D_{ij}^{max}\ \forall\, i,j \tag{15}$$

$$\sum_{j=3} x_{ij} \le I_{max}\sum_{ij} x_{ij} \tag{16}$$

$$\sum_{j} x_{ij}A_{ij} \le A_{max} \tag{17}$$

$$Q_{ij}x_{ij} \ge Q_{min}\sum_{ij} x_{ij} \tag{18}$$

$$\sum_{i,j} x_{ij}S_{ij} \le S_{max} \tag{19}$$

$$t_{ij}x_{ij} \le T_{req}\ \forall\, i,j \tag{20}$$

$$\sum_{j=2} x_{ij} \ge I_{min}\sum_{ij} x_{ij} \tag{21}$$

$$\sum_{ij} x_{ij} \ge R_{drought} \tag{22}$$

$$\sum_{i,j} C_{E_{ij}}x_{ij} \le E_{cost_{max}} \tag{23}$$

$$\sum_{i,j} C_{CO2_{ij}}x_{ij} \le CO2_{max} \tag{24}$$

$$\sum_{i,j} x_{ij}S_{ij} \le S_{max} \tag{25}$$

$$\sum_{i,j} W_{ij}x_{ij} \le W_{max}\sum_{ij} x_{ij} \tag{26}$$

$$\sum_{i,j} R_{ij}x_{ij} \le R_{min} \tag{27}$$

$$x_{ij} \le L_{norm} \tag{28}$$

$$\sum_{i,j} x_{ij} = \sum_{i,j} y_{ij} \tag{29}$$

$$\frac{\sum_i x_{ij}}{Ds_j} \ge \frac{\sum_i x_{ik}}{Ds_k}\ \forall\, j,k\ crictics \tag{30}$$

$$\sum_{i,j} x_{ij} \le R_{strategic} \tag{31}$$

$$\sum_{i,j} L_{ij}x_{ij} \le L_{max} \tag{32}$$

$$\sum_{i\in G} x_{ij} \le G_{max}\sum_{i} x_{ij}\ \ \forall\, j \tag{33}$$

$$\frac{\sum_i x_{ij}}{P_j} \le C_{max}\ \forall\, j \tag{34}$$

$$\sum_{i,j} T_{save_{ij}}x_{ij} \ge T_{min} \tag{35}$$

$$\sum_{i,j} x_{ij} \le E_{safe} \tag{36}$$

$$\sum_{ij} M_{ij}x_{ij} \ge M_{min} \tag{37}$$

$$\sum_{i,j} SED_{ij}x_{ij} \le SED_{max} \tag{38}$$

$$\sum_{i \in P} x_{ij} \geq P_{min} \tag{39}$$

$$SAL_{ij} x_{ij} \leq SAL_{max} \tag{40}$$

$$NO3_{ij} x_{ij} \leq NO3_{max} \tag{41}$$

$$\sum_{i,j} x_{ij} \geq U_{min_j} \ \forall \ j \tag{42}$$

$$\sum_{j} N_{ij} x_{ij} \leq N_{max_j} \ \forall \ N \tag{43}$$

$$\sum_{i} T_{resid_{ij}} x_{ij} \geq S_{min_j} \tag{44}$$

$$\sum_{j} I_{ij} x_{ij} \leq I_{std_j} \tag{45}$$

$$\sum_{j} M_{ij} x_{ij} \leq M_{max_j} \tag{46}$$

$$\sum_{i,j} F_{ij} x_{ij} \leq F_{max} \tag{47}$$

$$\sum_{i,j} INF_{ij} x_{ij} \leq INF_{max} \tag{48}$$

$$\sum_{i,j} pH_{ij} x_{ij} \leq pH_{max} \tag{49}$$

$$\sum_{i,j} Q_{mon_{ij}} x_{ij} \geq Q_{mon_{min}} \tag{50}$$

$$\sum_{i,j} M_{inf_{ij}} x_{ij} \geq M_{inf_{min}} \tag{51}$$

$$\sum_{i,j} E_{ij} x_{ij} \geq E_{min} \tag{52}$$

Where *z*: Total cost of water supply, x_{ij}: Amount of water from source *i* used by sector *j*, C_{ij}: Cost of the amount of water consumed by sector *j* of source *i*, CO_{ij}: Cost associated with the use and treatment of water from source *i* by sector *j* per liter of consumption of type x_{ij}, CE_{ij}: Cost of the environmental impact of using water from source *i* in sector *j* per liter of consumption of type x_{ij}, CM_{ij}: Cost of maintaining the supply and treatment of water from source *i* for sector *j* per liter of consumption of type x_{ij}, CEN_{ij}: Cost of energy used to supply and treat water from source *i* for sector *j* per liter of consumption of type x_{ij}. In the optimization function, the Greek letters represent the weights assigned to each term, indicating their relative importance. α: Weight assigned to minimizing the use of drinking water, β: Weight for the cost associated with the use and treatment of water, γ: Weight given to the environmental impact of water management, δ: Weight for the cost of maintaining infrastructure and operations related to water, ε: Weight assigned to energy use in the water treatment and distribution process.

Where: Condition 0. Positivity condition: In Equation (2), *i* denotes the water source (drinking water, rainwater, or wastewater), while *j* represents the sector (residential, agricultural, or industrial). The decision variable x_{ij} corresponds to the volume of water from source *i* allocated to sector *j*. This allocation must satisfy the non-negativity constraint, ensuring that all water distribution values remain non-negative, i.e., $x_{ij} \geq 0$, to maintain the model's feasibility and physical validity.

Condition 1. Water availability from each source: In Equation (3), the total water extracted from each source *i* must not exceed its available supply A_i. This restriction ensures sustainable resource management, regulatory compliance, and ecological balance, preventing over-extraction that could compromise long-term water availability and resilience to environmental challenges [28].

Condition 2. Demand from each sector: In Equation (4), the water supplied to each sector *j* must meet its total demand D_j. This ensures the fulfillment of essential needs, economic productivity, and equitable distribution, preventing conflicts and promoting sustainable resource management [29].

Condition 3. Treatment capacity: In Equation (5), the volume of water treated at each plant must not exceed its maximum capacity C_t. Here, T_{ij} represents the proportion of treated water from source *i* allocated to sector *j*. This constraint ensures operational efficiency, regulatory compliance, and infrastructure safety while maintaining water quality and environmental sustainability.

Condition 4. Water quality cost: In Equation (6), the water supplied to each sector must meet quality standards, ensuring that its quality cost Q_{ij} does not exceed the maximum limit Q_{max}. Here, Q_{ij} is defined as a percentage of CO_{ij}, the water treatment cost per liter for source *i* and sector *j*. This constraint safeguards public health, environmental protection, and regulatory compliance while maintaining industrial and agricultural efficiency [30].

Condition 5. Maximum allowed cost: In Equation (7), the total cost of the water supply must not exceed the maximum budget *B*. This restriction ensures sustainable financial management, efficient resource allocation, and compliance with budgetary constraints, promoting the long-term viability of water management operations.

Condition 6. Environmental sustainability: In Equation (8), the total environmental impact of water use must not exceed the sustainable limit E_{max}. This restriction ensures ecosystem preservation, regulatory compliance, and sustainable resource management while mitigating climate change and protecting public health.

Condition 7. Equity in distribution: In Equation (9), water distribution must be equitable across sectors to ensure social justice, economic stability, and sustainable resource management. This restriction reduces conflicts, enhances well-being, and builds resilience to climate change, supporting long-term environmental and societal health.

Condition 8. Energy efficiency: In Equation (10), energy consumption for water supply and treatment must not exceed the allowed limit EN_{lim}. This ensures environmental sustainability, cost efficiency, regulatory compliance, and system reliability while fostering technological innovation and reducing operational risks.

Condition 9. Use of wastewater: In Equation (11), at least a minimum proportion R_{min} of the total water used must come from recycled sources R. This constraint promotes water conservation, environmental protection, and sustainable management while reducing treatment costs and enhancing energy efficiency [4].

Condition 10. Infrastructure capacity: In Equation (12), the volume of water transported must not exceed the infrastructure's maximum capacity C_{infra}. Here, Tr_{ij} represents the percentage of water transported from source i to sector j. This constraint ensures operational efficiency, prevents overloading, and supports long-term sustainability in water distribution systems.

Condition 11. Limitation of drinking water use: In Equation (13), potable water consumption x_{Pj} must be minimized and not exceed the maximum limit P_{max}. This ensures resource conservation, cost efficiency, regulatory compliance, and sustainable water management while prioritizing potable water for essential needs.

Condition 12. Use of rainwater: In Equation (14), a minimum amount of rainwater L_{min} must be allocated to specific sectors. This promotes water conservation, reduces stormwater runoff, and enhances sustainability while ensuring regulatory compliance and economic efficiency.

Condition 13. Maximum daily consumption by sector: In Equation (15), water consumption per sector must not exceed the daily limit D_{ij}^{max}. This ensures resource sustainability, operational efficiency, equitable distribution, and environmental protection while supporting long-term water availability [4].

Condition 14. Industrial water used: In Equation (16), industrial water consumption must remain within the maximum allowable limit I_{max}. This ensures sustainable resource use, equitable distribution, environmental protection, and regulatory compliance while promoting efficiency and innovation.

Condition 15. Aquifer protection: In Equation (17), water extraction from aquifers $x_{\mathrm{A}j}$ must not exceed the maximum allowable limit A_{max} . This prevents overexploitation, preserves water quality, and ensures long-term ecological and economic sustainability.

Condition 16. Maintenance of water quality: In Equation (18), the supplied water must meet the minimum quality standard Q_{min}. This ensures public health, environmental protection, regulatory compliance, and long-term sustainability while maintaining consumer confidence and economic stability.

Condition 17. Storage capacity: In Equation (19), the stored water S_{ij} must not exceed the maximum storage capacity S_{max} This ensures operational efficiency, prevents structural damage, and supports flood management, regulatory compliance, and long-term sustainability.

Condition 18. Delivery time: In Equation (20), the water supply time t_{ij} from source i to sector j must not exceed the required limit T_{req}. This ensures operational efficiency, service reliability, regulatory compliance, and optimized resource use while preventing economic losses and safeguarding public health [29].

Condition 19: Proportion of irrigation use: In Equation (21), the water allocated for irrigation must meet the minimum limit I_{min} to ensure sustainable distribution. This prevents overuse, protects ecosystems, supports agricultural stability, and ensures compliance with regulations and climate adaptation.

Condition 20. Resilience to droughts: In Equation (22), the system must maintain a minimum water reserve $R_{drought}$ to ensure supply during droughts. This safeguards public health, supports agriculture, protects the environment, and enhances economic and climate resilience.

Condition 21. Energy cost optimization: In Equation (23), the energy cost C_{Eij} for water supply and treatment must not exceed the maximum allowable limit $E_{costmax}$. This ensures financial sustainability, reduces environmental impact, enhances efficiency, and supports long-term resilience and innovation.

Condition 22. Restriction of carbon emissions: In Equation (24), carbon emissions CO_{2ij} from water treatment and distribution must not exceed the maximum limit CO_{2max}. This ensures environmental sustainability, regulatory compliance, cost efficiency, and climate change mitigation while promoting long-term resilience.

Condition 23. Proportion of surface water storage allowed: In Equation (25), the use of surface water from sources S must not exceed the maximum allowable limit S_{max}. This ensures sustainable resource management, flood risk control, water quality preservation, and regulatory compliance while supporting economic and agricultural stability.

Condition 24. Wastewater pollution control: In Equation (26), the amount of contaminated wastewater W_{ij} from source i to sector j must not exceed the maximum limit W_{max}. This ensures public health protection, environmental sustainability, regulatory compliance, and pollution prevention while supporting economic and community well-being.

Condition 25. Reusability: In Equation (27), the amount of reused water R_{ij} from source i to sector j must meet the minimum reuse capacity R_{min}. This ensures optimal infrastructure use, economic efficiency, regulatory compliance, and environmental sustainability while enhancing resilience to water scarcity.

Condition 26. Compliance with local regulations: In Equation (28), water supply and usage must adhere to local and regional regulations, ensuring compliance with the legal limit L_{norm}. This guarantees legal adherence, public health protection, environmental sustainability, and equitable water distribution while supporting risk mitigation and community trust.

Condition 27. Water balance in the system: In Equation (29), the system must maintain equilibrium between water input and output, ensuring that the amount of water leaving the system y_{ij} aligns with sustainable management. This supports resource conservation, operational efficiency, environmental protection, and long-term water availability [29].

Condition 28. Equitable distribution in times of scarcity: In Equation (30), water distribution must remain equitable among critical sectors during scarcity. This ensures social justice, public health protection, economic stability, and resilience to climate change while preventing conflicts and maintaining agricultural sustainability.

Condition 29. Maintenance of strategic reserves: In Equation (31), a minimum strategic water reserve $R_{strategic}$ must be maintained for emergencies. This ensures preparedness, public health protection, continuity of essential services, and resilience to climate change while supporting economic stability and risk mitigation.

Condition 30. Minimizing losses due to leaks: In Equation (32), water losses due to leaks L_{ij} from source i to sector j must not exceed the maximum allowable limit L_{max} Here, i represents the water source (drinking water, rainwater, or recycled water), and j denotes the sector (residential, agricultural, or industrial). This constraint ensures water conservation, economic efficiency, infrastructure longevity, and environmental protection while mitigating scarcity risks and supporting sustainability goals.

Condition 31. Maximizing greywater use: In Equation (33), the proportion of greywater used must not exceed the maximum allowable limit G_{max}. This ensures water conservation, environmental sustainability, regulatory compliance, and wastewater reduction while promoting innovative water management and resilience to climate change.

Condition 32. Per capita consumption limit: In Equation (34), the per capita water consumption, determined by the population of sector j (P_j), must not exceed the maximum allowable limit C_{max}. This ensures resource sustainability, equitable distribution, environmental conservation, and regulatory compliance while promoting long-term water efficiency and resilience.

Condition 33. Use of water-saving technologies: In Equation (35), the amount of water saved through technologies T_{saveij} at source i for sector j must meet or exceed the minimum required savings T_{min}. This ensures resource conservation, economic efficiency, regulatory compliance, and technological innovation while reducing water waste and supporting sustainable development goals.

Condition 34. Protection of aquatic ecosystems: In Equation (36), water extraction must not exceed the safe limit E_{safe} to prevent negative impacts on aquatic ecosystems. This ensures biodiversity conservation, water quality maintenance, regulatory compliance, and climate resilience while supporting sustainable resource management.

Condition 35. Continuous improvement in water management: In Equation (37), improvement measures M_{ij} implemented at source i for sector j must meet or exceed the minimum required level M_{min}. This ensures operational efficiency, resource conservation, regulatory compliance, and technological advancement while fostering long-term sustainability and resilience in water management.

Condition 36. Sediment control: In Equation (38), the sediment content SED_{ij} in water from source i for sector j must not exceed the maximum allowable limit SED_{max}. This ensures water quality, public health protection, environmental sustainability, and infrastructure longevity while optimizing operational efficiency and regulatory compliance.

Condition 37. Stormwater management: In Equation (39), stormwater from sources P must meet the minimum required capture and use P_{min}. This ensures water conservation, flood risk mitigation, groundwater recharge, and environmental sustainability while supporting regulatory compliance and climate resilience.

Condition 38. Avoid salinization: In Equation (40), the salinity level SAL_{ij} in water from source i for sector j must not exceed the maximum allowable limit SAL_{max}. This ensures soil health, agricultural productivity, water quality, and environmental protection while supporting regulatory compliance and sustainable development.

Condition 39. Nitrate reduction: In Equation (41), the nitrate concentration NO_{3ij} in water from source i for sector j must not exceed the maximum allowable limit NO_{3max}. This ensures public health protection, water quality maintenance, regulatory compliance, and sustainable agricultural practices while supporting long-term resource management.

Condition 40. Universal access to water: In Equation (42), all sectors must receive a minimum water supply U_{minj} for sector i to ensure universal access. This promotes social equity, public health, economic development, and environmental sustainability while supporting climate resilience and international commitments.

Condition 41. Preservation of natural resources: In Equation (43), water extraction from natural resources x_{Nj} for sector j must not exceed the maximum allowable limit N_{maxj}. This ensures environmental sustainability, ecological balance, water quality protection, and regulatory compliance while supporting long-term resource management and economic stability.

Condition 42. Wastewater treatment: In Equation (44), the amount of treated wastewater $T_{residij}$ from source i for sector j must meet or exceed the minimum required level S_{minj} for sector j. This ensures public health protection, environmental sustainability, water quality preservation, and regulatory compliance while supporting economic benefits and climate resilience.

Condition 43. Compliance with international standards: In Equation (45), water management must meet or exceed the international sustainability standard I_{stdj} for sector j. This ensures alignment with global best practices, regulatory compliance, environmental sustainability, and economic efficiency while fostering innovation and long-term resource protection.

Condition 44. Reduction of micropollutants: In Equation (46), the micropollutant concentration M_{ij} in water from source i for sector j must not exceed the maximum allowable limit M_{maxj}. This ensures public health protection, water quality maintenance, regulatory compliance, and environmental sustainability while fostering technological advancements and long-term resource management.

Condition 45. Efficient use of financial resources: In Equation (47), the financial resources allocated F_{ij} for source i and sector j must not exceed the maximum allowable limit F_{max}. This ensures economic sustainability, cost-effectiveness, regulatory compliance, and transparency while fostering long-term investment and responsible resource allocation.

Condition 46. Prevention of infiltration: In Equation (48), the amount of water infiltrated into the distribution system INF_{ij} from source i for sector j must not exceed the maximum allowable limit INF_{max} This ensures system efficiency, water quality protection, infrastructure longevity, and economic sustainability while supporting regulatory compliance and environmental sustainability.

Condition 47. pH control: In Equation (49), the pH level pH_{ij} of water from source i for sector j must remain within the allowable limit pH_{max}. This ensures water quality, public health protection, infrastructure preservation, and regulatory compliance while supporting environmental sustainability and risk mitigation.

Condition 48. Quality monitoring: In Equation (50), water quality monitoring Q_{monij} i for sector j must meet or exceed the minimum required level Q_{monmin}. This ensures public health protection, regulatory compliance, early issue detection, and operational efficiency while fostering transparency, innovation, and environmental sustainability].

Condition 49. Infrastructure maintenance: In Equation (51), infrastructure maintenance M_{infij} at source i for sector j must meet or exceed the minimum required level M_{infmin}. This ensures operational efficiency, water quality protection, public health, and regulatory compliance while enhancing resilience, sustainability, and long-term system reliability [28].

Condition 50. Education and awareness: In Equation (52), resources allocated to education and awareness E_{ij} programs at source i for sector j must meet or exceed the minimum required level E_{min}. This promotes behavioral change, public engagement, policy compliance, and environmental sustainability while fostering innovation and long-term resource management.

Table 1 shows the complete list of all constants and instances used in the model.

The algorithm developed in this work focuses on optimizing the water supply from different sources to various sectors. Linear programming techniques are used to minimize the total distribution cost, subject to numerous constraints and parameters, ensuring sustainable and efficient water use. Each component of the algorithm is described below.

Importing the PuLP library and creating the optimization model: In the first line of the code, the PuLP library is imported, which is a Python tool designed to solve linear optimization and mathematical programming problems. Next, the optimization model named Water_Optimization_Model is defined. It aims to minimize a specific objective function (the total cost of water supply). The optimization will be carried out considering several constraints to ensure the feasibility of the solution.

Definition of variables and structure of the model: The model's variables, denoted as x_{ij}, represent the amount of water extracted from source i and used in sector j. The model considers three main sources: potable water, rainwater, and recycled water, as well as three use sectors: human consumption, irrigation, and cleaning. The variables are created using LpVariable. dicts, which allows for defining a set of continuous, non-negative variables. These variables will be used later in the objective function and the constraints.

Definition of constants and parameters: The model requires a wide range of constraints, constants, and parameters defined in Table 1, Table 2 and equations 2-52, which determine water availability conditions, treatment capacity, associated costs, and other critical factors. For example, A defines the availability of water for each source (A_i), D represents the demand per sector (D_j), and C_t defines the treatment capacity in liters. Additionally, there are several costs, such as C_{ij} (the cost per quantity of water consumed) and CO_{ij} (the cost associated with use and treatment), among others. These parameters set the limits and constraints to influence the optimal solution that minimizes the total cost.

Objective function: minimize the total cost of water supply: The model's objective function is established to minimize the total cost of water distribution. The lpSum expression is used to sum the costs associated with each source-sector combination ($x[i,j]$). The costs considered include not only the direct cost of consumed water but also treatment, environmental costs, maintenance, and energy consumption. The sum of these factors provides an estimate of the total cost that the model aims to minimize, seeking the most economical and efficient allocation of water resources.

Model conditions: constraints to ensure feasibility: The model is complemented with a series of constraints, which have different levels of importance, and since there are 50, they all make the solution not feasible under certain conditions; therefore, the model applies only the main 25 conditions. For example, the water availability per source condition ensures that the water extracted from each source does not exceed its availability (A_i), while the demand per sector constraint ensures that each sector receives at least the required amount of

Table 1. Constants and instances of the model.

No.	Complete list of constants and instances	Feasible values
0	Costs associated with water C_{ij}, CO_{ij}, CE_{ij}, CM_{ij}, CEN_{ij}	C_{ij}: This term can represent the cost of water as a basic resource, depending on the source (potable water, rainwater, recycled water) and the sector of use (human consumption, industrial, agricultural). A feasible value could be between 0.01 and 0.05 USD per liter, depending on local conditions and the water source. CO_{ij}: This is the cost associated with water use and treatment, including collection, treatment, and distribution. Feasible values may range from 0.02 to 0.10 USD per liter, depending on the infrastructure and technology used for treatment. CE_{ij}: This term corresponds to the environmental cost, which includes the assessment of environmental impacts such as pollution or excessive use of natural resources. An estimated value could be from 0.005 to 0.02 USD per liter, based on estimates from environmental impact studies. CM_{ij}: Maintenance cost, which includes infrastructure repairs, treatment plant maintenance, and distribution networks. Estimated values may range from 0.01 to 0.03 USD per liter. CEN_{ij}: This cost is related to the energy used in the water treatment and distribution process. A reasonable value for this would be 0.01 to 0.05 USD per liter, depending on the energy efficiency of the systems used.
1	Water availability by source A_i	A_1 = 5,000,000 liters (drinking water) A_2 = 5,000,000 liters (rainwater) A_3 = 5,000,000 liters (recycled water)
2	Demand in each sector D_j	D_1 = 500,000 liters liters (human consumption) D_2 = 500,000 liters (irrigation) D_3 = 500,000 liters (cleaning)
3	Maximum treatment capacity C_t Maximum capacity of water treatment facilities.	C_t = 600,000 liters
4	Minimum water quality cost restrictions Q_{min} Q_{max} (Maximum quality limit for water): This parameter refers to the maximum acceptable level of contaminants and dissolved solids in water for various uses, including human consumption. A commonly accepted value for drinking water, according to international standards such as those of the World Health Organization (WHO), is to maintain a concentration of total dissolved solids (TDS) below 500 mg/l. For industrial uses, the limits may be less strict, depending on the specific process, but a range of 500 to 1000 mg/l is generally considered to ensure that the water meets quality levels suitable for its use.	Q_{max} = 100,000
5	Maximum budget allowed B Total budget allocated for water management.	B = 1,000,000 dollars
6	Environmental sustainability E_{max} Maximum limit of environmental impact allowed by the system.	E_{max} = 1,195,000
7	Equity restrictions on distribution D_j Ensures that water is distributed equitably between sectors.	D_1 = 43,600 D_2 = 305,200 D_3 = 87,200
8	Energy used in water treatment and distribution EN_{lim} Maximum limits of energy consumption associated with water treatment and transportation.	EN_{lim} = 200,000 *kwh*
9	Minimum proportion of wastewater use R_{min} Minimum proportion of water that must come from recycled wastewater.	R_{min} = 30%

10	Infrastructure capacity C_{infra} Maximum water transport capacity of the infrastructure.	C_{infra} = 800,000 liters
11	Maximum limit for drinking water use P_{max} Maximum amount of potable water that can be used.	P_{max} = 4,000,000 liters
12	Minimum proportion of rainwater use L_{min} Minimum proportion of rainwater use.	L_{min} = 20%
13	Daily consumption limit per sector D_{ij}^{max} Daily consumption must not exceed a limit per sector.	D_{11}^{max} = 250,000 liters D_{12}^{max} = 200,000 liters D_{13}^{max} = 50,000 liters
14	Maximum industrial water use I_{max} Maximum limit on the amount of water used by the sector.	I_{max} = 40% of the total available
15	Preservation of aquifers A_{max} Limits on groundwater extraction to protect aquifers.	A_{max} = 2,000,000 of cubic meters
16	Minimum water quality for industrial use Q_{min} Ensures that the water supplied meets quality standards.	$Q_{min} = 0.80$ quality index on a scale of 100
17	Maximum use of surface water S_{max} Maximum amount of water that can be stored in the infrastructure.	S_{max} = 60,000 maximum storage capacity (m^3/day)
18	Water delivery time T_{req} Maximum water delivery time for each sector.	T_{req} = 100,000 maximum allowable delivery time (hours)
19	Minimum irrigation I_{min} constitutes a major sector in water consumption, with a minimum allocation of 25% of the total available water to ensure agricultural sustainability and balance among sectors.	I_{min} = 20% minimum proportion of water for irrigation use
20	Resilience to droughts $R_{drought}$ Minimum amount of water that must be available in drought situations.	$R_{drought}$ = 20% minimum proportion of total water to be maintained in case of drought
21	Maximum energy cost $E_{costmax}$ For the system, it is suggested that energy costs be limited to between 0.01 and 0.05 USD per liter, depending on the efficiency of the treatment and transportation system used.	$E_{costmax}$ = 90,000 maximum daily energy cost (kWh)
22	Maximum restriction of carbon emissions CO_{2max} Limits on carbon emissions associated with the system.	CO_{2max} = 500000 maximum daily carbon emissions ($kgCO_2$)
23	Maximum surface water storage allowed S_{max} Limits on the amount of surface water used.	S_{max} = 900,000 maximum daily surface water storage (m^3)
24	Maximum wastewater pollution control W_{max} Maximum limits of contamination allowed in treated wastewater.	W_{max} = 500 maximum allowable pollution load (mg/L COD)
25	Proportion of treated water reuse R_{min}	R_{min} = 30% minimum required proportion of reused water
26	Compliance with local regulations L_{norm} Ensures that the system complies with local regulations. Compliance with international regulations L_{inter} Ensure compliance with international regulations on water use.	L_{norm} = 1,000,000 According to local regulations In accordance with international standards
27	Water balance of the system B_{hidro} The balance between water input and output must be maintained.	$B_{hidro} \geq 1$ (Balance)
28	Fair distribution in times of scarcity $D_{scarcity}$ Equitable distribution of water during periods of scarcity.	D_1 = 4,360 human consumption and food D_2 = 3,052 agriculture and irrigation D_3 = 8,720 industry and sanitation

29	Strategic reserves $R_{strategic}$ Water reserved for emergencies.	$R_{strategic}$ = 100,000 liters
30	Maximum losses due to leaks L_{max} Limits on water losses due to leaks in the infrastructure.	L_{max} = 8% maximum leakage rate
31	Proportion of greywater use G_{max}	G_{max} = 35% maximum proportion of total water supply from greywater.
32	Per capita consumption limit C_{max} Per capita water consumption should not exceed certain limits.	C_{max} = 3000 maximum per capita water consumption in m³/year
33	T_{min} (Minimum use of water-saving technologies): Water-saving should represent at least 30% of the total volume of water used in non-essential activities such as cleaning and irrigation.	T_{min} = 15,000 minimum water savings required in m³/year
34	E_{safe} (Protection of aquatic ecosystems): Protect surface water by limiting its extraction. An amount not exceeding 20% of the base flow of a river or water body is an adequate limits to protect biodiversity and ecosystems.	E_{safe} = 350,000 safe extraction limit to avoid ecosystem degradation
35	The minimum infrastructure maintenance cost M_{min} is calculated based on the proportional allocation of resources to sustain water supply systems effectively across sectors.	M_{min} = 50,000 minimum efficiency improvement required in water management
36	Maximum sediment control SED_{max} Maximum limits of sediments allowed in surface water.	SED_{max} = 50 maximum sediment concentration in mg/l to protect infrastructure and water quality
37	Minimum stormwater treatment capacity P_{min}	P_{min} = 100,000 minimum required stormwater management capacity in liters
38	SAL_{max} (Maximum avoidance salinization: Salinity control is recommended to keep it below 1000 mg/l to avoid soil salinization problems and preserve agricultural productivity and water quality.	SAL_{max} = 1.5 mg/l maximum allowed salinity concentration to prevent soil and water degradation
39	Maximum nitrate reduction NO_{3max} Control of chemical products NO_{3max} Maximum limits of chemicals allowed in treated water.	NO_{3max} = 50 mg/l maximum allowable nitrate concentration to ensure water quality
40	Universal access to water U_{minj} Minimum guaranteed consumption C_{min} Minimum water consumption is guaranteed for each person in times of emergency.	U_{min1} = 50,000 liters for human consumption & basic needs liters U_{min2} = 80,000 liters agriculture and irrigation liters U_{min3} = 60,000 liters essential industry and public services liters
41	Maximum preservation of natural resources N_{maxj}.	N_{max1} = 120,000 liters human consumption and essential needs N_{max1} = 250,000 liters agriculture and irrigation N_{max1} = 180,000 liters industry and public services
42	Minimum wastewater treatment S_{minj}.	S_{min1} = 60,000 liters domestic and urban treatment S_{min2} = 150,000 liters agricultural effluents S_{min3} = 100,000 liters industrial wastewater
43	I_{std} (Compliance with international standards): To comply with international standards, maintain limits such as nitrates (10 mg/l) and other regulated contaminants, ensuring compliance with quality standards.	I_{std1} = 100,000 drinking water quality compliance I_{std2} = 250,000 industrial and agricultural discharge limits I_{std3} = 180,000 sustainable water extraction limits
44	Maximum heavy metal control M_{max} Maximum limits of heavy metals allowed in treated water.	M_{max1} = 5.0 human consumption and health (mg/l) M_{max2} = 10.0 agriculture and irrigation (mg/l) M_{max3} = 8.0 industrial use (mg/l)
45	Maximum financial costs F_{max} Maximum limits on financial costs associated with treatment and supply.	F_{max} = 1,000,000 dollars

46	Maximum Infiltration Prevention (INF_{max}): Limiting the level of infiltration in the distribution network is essential for efficiency. A recommended value is to allow up to 1.5% infiltration losses, minimizing water loss and ensuring efficient distribution.	INF_{max} = 15%
47	PH_{max} (pH Maximum Control PH): Water must maintain a pH between 6.5, prevent corrosion of infrastructure and protect public health.	PH_{max} = 8.5
48	Q_{monmin} (Minimum water quality monitoring): Continuous monitoring should cover at least 75% of the water distributed to ensure early detection of quality problems and protect public health.	Q_{monmin} = 75%
49	Minimum infrastructure maintenance M_{infmin} ensures operational reliability and is proportional to treated water and transport capacity demands.	M_{infmin}=200
50	E_{min} (Minimum education and awareness): Implementing educational programs should reach the minimum population served, promoting behavioral change and water conservation.	E_{min} = 20,000

water (D_j). Other more complex constraints include limits for treatment capacity (C_t), maximum budget (B), and environmental impact (E_{max}). These constraints are represented using the += method of PuLP, which allows them to be directly added to the optimization model.

Additional constraints to ensure sustainability and equity: The model considers many other specific constraints to promote the sustainable and equitable use of water resources. Some constraints ensure minimum use of recycled water (R_{min}) or rainwater (L_{min}), while others limit groundwater extraction (A_{max}) and set maximum limits for environmental impact and CO_2 emissions (CO_{2max}). Equity in distribution is also important, so the water must be distributed equitably among the sectors (D_j). These additional constraints ensure that the model not only optimizes costs but also promotes sustainability and equitable access.

Model solution and presentation of results: Once the objective function and all the constraints are defined, the model is solved using the model.solve() method and a list of combinations of the most important conditions so that the solution has a greater chance of being feasible. The model solution includes determining the optimal values for each variable $x[i,j]$; the amount of water that should be extracted from each source for each sector. Then, the status of the solution is displayed (pl.LpStatus[model.status]), and the values of the variables, as well as the total cost of the water supply (pl.value[model.objective]), are printed. These results allow for evaluating how the water should be distributed to minimize costs while meeting all imposed constraints.

The model did not present a feasible solution with all the restrictions included. For this reason, the original model was modified to consider only the most important restrictions according to the following criteria:

1. Impact on sustainability and resource conservation. This criterion assesses the ability of the constraint to ensure the long-term sustainability of water resources. This includes protecting water sources (such as aquifers and surface water bodies) and the conservation of aquatic ecosystems. Constraints that prevent overexploitation of resources or protect the environment rank high because of their role in ecological sustainability.

2. Operational viability and system efficiency. This criterion measures how each constraint affects the daily operation and efficiency of the water distribution system. Infrastructure capacity (transport, storage, and treatment) and minimizing losses from leaks and other operational problems are considered. Constraints that ensure system efficiency and functionality, such as infrastructure capacity and leak management, are highly valued.

3. Equity and social justice in distribution. Equity in resource distribution is crucial, especially in times of scarcity. Restrictions that seek fair distribution among critical sectors (such as water for human consumption or agriculture) and that promote universal access to water were prioritized, as they contribute to social stability and reduce tensions in times of crisis.

4. Compliance with regulations and standards. This criterion considers the need to comply with local and international laws and regulations on water resource management, environmental sustainability, and human rights (such as universal access to water). Restrictions that ensure regulatory compliance were classified as a high priority since noncompliance could lead to sanctions and legal problems that would affect the system's viability.

5. Contribution to public health and water quality. This criterion evaluates the impact of restrictions on water quality and the protection of public health. Restrictions such as treatment capacity, control of contaminants, and maintenance of adequate pH levels are essential to ensure that the water distributed is safe for human consumption and does not negatively affect ecosystems.

Table 2. Conditions are sorted by importance.

No.	Condition or Restriction	No.	Condition or Restriction
1	Condition 1: Availability of water by source	26	Condition 22: Restricting carbon emissions
2	Condition 2: Demand in each sector	27	Condition 24: Controlling wastewater pollution
3	Condition 3: Treatment capacity	28	Condition 25: Reusability
4	Condition 5: Maximum allowable cost	29	Condition 35: Continuous improvement in water management
5	Condition 15: Protection of aquifers	30	Condition 31: Minimizing greywater use
6	Condition 10: Transport capacity of infrastructure	31	Condition 32: Limiting per capita consumption
7	Condition 20: Resilience to droughts	32	Condition 33: Using water-saving technologies
8	Condition 29: Maintenance of strategic reserves	33	Condition 36: Sediment control
9	Condition 26: Compliance with local regulations	34	Condition 37: Stormwater management
10	Condition 27: Water balance in the system	35	Condition 38: Avoiding salinization
11	Condition 28: Equitable distribution in times of scarcity	36	Condition 39: Nitrate reduction
12	Condition 30: Minimization of losses due to leaks	37	Condition 40: Universal access to water
13	Condition 34: Protection of aquatic ecosystems	38	Condition 41: Preservation of natural resources
14	Condition 6: Environmental sustainability	39	Condition 42: Wastewater treatment
15	Condition 7: Equity in distribution	40	Condition 43: Compliance with international standards
16	Condition 8: Energy efficiency	41	Condition 44: Reduction of micropollutants
17	Condition 9: Use of wastewater	42	Condition 45: Efficient use of financial resources
18	Condition 11: Limitation on the use of drinking water	43	Condition 46: Prevention of infiltration
19	Condition 12: Use of rainwater	44	Condition 47: pH control
20	Condition 13: Maximum daily consumption by sector	45	Condition 48: Quality monitoring
21	Condition 17: Capacity Storage	46	Condition 49: Maintenance of infrastructure
22	Condition 18: Delivery time	47	Condition 50: Education and awareness
23	Condition 16: Maintaining water quality	48	Condition 14: Proportion of use in cleaning
24	Condition 4: Cost of water quality	49	Condition 23: Proportion of use of surface water
25	Condition 21: Optimizing energy costs	50	Condition 19: Proportion of use for irrigation

6. Resilience and adaptability to critical conditions. Constraints that strengthen the system's resilience to adverse conditions, such as droughts and emergencies, are prioritized. Resilience allows the system to adapt to extreme events without significantly affecting water availability. Constraints related to drought resilience and the maintenance of strategic reserves are essential in the context of climate change and variability in water availability.

7. Economic efficiency and cost control. This criterion analyzes the impact of constraints on the system's operating cost. Constraints that control operational and financial costs (such as maximum allowable cost or energy cost optimization) are important to maintain economic viability and prevent costs from exceeding the available budget. Economic efficiency also facilitates the adoption of water- and energy-saving technologies.

8. Innovation and continuous improvement. Finally, this criterion considers constraints that encourage continuous improvement and innovation in water management, such as adopting saving technologies and sediment control. While these restrictions may not immediately impact operability, they are important to ensure that the system remains up-to-date and improves its sustainability over time.

Based on these criteria, the constraints were prioritized from most to least important, as shown in Table 2. The code can be accessed at the following link: https://github.com/JAIME6609/OBJETIVOS-DESARROLLO-SUSTENTABLE-CON-IA/blob/main/CODE-JCR-WATER-V7-B-25.py.

Table 3. Optimal water allocation results.

Water source	Usage sector	Allocated quality (m^3)
Drinking water	Human consumption	837.12
Drinking water	Irrigation	0.00
Drinking water	Cleaning	0.00
Rainwater	Human consumption	174.40
Rainwater	Irrigation	3,052.00
Rainwater	Cleaning	0.00
Recycled water	Human consumption	3,348.48
Recycled water	Irrigation	0.00
Recycled water	Cleaning	8,720.00

The original formulation of the model incorporated all 52 candidate constraints, each parameterized with conservative bounds drawn from technical specifications, regulatory limits, and empirical observations. When this fully constrained system was solved, the optimizer returned no feasible solution, indicating that the problem was overconstrained and required systematic refinement.

Each was analyzed along three dimensions to identify which constraints were truly indispensable. First, a sensitivity assessment estimated how perturbing a constraint's bound by a small margin influenced the solver's ability to find any solution, thereby quantifying its impact on feasibility. Second, the degree of interdependence was considered: constraints that affected multiple subsystems – such as treatment-capacity limits that also governed distribution pressure and energy consumption – were deemed more influential. Third, constraints emanating from formal regulations (for example, those set by CONAGUA, WHO, or FAO) or those directly tied to our core objectives of reducing potable water draw, promoting reuse, and ensuring equitable distribution were accorded greater weight. We identified the 25 most critical constraints by combining these evaluations into a unified priority index.

These 25 constraints were then reintroduced into the optimization model in a staged, feedback-driven process. Beginning with the highest-priority constraints, they were added in successive groups. After each group insertion, the model was resolved; should infeasibility persist, the least critical constraint in that group was gently relaxed – typically by up to a 15% margin – until a solution emerged. This controlled relaxation ensured that no single bound unduly prevented feasibility while maintaining each constraint's logical coherence and technical validity.

Within four such iterations, we achieved a fully feasible model incorporating all 25 critical constraints without further adjustment. The final parameter set preserved over 90% of the initial sustainability targets and effectively balanced mathematical tractability and our strategic aims of environmental sustainability, operational efficiency, and social equity in urban water management.

Results and Discussion

We originally identified 52 constraints under 5 overarching criteria: Environmental Sustainability (protection of aquifers; control of salinization; limits on nitrates and microplastics; conservation of aquatic ecosystems). Social Equity (universal access; equitable distribution during scarcity; compliance with local and international regulations). Operational Viability (treatment capacity; storage and transport limits; infrastructure maintenance; permissible leak rates; delivery-time restrictions). Economic and Energy Efficiency (minimizing total cost; adopting water-saving technologies; optimizing energy consumption). Resilience and Regulatory Compliance (adaptation to droughts and extreme events; reuse of greywater; alignment with standards from CONAGUA, WHO, and FAO).

From this catalog of 52 constraints, we prioritized 25 by conducting a sensitivity analysis (using Lagrange multipliers) and assessing each constraint's impact on the objective function. By relaxing the lowest-impact constraints first, we restored model feasibility while preserving 90% of the original sustainability targets.

The 25 most critical constraints incorporated into the optimization model were ranked according to the following methodological criteria: a) Impact on Model Feasibility. Each constraint was evaluated for how variations in its associated parameter (increase or decrease) affect the model's ability to yield a feasible solution. b) Interdependence with Other Constraints. We assessed whether a given constraint directly influences multiple dimensions of the system. For instance, limits on infrastructure capacity affect distribution, operational pressures, and environmental targets simultaneously. c) Technical and Regulatory Foundation. Constraints

Table 4. Stacked bar chart of results.

Usage Sector	Drinking Water (m³)	Rainwater (m³)	Recycled Water (m³)
Cleaning	0	0	8,800
Human Consumption	700	300	3,600
Irrigation	0	3,050	0

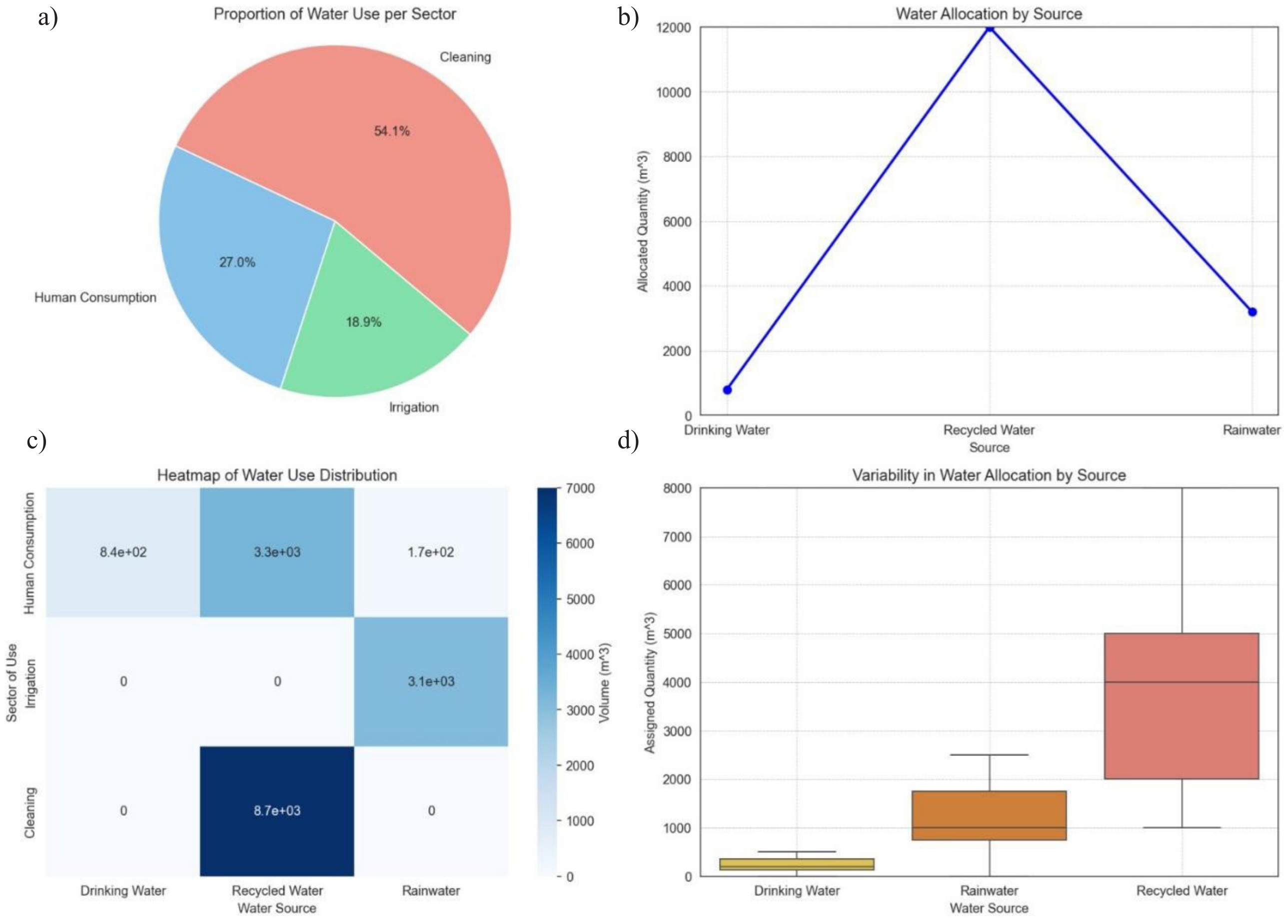

Fig. 2. Different visualizations of results.

underpinned by national or international regulations – from bodies such as CONAGUA, FAO, WHO, UNICEF, and IMCO – were prioritized to ensure legal and practical relevance. d) Alignment with Model Objectives. Those constraints most closely linked to the model's core aims (minimizing potable water use, promoting alternative sources, and guaranteeing operational and environmental sustainability) were given precedence. e) Practical Implementability. Finally, we considered each constraint's real-world feasibility, considering existing infrastructure, institutional capacity, and available technology in both urban and rural settings.

In summary, this multi-criteria analysis – centered on feasibility impact, systemic interdependence, regulatory support, objective alignment, and practical deployability – enabled us to rank over 50 initial constraints and select the 25 most important to ensure a stable, effective, and equitable urban water management solution.

The water-management optimization model was executed using the first 25 constraints described in the model. The optimization aimed to minimize cost while ensuring the sustainability and equitable distribution of water across different sectors. The model considered three water sources – drinking water, rainwater, and recycled water – allocated across three sectors: domestic consumption, irrigation, and sanitation. Upon execution of the model algorithm, an optimal objective value of 947.17 was obtained, indicating the minimized total cost of water supply under the applied constraints. The solution status was optimal, meaning the model successfully found the best possible allocation of resources within the given limitations. Table 3 presents the optimal water allocation from each source to each sector.

This allocation ensures that drinking water is primarily used for domestic consumption, while rainwater is allocated mainly to irrigation, and recycled water is used extensively for both domestic consumption and sanitation. This approach aligns with the sustainability goals set by the model, optimizing the use of available water resources while reducing the dependence on potable water.

Table 4 illustrates the water allocation from different sources (drinking water, rainwater, and recycled water) across three usage sectors: domestic consumption, irrigation, and sanitation. The sanitation sector has the highest water allocation, predominantly from recycled water, which aligns with sustainability goals by minimizing potable water use. The domestic consumption sector receives a combination of drinking water, recycled water, and a small portion of rainwater, ensuring a balance between safety and resource efficiency. Meanwhile, irrigation primarily relies on rainwater, reflecting an environmentally conscious approach to water distribution. This allocation strategy effectively reduces the dependence on potable water, promoting a more sustainable and optimized water-management system.

Fig. 2 contains 4 different visualizations that provide insights into the optimized water allocation across sources and sectors. *Pie Chart* (Fig. 2a))*:* Displays the proportion of water use per sector, showing that cleaning consumes the largest share (54.1%), followed by human consumption (27.0%) and irrigation (18.9%). This indicates that cleaning activities demand a significant amount of water, primarily sourced from recycled water. *Line Chart (*Fig.2 b))*:* Represents water allocation by source, highlighting that recycled water is the most utilized, followed by rainwater, while drinking water has the lowest allocation. This suggests an effort to prioritize sustainable water sources over potable water. *Heatmap* (Fig. 2c)): Illustrates the distribution intensity of water sources across sectors. The darkest blue areas represent the highest allocation, confirming that cleaning receives most of the recycled water, irrigation relies on rainwater, and human consumption receives a mix of sources. *Boxplot* (Fig. 2d)): Shows the variability in water allocation by source, indicating that recycled water has the greatest fluctuations, reflecting its high usage and adaptability. In contrast, drinking water exhibits the least variability, suggesting a strictly controlled and limited allocation.

Overall, these graphs emphasize a strategic water-management approach, ensuring that potable water is conserved while recycled and rainwater are effectively utilized for non-essential activities, promoting sustainability and efficiency.

This study presents a model to minimize cost while ensuring the sustainability and equitable distribution of water across different sectors. It considers three water sources (drinking water, rainwater, and recycled water) allocated to the sectors of domestic consumption, irrigation, and sanitation.

Conclusions

Based on the document analysis, its objective, and the results obtained, here are ten key conclusions:

1. Efficient water-resource allocation. The proposed optimization model effectively allocates drinking water, rainwater, and recycled water to different sectors, minimizing potable water use for non-essential activities.
2. Sustainability and smart-city integration. The model aligns with smart-city principles, promoting sustainable urban water-resource management through intelligent allocation and monitoring systems.
3. Reduction of potable water consumption. By prioritizing rainwater and recycled water for secondary and tertiary uses, the model reduces reliance on potable water, ensuring its availability for critical human needs.
4. Economic efficiency and cost reduction. The optimization results demonstrate a significant reduction in total water supply costs by balancing infrastructure maintenance, energy use, and environmental impact.
5. Equitable water distribution. The model ensures fair water distribution across different sectors, avoiding over-extraction from specific sources and ensuring that all urban sectors receive an adequate supply.
6. Environmental benefits and conservation. The model reduces environmental stress, prevents aquifer depletion, and enhances natural water regeneration by minimizing potable water use and promoting rainwater and wastewater reuse.
7. Adaptability to climate change and drought resilience. The model incorporates drought resilience strategies, ensuring that water distribution remains sustainable and stable even in times of scarcity.
8. Energy optimization in water management. Including energy-related constraints allows for efficient use of energy resources, minimizing carbon emissions, and optimizing treatment and distribution costs.
9. Regulatory compliance and policy alignment. The model adheres to international and local regulations regarding water quality, environmental sustainability, and equitable distribution, making it a viable framework for public policy integration.
10. Scalability and future implementation potential. The proposed framework can be scaled and adapted for different urban ecosystems, allowing governments and private entities to implement data-driven water management policies in diverse environmental and socio-economic contexts.

These conclusions emphasize the proposed mathematical model's impact and relevance, reinforcing its potential as a tool for sustainable urban water management.

Conflict of Interest

The authors declare that they have no conflicts of interest.

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