

Original Research

Municipal Solid Waste Incineration Fly Ash for Sustainable Building Materials: A Novel Decision Support System Incorporating Multi-Source Information and Missing Data

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Abstract

Municipal solid waste incineration (MSWI) fly ash poses environmental risks and demands sustainable management strategies. Reusing this byproduct in building materials offers a promising pathway for waste reduction and resource recovery. However, selecting the most sustainable fly ash (FA)-to-building material (BM) technology requires balancing diverse environmental, economic, and technical criteria. This task is further complicated by data gaps and inherent uncertainties, which often exclude emerging technologies, introduce bias through subjective weighting, and limit the reliability of sustainability assessments. To address these gaps, this study develops a novel decision support system (DSS) that imputes missing data using expert judgments and partial datasets, derives balanced weights by integrating subjective preferences with objective data features, and employs the PROMETHEE II outranking method to rank alternatives amid uncertainty and incompleteness. The proposed DSS enhances existing assessment methods by effectively managing information gaps and supporting robust sustainability evaluations. A case study involving seven FA-to-BM alternatives and twelve criteria demonstrates the practical applicability and decision-making capacity of the framework. The results highlight the system's potential to assist complex decision-making processes and provide methodological insights for the sustainable reuse of MSWI fly ash in building materials.

Keywords: MSWI fly ash, building materials, missing data imputation, combined weighting method, outranking approach

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Introduction

The global generation of municipal solid waste (MSW) has increased significantly, creating an urgent need for more effective treatment solutions. Although landfilling remains a widely used disposal method, it consumes valuable land resources and poses significant risks to soil and groundwater. In contrast, incineration has become more popular because it effectively reduces waste volume while recovering energy [1]. However, municipal solid waste incineration (MSWI) produces secondary pollutants, including fly ash, which is classified as hazardous waste because of its heavy metal, chloride, and dioxin content [2].

When treated as hazardous waste, MSWI fly ash typically requires solidification with cement or stabilization with chemical agents before landfilling. Yet, its composition shows substantial amounts of silica (SiO_2), calcium oxide (CaO), alumina (Al_2O_3), and other mineral phases commonly found in construction materials [3]. This composition suggests significant potential for repurposing the fly ash as a resource for producing building materials. Indeed, converting MSWI fly ash into building materials presents a promising, sustainable solution. It not only reduces the environmental impact of landfilling but also supports the development of greener construction products, offering dual benefits for waste management and the construction industry [4, 5].

Typical methods for reusing MSWI fly ash in the construction industry can be categorized into four main approaches: cement clinker co-combustion, high-temperature sintering/melting, low-temperature curing, and metal resource recovery, as illustrated in Fig. 1 [6]. In these processes, chemical or thermal treatments, or their combination, can reduce hazardous components, such as heavy metals, and ensure compliance with environmental and construction standards.

For instance, a real-scale test by Wang et al. [7] demonstrated the successful incorporation of MSWI fly ash into ordinary Portland cement. The results showed that adding fly ash had little effect on the mechanical strength of concrete, though it slightly shortened the setting time compared with cement without fly ash. Advanced processing techniques further increase flexibility and control in converting MSWI fly ash into high-value materials. For example, the strength of metakaolin-based geopolymers can be improved when MSWI fly ash is added at levels below 5 wt.%, underscoring its potential as a sustainable additive [8]. Similarly, when 30% MSWI fly ash by mass is incorporated, the bulk density of the resulting lightweight aggregates can reach 212 kg/m^3 , while other properties, including granule strength, one-hour water absorption, and shape factor, still meet established standards [9]. In addition, acid leaching can efficiently recover metal resources, achieving optimal recovery rates of 67% for Zn and 30% for Cu [10]. Building on these process-level investigations, recent studies

increasingly [11, 12] focus on the sustainable utilization of MSWI fly ash from integrated environmental, economic, and technical perspectives, emphasizing its role in circular economy strategies. These works demonstrate the feasibility of converting MSWI fly ash into value-added construction materials while enabling the recovery of valuable components, thereby highlighting its potential as a viable secondary resource.

Given the wide range of recycling methods and end-use applications, identifying the most sustainable pathway for reusing MSWI fly ash remains a major challenge. Pursuing this optimal solution not only stimulates innovation in waste management but also promotes the development of greener construction materials, supporting the goals of a circular economy.

To successfully reuse MSWI fly ash as building materials, it is essential to evaluate and compare the advantages and disadvantages of different processes from various perspectives. In existing literature, numerous studies have employed life cycle assessment (LCA) methods to compare the environmental benefits of different reuse routes. For example, one study compared three options: partial substitution in cement production, partial substitution in burnt brick manufacturing, and use as an alkali in Waelz steelmaking [13]. The results indicated that using fly ash in brick production is more environmentally friendly. Some studies combined LCA with economic analysis [14], energy analysis [15], or other methods to evaluate technologies from multiple angles, including environmental, economic, and energy considerations. These works show that fly ash reuse technologies perform differently across these dimensions. No single method is superior in all aspects. Consequently, recent research [16] has employed multi-criteria decision-making (MCDM) to address the conflicting and often incommensurable results of multi-criteria evaluations when assessing the sustainability of fly ash-based building materials. Methodological surveys indicate a rapid development of hybrid MCDM frameworks that integrate multiple sustainability dimensions through the combined use of criteria weighting and alternative ranking, thereby improving decision rationality in complex assessment contexts [17, 18]. In parallel, reviews of decision support systems (DSS) emphasize the importance of incorporating both quantitative and qualitative criteria, as well as a broad spectrum of alternatives covering established industrial processes and emerging technologies, to ensure comprehensive and defensible sustainability rankings [19].

However, the practical implementation of such comprehensive evaluation frameworks is challenged by the inherent complexity of MSWI fly ash reuse processes. These processes simultaneously involve technical, economic, and environmental considerations, while the corresponding technologies are at markedly different stages of development, ranging from large-scale industrial applications to small demonstration projects [6, 20]. As a result, obtaining complete

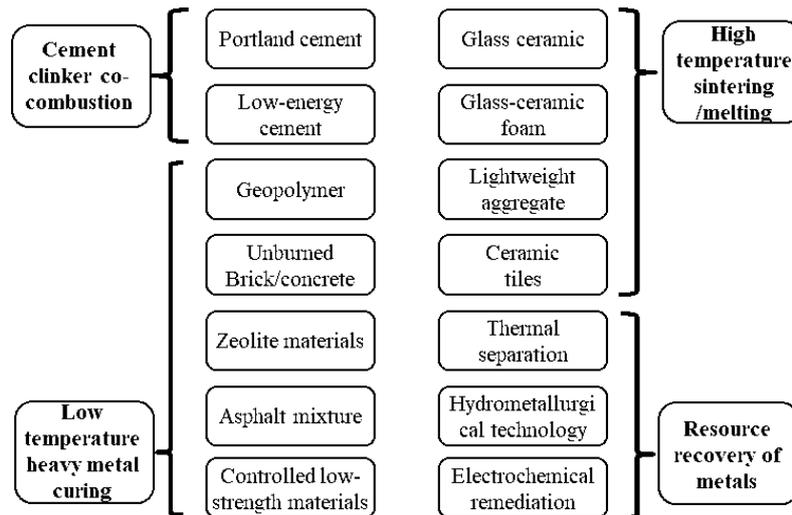


Fig. 1. Possible routes for reusing MSWI fly ash as building materials [6].

evaluation data for all alternatives is often unfeasible. In particular, many studies face challenges due to incomplete information, especially for emerging processes. For example, estimating the total annual revenue of novel technologies, such as fly ash-based geopolymer production, is difficult because of limited commercial implementation and scarce data.

Traditional treatments of missing data in MCDM can generally be categorized into two types. The first involves direct imputation without mathematical modeling, such as mean substitution [21], which often leads to biased estimates. The second relies on expert-based judgments through methods like the analytic hierarchy process (AHP) [22] or best-worst method (BWM) [23], where linguistic terms and level grading are commonly used in soft computing applications. However, these approaches depend heavily on subjective assessments, introducing human bias and limiting their practical applicability. In reality, evaluation problems often involve a mix of complete and incomplete data, making it difficult to effectively handle missing information in sustainability assessments [24]. To address this challenge, it is crucial to select suitable multi-criteria decision-making (MCDM) methods that can handle hybrid information for both weighting and ranking. In the context of fly ash treatment and utilization, commonly used MCDM weighting methods include AHP and BWM. These methods determine the relative importance of criteria based on expert judgment, which may be subject to manipulation and lead to biased weights. In contrast, ranking methods such as weighted sum method (WSM) and technique for order preference by similarity to ideal solution (TOPSIS) combine the weighted performance of alternatives using operations like summation or multiplication. Although these methods perform adequately when complete datasets are available, they are inherently limited or inapplicable in scenarios with missing values, as they typically require a fully populated decision matrix. This limitation is

particularly critical in the sustainability evaluation of MSWI fly ash reuse pathways, where emerging technologies often suffer from incomplete performance data across environmental, economic, and technical criteria.

In general, the research gaps identified in existing literature can be summarized as follows: Existing sustainability assessments of MSWI fly ash reuse predominantly assume complete and fully quantified datasets, leading to the frequent exclusion or oversimplification of alternatives with incomplete information. This practice systematically favors mature, well-documented technologies and limits the inclusion of emerging but data-scarce options, thereby reducing the inclusiveness and forward-looking capacity of decision outcomes. Moreover, although various MCDM frameworks have been applied, weighting strategies remain largely confined to either subjective judgment-based methods (e.g., AHP, BWM) or purely data-based approaches (e.g., entropy), both of which are sensitive to expert bias, data availability, and underlying data structure. In particular, the influence of criterion removal and uneven information quality on overall system performance is rarely considered. Consequently, few studies have established a unified and mathematically consistent framework integrating missing data imputation, hybrid qualitative–quantitative processing, balanced weight determination, and alternative prioritization. This limitation reduces the objectivity, inclusiveness, and practical applicability of traditional complete-data MCDM or expert-dependent tools in sustainability assessments of diverse MSWI fly ash reuse systems.

To overcome the above limitations, this study proposes a decision support system (DSS) that is distinguished from existing MCDM-based frameworks by its integrated treatment of data incompleteness, weight derivation, and alternative ranking. The main

contributions of this work can be summarized as follows:

- Inspired by the full consistency method (FUCOM) principles [25], a novel method for imputing missing data is proposed. It quantitatively estimates missing performance values by integrating expert-provided qualitative rankings with adjacent known data, while preserving ordering consistency and proportional relationships. This approach enables the systematic inclusion of data-scarce MSWI fly ash reuse technologies without relying on biased substitutions or exclusion-based treatments.

- A hybrid weighting strategy combining subjective importance and objective influence is proposed. It integrates level-based weight assessment (LBWA)-based subjective weights, which require minimal comparisons and reduce inconsistency, with the method based on the removal effects of criteria (MERECE)-based objective weights, and combines them via deviation-minimizing programming. This ensures that the criterion weights capture both expert priorities and actual impacts, enhancing robustness and transparency when applied to MSWI fly ash reuse sustainability assessment under hybrid information conditions.

- A pioneering decision support system (DSS) framework has been developed. The DSS mathematically integrates missing data handling, hybrid weighting, and alternative prioritization within a unified decision pipeline. This integration ensures consistent sustainability evaluation across environmental, economic, and technical dimensions, providing a more inclusive, objective, and practically implementable tool for identifying sustainable MSWI fly ash reuse pathways under incomplete and heterogeneous information.

Besides the first section of the Introduction, the rest of this work is organized as follows: Section 2 describes the materials and methods used in the DSS framework, Section 3 presents the results and discussion of a case study involving seven alternative options for reusing MSWI fly ash in building materials, and Section 4 provides the study's conclusions and future directions.

Materials and Methods

This section outlines the structure of the proposed decision support system (DSS). As shown in Fig. 2, the process begins with identifying the fly ash-to-building material (FA-to-BM) alternative technologies to be evaluated and defining the environmental, economic, and technical criteria for comparison. These preparatory steps ensure that the assessment framework matches user needs and application requirements. Next, performance data for each criterion are collected to form a decision matrix, which may contain missing values. For MSWI fly ash utilization systems, data gaps should be minimized through literature surveys, industrial investigations, software simulations, or other empirical approaches. Based on the compiled dataset, the DSS

proceeds through three sequential stages: missing data imputation, combined weight determination, and alternative prioritization. The final output identifies the most sustainable technology for reusing MSWI fly ash in building materials.

Missing Data Imputation Method

Missing data in decision-making often occurs when quantitative measurements for certain technologies are unavailable. However, expert knowledge can provide qualitative comparisons to compensate for these gaps. For example, consider the carbon emissions of three technologies: A_1 (2.2), A_3 (1.3), and A_2 (unknown). Based on expert judgment, the carbon emissions of these alternatives can be ranked as $A_1 > A_2 > A_3$. In this case, adjacent pairwise comparisons, inspired by the full consistency method (FUCOM) [25], are applied to impute the missing data. This approach uses expert-provided qualitative rankings to estimate a plausible value for the missing data, aligning it with surrounding known values while preserving qualitative ordering consistency. However, it is important to note that the missing data imputation method proposed in this paper is suitable for situations where the amount of missing data is relatively small. If missing entries reach or exceed one-third of the total data, additional data should be collected or alternatives with excessive gaps should be excluded. As outlined below, the missing data imputation method involves four steps.

Step 1.1 Rank alternatives based on performances.

When performance data are missing for some technologies under a given criterion, known data are combined with expert judgment to rank all alternatives from highest to lowest. For example, with seven technologies (A_1 to A_7), where data are available for A_1 - A_5 but missing for A_6 and A_7 , their performances may be ranked by integrating collected data for known cases and expert judgment for unknown ones, as follows: x_{2j} (23) $>$ x_{5j} (20) $>$ x_{1j} (18) $>$ x_{6j} (unknown) $>$ x_{4j} (14) $>$ x_{3j} (9) $>$ x_{7j} (unknown). For simplicity, this sequence is expressed as $S_{1j} > S_{2j} > S_{3j} > S_{4j}$ (unknown) $>$ $S_{5j} > S_{6j} > S_{7j}$ (unknown); where, for example, S_{1j} corresponds to the value 23 from A_2 under criterion C_j , and so on.

Step 1.2 Locate the missing data and list adjacent known data.

In this step, each missing point should be paired with its two closest known values. This may involve cases with the missing value in the middle (e.g., [S_{3j} (18), S_{4j} (unknown), S_{5j} (14)]) or at the end (e.g., [S_{5j} (14), S_{6j} (9), S_{7j} (unknown)]). If two adjacent values are missing, such as [$S_{(i-1)j}$ (known), S_{ij} (unknown), $S_{(i+1)j}$ (unknown), $S_{(i+2)j}$ (known)], create two subsets, [$S_{(i-1)j}$, S_{ij} , $S_{(i+2)j}$] and [$S_{(i-1)j}$, $S_{(i+1)j}$, $S_{(i+2)j}$], then apply the subsequent estimation procedure separately to each. However, if three consecutive values are missing, imputation becomes unreliable. In this case, for instance, [S_i (known), S_2 (unknown), S_3 (unknown), S_4 (unknown), S_5 (known)], the middle missing value (S_3) lacks directly adjacent

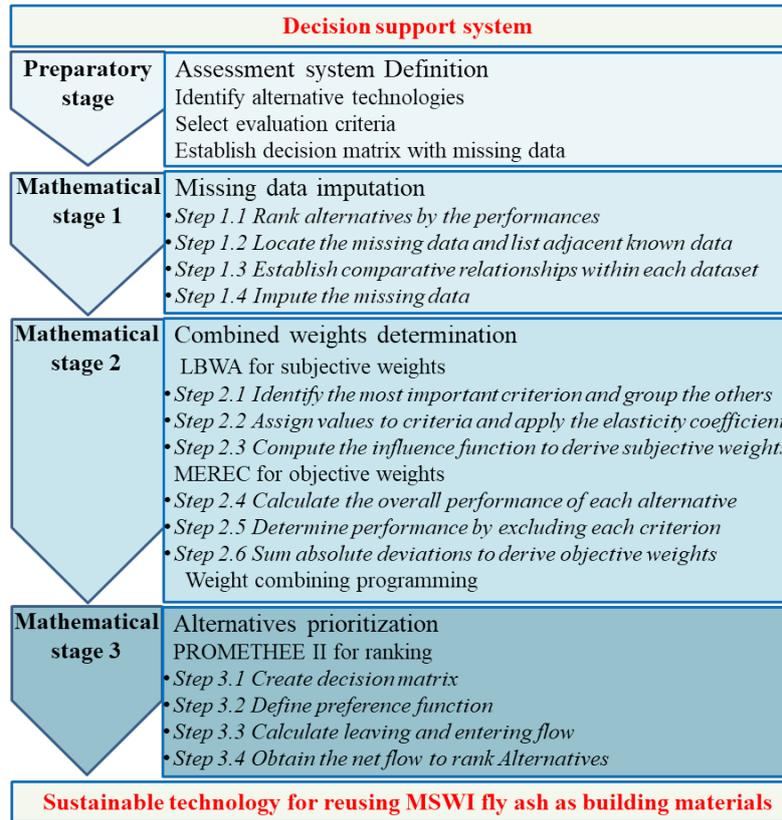


Fig. 2. The structure of the proposed decision support system for reusing MSWI fly ash building materials.

known data on either side. As a result, estimates must rely on distant values, increasing error and potential bias. Therefore, alternative data collection should be used when three or more consecutive values are missing.

Step 1.3 Establish comparative relationships within each dataset.

Each dataset requires pairwise comparisons among three elements, using either actual data or scale-based estimates. For example, as outlined in Step 1.1, with known data for S_{5j} (14) and S_{6j} (9), the comparative relationship is calculated as $\varphi_{5j/6j} = 14/9 = 1.56$. If data is missing, expert judgment can assign scale values (e.g., 1 to 9). For instance, $S'_{5j} = 3$ and $S'_{6j} = 2$ yield $\varphi'_{5j/6j} = 3/2 = 1.50$. This demonstrates that comparative relationships between ranked alternatives can be derived either from real data or scale-based expert assessments. If two alternatives perform equally, $\varphi = 1$ is used.

Based on the above information, comparative relationships can be established as follows: When the missing data is located in the middle, the required relationships are $S_{(i-1)j}/S_{ij} = \varphi_{(i-1)j/ij}$ and $S_{ij}/S_{(i+1)j} = \varphi_{ij/(i+1)j}$, while the direct relationship between known values, $S_{(i-1)j}/S_{(i+1)j} = \varphi_{(i-1)j/(i+1)j}$, can be calculated from available data. When the missing data is at either end, the required comparative relationships include $S_{(i-1)j}/S_{(i+1)j} = \varphi_{(i-1)j/(i+1)j}$, $S_{ij}/S_{(i+1)j} = \varphi_{ij/(i+1)j}$ (when $S_{(i+1)j}$ is unknown), or $S_{(i-1)j}/S_{ij} = \varphi_{(i-1)j/ij}$, $S_{(i-1)j}/S_{(i+1)j} = \varphi_{(i-1)j/(i+1)j}$ (when $S_{(i-1)j}$ is unknown). In either case, the calculated priority should be $S_{(i-1)j}/S_{ij}$

= $\varphi_{(i-1)j/ij}$ if $S_{(i+1)j}$ is unknown, or $S_{ij}/S_{(i+1)j} = \varphi_{ij/(i+1)j}$ if $S_{(i-1)j}$ is unknown.

Step 1.4 Impute the missing data.

Inspired by the FUCOM solving process [25], the pairwise relationships from Step 1.3 are used to build the programming model in Eq. (1). All values, known and unknown, are first normalized (i.e.,

$$\bar{S}_{ij} = \frac{S_{ij}}{(S_{(i-1)j} + S_{ij} + S_{(i+1)j})}).$$

The model then applies ratio and

transitivity constraints to estimate missing entries by minimizing a deviation parameter (θ). This ensures the imputed values align with both the available data and expert rankings, resulting in a complete decision matrix for later analysis.

Minimize θ

s.t.

$$\left| \frac{\bar{S}_{(i-1)j}}{\bar{S}_{ij}} - \varphi_{(i-1)j/ij} \right| \leq \theta, \left| \frac{\bar{S}_{ij}}{\bar{S}_{(i+1)j}} - \varphi_{ij/(i+1)j} \right| \leq \theta, \left| \frac{\bar{S}_{(i-1)j}}{\bar{S}_{(i+1)j}} - \varphi_{(i-1)j/(i+1)j} \otimes \varphi_{ij/(i+1)j} \right| \leq \theta,$$

$$\bar{S}_{(i-1)j} + \bar{S}_{ij} + \bar{S}_{(i+1)j} = 1, S_{(i-1)j} + S_{ij} + S_{(i+1)j} = T,$$

$$\bar{S}_{i-1} = S_{i-1}/T, \bar{S}_i = S_i/T, \bar{S}_{i+1} = S_{i+1}/T$$

(1)

According to Steps 1.1-1.4, Table 1 presents a detailed numerical example of the missing data imputation process. The first row lists the seven alternatives, where

Table 1. Numerical example of the missing data imputation procedure.

Original value	$A_1(18), A_2(23), A_3(9), A_4(14), A_5(20), A_6$ and A_7 unknown	
Ranks	$S_{1j}(23) > S_{2j}(20) > S_{3j}(18) > S_{4j}(\text{unknown}) > S_{5j}(14) > S_{6j}(9) > S_{7j}(\text{unknown})$	
Adjacent known data	For $A_6(S_{4j})$	For $A_7(S_{7j})$
	$[S_{3j}(18), S_{4j}(\text{unknown}), S_{5j}(14)]$	$[S_{5j}(14), S_{6j}(9), S_{7j}(\text{unknown})]$
Comparative relationships	Reference ratio: $\varphi_{3j/5j}=18/14=1.29$ Expert judgment: $\varphi_{3j/4j}=1.2$, and $\varphi_{4j/5j}=1.1$	Reference ratio: $\varphi_{5j/6j}=14/9=1.56$ Expert judgment: $\varphi_{5j/7j}=2.4$, and $\varphi_{6j/7j}=1.5$
Programming	Minimize θ s.t. $ \bar{S}_{3j}/\bar{S}_{4j}-1.2 \leq \theta, \bar{S}_{4j}/\bar{S}_{5j}-1.1 \leq \theta, \bar{S}_{3j}/\bar{S}_{5j}-1.2 \times 1.1 \leq \theta,$ $\bar{S}_{3j} + \bar{S}_{4j} + \bar{S}_{5j} = 1, S_{3j} + S_{4j} + S_{5j} = T,$ $\bar{S}_{3j} = S_{3j}/T, \bar{S}_{4j} = S_{4j}/T, \bar{S}_{5j} = S_{5j}/T$	Minimize θ s.t. $ \bar{S}_{5j}/\bar{S}_{6j}-1.56 \leq \theta, \bar{S}_{6j}/\bar{S}_{7j}-1.5 \leq \theta, \bar{S}_{5j}/\bar{S}_{7j}-2.4 \leq \theta,$ $\bar{S}_{5j} + \bar{S}_{6j} + \bar{S}_{7j} = 1, S_{5j} + S_{6j} + S_{7j} = T,$ $\bar{S}_{5j} = S_{5j}/T, \bar{S}_{6j} = S_{6j}/T, \bar{S}_{7j} = S_{7j}/T$
Patched data	15	6

A_1 - A_5 contain known values and A_6 - A_7 include missing data. These alternatives are reordered by performance and expert judgment, becoming S_{1j} - S_{7j} (from best to worst) in the second row. The third row identifies two data segments that include the missing values and their adjacent known points. The fourth row specifies the pairwise comparative relations constructed from existing data and expert inputs. Based on these relations, the fifth row formulates the corresponding optimization model, which outputs the imputed values summarized in the sixth row.

It should be noted that the present study primarily focuses on the mathematical formulation of the proposed imputation method. Nevertheless, the quality and reliability of the imputed results in practical applications are inevitably influenced by expert-related factors, particularly expert selection and judgment consistency. In this respect, well-established practices in group decision-making can be referred to, such as the use of predefined qualification criteria to identify suitable experts based on domain expertise (e.g., professional background, relevant experience, and research contributions) [26], and the adoption of appropriate aggregation strategies to derive coherent collective assessments from potentially divergent individual judgments [27]. Based on the assumption that experts provide rational and reasonable judgments, the proposed adjacent pairwise comparison approach leverages expert rankings to estimate missing values. By constraining the semantic comparison scales within a realistic and practical range, the method ensures that the imputed entries align with both adjacent known data and expert-based qualitative rankings, thereby enhancing their plausibility and accuracy.

Combined Weights Determination

Evaluating the sustainability of MSWI fly ash reuse for building materials requires considering multiple criteria, with weights assigned to reflect their

importance. Common weighting approaches include subjective weighting, based on expert preferences and judgments, objective weighting, derived from actual data, and combined methods that integrate both. While subjective weighting is widely used, it risks human bias; objective weighting is often constrained by data availability and quality. Thus, combined methods offer a more balanced, reliable solution [28]. In light of the incomplete evaluation information in this study, a novel integrated weighting method is proposed. This method simplifies converting expert judgments to subjective weights while using available data for objective weights, yielding a comprehensive, balanced set.

Level-based Weight Assessment (LBWA) Model for Subjective Weights

The LBWA is an updated approach designed to overcome the challenges of pairwise comparisons in subjective weighting techniques. Traditional methods, such as the analytic hierarchy process (AHP) and the best-worst method (BWM), require $n(n-1)/2$ and $(2n-3)$ comparisons, respectively. In contrast, the LBWA method requires only $(n-1)$ comparisons, significantly increasing computational efficiency. Moreover, LBWA's streamlined hierarchical structure enables straightforward mathematical operations for weight assignment, thereby minimizing inconsistencies that may arise from complex comparison procedures [29]. According to the literature [30], the following steps are required for implementing the LBWA.

Step 2.1 Identify the most important criterion and group the others.

Experts first select the most important criterion (C_B). The remaining criteria are grouped by their relative importance to C_B , using the levels in Table 2. Here, the significance of a criterion (C) is denoted by $G(C)$. Elements are categorized into levels L_1, L_2, \dots, L_k , where each level L_i contains elements satisfying

Table 2. Levels of significance in the level-based weight assessment (LBWA) [30].

Level	Formula	Brief description
1	$C_j \in [1,2)$	At group L_1 , the element whose significance is equal to the significance of C_B , or up to two times less significant than the significance of C_B
2	$C_j \in [2,3)$	At group L_2 , the element whose significance is exactly half the significance of C_B , or up to three times less significant than the significance of C_B
3	$C_j \in [3,4)$	At group L_3 , the element whose significance is exactly one-third the significance of C_B , or up to four times less significant than the significance of C_B
k	$C_j \in [k, k+1)$	At group L_k , the element whose significance is exactly $1/k$ times less than the significance of C_B , or up to $k+1$ times less significant than the significance of C_B

$L_t = \{C_j \in L : t \leq G(C_j) < t+1\}$. Thus, C_j in level L_t has a significance between t and $t+1$.

Step 2.2 Assign values to criteria and introduce the elasticity coefficient.

In each significance level L_t , criteria are compared based on their relative importance. The most significant criterion (in L_t) is assigned a value of $I_B = 0$. For other criteria within the same level L_t , if $I_{iq} \geq I_{ip}$, then C_{ip} is considered more significant than or equal to C_{iq} . Subsequently, the maximum scale value λ for comparison is determined as $\lambda = \max\{L_1, L_2, \dots, L_k\}$. To enhance discrimination between levels, an elasticity coefficient $\Delta\lambda$ is introduced, where $\Delta\lambda > \lambda$.

Step 2.3 Compute the influence function to derive subjective weights.

For each criterion belonging to L_t , the influence function is computed using Eq. (2). Here, t represents the level to which C_j belongs, and I_{ip} is the assigned significance value of C_j within level L_t . Next, for the most significant criterion, the weight (ws_B) is obtained with Eq. (3). For all other criteria, the influence function is applied to generate the corresponding weights, i.e., $ws_j = f(C_j) ws_B$.

$$f(C_j) = \Delta\lambda / (t \cdot \Delta\lambda + I_{ip}) \quad (2)$$

$$ws_B = 1 / \left[1 + \sum_{j=1, j \neq B}^n f(C_j) \right] \quad (3)$$

Method Based on the Removal Effects of Criteria (MEREK) for Objective Weights

The MEREK method determines the relative importance of criteria by evaluating the impact of their removal from the objective dataset [31]. Unlike Entropy and similar methods, which assign objective weights based on data dispersion or variability, MEREK evaluates the impact of excluding each criterion on the overall performance of alternatives. Criteria that produce a larger change when removed are considered more influential and consequently receive higher

weights. This approach provides a more targeted and robust method for objective weighting, ensuring that criteria with significant influence on performance are prioritized. According to the literature [31], MEREK involves the following three steps.

Step 2.4 Calculate the overall performance of each alternative.

A non-linear aggregation function is employed to calculate the overall performance of the i -th alternative, taking into account all n criteria, as expressed in Eq. (4).

$$Q_i = \ln \left(1 + \left(\frac{1}{m} \sum_j |\ln(z_{ij})| \right) \right) \quad (4)$$

where z_{ij} represents the standardized data reflecting the relative performance of the i -th alternative with respect to the j -th criterion. This is achieved by processing the original collected data x_{ij} using a specific formula for benefit criteria ($z_{ij} = \min_{i=1,2,\dots,m} x_{ij} / x_{ij}$) and a different formula for cost criteria ($z_{ij} = x_{ij} / \max_{i=1,2,\dots,m} x_{ij}$).

Step 2.5 Determine performances by excluding each criterion.

A similar logarithmic algorithm is applied in this step. The performance of the i -th alternative, considering the removal of the j -th criterion, is calculated using Eq. (5).

$$Q_{ij} = \ln \left(1 + \left(\frac{1}{m} \sum_{h, h \neq j} |\ln(z_{ih})| \right) \right) \quad (5)$$

Step 2.6 Sum absolute deviations to derive objective weights.

The effect of removing the j -th criterion (E_j) is determined using Q_i and Q_{ij} , see Eq. (6). The objective weights (wo_j) are then calculated by normalizing all E_j values, as shown in Eq. (7).

$$E_j = \sum_i |Q_{ij} - Q_i| \quad (6)$$

$$wo_j = E_j / \sum_j E_j \quad (7)$$

Weight Combining Programming

To effectively integrate subjective and objective weights, a novel weight combining programming is proposed. The key feature of this approach, as described in Eq. (8), lies in minimizing the deviation of the combined weight w_j from both subjective weight (ws_j) and objective one (wo_j), while preserving the distribution and characteristics of both ws_j and wo_j . By using solvers such as Lingo, the final weight for each criterion can be obtained.

$$\begin{aligned} \text{Minimize } D &= \sum_j \frac{\sqrt{\left[(ws_j - w_j)^2 + (wo_j - w_j)^2 \right] / 2}}{\sqrt{(ws_j - wo_j)^2}} \\ \text{s.t.} \\ \sum_j w_j &= 1 \\ \min(ws_j, wo_j) &\leq w_j \leq \max(ws_j, wo_j) \end{aligned} \quad (8)$$

Alternatives Prioritization

Various MCDM ranking methods are available for ranking alternative technologies. According to the review [32], validated methods can be categorized into the following types: full aggregation (e.g., UTADIS), goal/aspiration/reference-level (e.g., TOPSIS), and outranking (e.g., ELECTRE and PROMETHEE). The full aggregation approach evaluates each criterion and synthesizes them into a global score. In this approach, a low score on one criterion can be compensated by a

high score on another. The goal/aspiration/reference-level approach defines a reference level for each criterion and identifies the option closest to this ideal, such as TOPSIS, which minimizes the distance from an ideal solution and is well suited to problems with clearly defined targets. The outranking approach is a non-compensatory method that enables partial ordering of alternatives, handling incomparability effectively. ELECTRE and PROMETHEE use outranking relations to classify alternatives and suit complex decisions with incomplete or uncertain information [32].

Although a missing data imputation method is incorporated into the framework, the decision-making problem may still involve incomplete or uncertain information. As a result, the outranking approach, which relies on non-compensatory logic, is chosen for prioritizing the alternatives. Among the families of ELECTRE and PROMETHEE, PROMETHEE II stands out for its ability to provide a complete ranking of alternatives, even when information is incomplete or uncertain [33]. Unlike some other outranking approaches that may produce partial rankings due to incomparability among alternatives, PROMETHEE II generates a complete ranking based on net preference flows, allowing clear and direct prioritization. The method employs preference functions to quantify the degree of preference between alternatives and calculates each alternative's net flow to determine their overall ranking. Its transparent preference structure and relatively low computational requirements make it practical for real-world applications. Moreover, PROMETHEE II can effectively handle imprecise or incomplete data, which makes it particularly suitable for complex decision-making scenarios involving uncertainty and missing information. Given its widespread use, only a concise

Table 3. A brief overview of the PROMETHEE II steps [34].

Steps	Description	Mathematical formula
Step 3.1 Create a decision matrix	Establish a decision matrix with normalized data y_{ij}	$y_{ij} = \left(x_{ij} - \min_j x_{ij} \right) / \left(\max_j x_{ij} - \min_j x_{ij} \right), j \in \text{benefit}$ $y_{ij} = \left(\max_j x_{ij} - x_{ij} \right) / \left(\max_j x_{ij} - \min_j x_{ij} \right), j \in \text{cost}$ <p style="text-align: center;">(9)</p>
Step 3.2 Define preference function	Define preference functions to express the preference of one alternative over another based on the criteria	$P_j(A_i, A_k) = \begin{cases} 0, & \text{if } y_{ij} \leq y_{kj} \\ y_{ij} - y_{kj}, & \text{otherwise} \end{cases}$ <p style="text-align: center;">(10)</p>
Step 3.3 Calculate the leaving and entering flows	Calculate the leaving and entering flows by summing the weighted preferences across all criteria	<p>Leaving flow: $\phi^+(A_i) = \frac{1}{m-1} \sum_k w_j \cdot P_j(A_i, A_k)$</p> <p>Entering flow: $\phi^-(A_i) = \frac{1}{m-1} \sum_k w_j \cdot P_j(A_k, A_i)$</p> <p style="text-align: center;">(11)</p>
Step 3.4 Obtain the net flow to rank alternatives	Rank the alternatives based on their net flow values	<p>Net flow: $\phi(A_i) = \phi^+(A_i) - \phi^-(A_i)$ (12)</p>

Table 4. Seven fly ash-to-building material technologies evaluated in the case study.

Alternative	Brief description
FA-based cement (A_1)	MSWI FA can partially replace raw materials in cement production through a process of washing, drying, component extraction, calcination at 800–1300°C, clinker milling, and cement application [20]
FA-based ceramsite (A_2)	FA is mixed with a specific ratio of additives and sintered at 1150–1350°C to produce ceramsite, which can be used in lightweight concrete blocks and as a component in road base materials [20]
FA-based artificial aggregate (A_3)	It involves mixing FA and other solid wastes with cement through a cold bonding pelletization process to produce artificial aggregates, offering a sustainable alternative to natural aggregates [35]
FA-based geopolymer (A_4)	FA is used as a precursor to synthesize geopolymers by combining it with other materials and an alkali activator. The mixture is then cured under controlled conditions, providing a potential replacement for cement-based building materials [36]
FA-based glassy slags (A_5)	It involves mixing water-washed FA with silica sand and heating the mixture above 1500°C to produce glassy slags, which can replace traditional shale stones in construction applications [20]
FA-based bricks (A_6)	FA is recycled into eco-friendly bricks through a process of water extraction and grinding stabilization, mixed with other materials, and fired at 1300°C in a kiln [37]
FA-recovered metals (A_7)	It uses a multi-stage cascading system with acid-base scrubbing water to leach fly ash at an optimized pH of 3.8, yielding a metal-rich solution for the recovery of metals like zinc and copper, which are commonly used in construction [10]

overview of the PROMETHEE II steps (Eqs. (9)-(12)) is presented (see Table 3). For a more detailed explanation of the PROMETHEE II process, readers are encouraged to refer to the relevant literature [34].

Results

Case Study Results

To validate the proposed framework, a case study was conducted to evaluate seven feasible technologies for reusing MSWI fly ash as building materials.

Assessment System for the Case Study

The assessment system in the case study is based on seven alternatives (A_1 to A_7), as outlined in Table 4) and twelve criteria, which are derived from key dimensions of sustainability evaluation: environmental (C_1 – C_4), economic (C_5 – C_8), and technical (C_9 – C_{12}), as detailed in Table 5.

Based on the established assessment system, the relative performance of each alternative with respect to the criteria is collected. The functional unit is defined as the treatment of 1 ton of MSWI fly ash. Due to data limitations, some information could not be obtained and is denoted as "M.S" in Table 6.

Missing Data Imputation for the Case Study

As shown in Table 6, seven criteria involve one or two alternative technologies with missing data. To illustrate the imputation process, C_5 (capital cost) is used as an

example. As described in the Materials and Methods section, expert knowledge can provide qualitative comparisons to address data gaps, while the number of experts and the procedures for eliciting their judgments may vary depending on the specific decision context. In this case study, five experts evaluated the alternatives associated with missing data and provided scale-based assessments for the unknown pairwise comparisons through a structured group discussion. It should be emphasized that these judgments were reached through a consensus-oriented expert panel, aiming to illustrate the practical applicability and operational feasibility of the proposed imputation method. Table A1 (Appendix) presents the background information of the participating experts, thereby supporting the transparency and traceability of the imputation process.

Specifically, in Step 1.1, based on actual data and expert judgment, the seven alternatives for C_5 are ranked from highest to lowest as follows: x_{15} (543.7) > x_{45} (510.4) > x_{55} (445.2) > x_{75} (217.3) > x_{35} (unknown) > x_{65} (unknown) > x_{25} (129.6). Among these, x_{35} (for A_3) and x_{65} (for A_6) are unknown and adjacent, making them ideal for demonstrating the proposed method, particularly for missing data in the middle. Notably, the rankings are renamed, with the unknown points denoted as S_{55} and S_{65} . Step 1.2 involves constructing datasets that include a specific unknown point and the two adjacent known data points, i.e., [$S_{45} = 217.3$, S_{55} (unknown), $S_{75} = 129.6$] and [$S_{45} = 217.3$, S_{65} (unknown), $S_{75} = 129.6$]. Step 1.3 calculates the ratios within these datasets, where $S_{45}/S_{75} = \varphi_{45/75} = 217.3/129.6 = 1.6767$ serves as a reference. Accordingly, the ratios S_{45}/S_{55} , S_{55}/S_{75} , and S_{45}/S_{65} , S_{65}/S_{75} must all be ≥ 1 , consistent with the ranking. Based on the five experts' judgments, values for the

Table 5. Twelve criteria from the sustainability dimensions in the case study.

Criterion	Brief description
Climate change (C_1)	Emissions of greenhouse gases into the atmosphere [38]
Water depletion (C_2)	Depletion of nonrenewable freshwater resources [20]
Acidification (C_3)	Emissions of acidifying substances into the air [38]
Primary energy consumption (C_4)	Total primary energy used in a process [20]
Capital cost (C_5)	One-time fixed costs for purchasing land, construction, equipment, etc. [20]
Operation cost (C_6)	Costs incurred during the operation stage [20]
Total annual income (C_7)	Revenue generated annually from using fly ash in building materials [20]
Break-even year (C_8)	The year when total revenues equal total costs, marking profitability [16]
Technology maturity (C_9)	A metric that assesses technology maturity on a scale of 1 to 9 [39]
Technology complexity (C_{10})	A scaling system used to evaluate the complexity of the technology [40]
Safety index (C_{11})	A scoring system for assessing the inherent safety of a technology [41]
Land occupation (C_{12})	The amount of land area required to build and implement the process [16]

unknown comparisons can be assigned by referencing the benchmark ratio S_{45}/S_{75} . Specifically, $S_{45}/S_{55} = \varphi_{45/55} = 1.2$ and $S_{55}/S_{75} = \varphi_{55/75} = 1.5$ are used to impute the data for S_{55} , while $S_{45}/S_{65} = 1.4$ and $S_{65}/S_{75} = 1.3$ are applied for patching the data for S_{65} . Step 1.4 involves establishing programming models, as shown in Eq. (11a) and (11b). By applying the solver software Lingo, S_{55} (i.e., x_{35}) is calculated as 181.1, and S_{65} (i.e., x_{65}) as 155.2. Using the same method, all missing data can be imputed (see Table A2 in the Appendix, which summarizes the imputation settings and ratio assignments for each criterion to support reproducibility and transparency).

For [S_{45} , S_{55} , S_{75}] (Eq.11a)

Minimize θ

s.t.

$$\left| \bar{S}_{45} / \bar{S}_{55} - 1.2 \right| \leq \theta, \quad \left| \bar{S}_{55} / \bar{S}_{75} - 1.5 \right| \leq \theta, \quad \left| \bar{S}_{45} / \bar{S}_{75} - 1.2 \times 1.5 \right| \leq \theta,$$

$$\bar{S}_{45} + \bar{S}_{55} + \bar{S}_{75} = 1, \quad S_{45} + S_{55} + S_{75} = T,$$

$$\bar{S}_{45} = S_{45}/T, \quad \bar{S}_{55} = S_{55}/T, \quad \bar{S}_{75} = S_{75}/T$$

For [S_{45} , S_{65} , S_{75}] (Eq.11b)

Minimize θ

s.t.

$$\left| \bar{S}_{45} / \bar{S}_{65} - 1.4 \right| \leq \theta, \quad \left| \bar{S}_{65} / \bar{S}_{75} - 1.3 \right| \leq \theta, \quad \left| \bar{S}_{45} / \bar{S}_{75} - 1.4 \times 1.3 \right| \leq \theta,$$

$$\bar{S}_{45} + \bar{S}_{65} + \bar{S}_{75} = 1, \quad S_{45} + S_{65} + S_{75} = T,$$

$$\bar{S}_{45} = S_{45}/T, \quad \bar{S}_{65} = S_{65}/T, \quad \bar{S}_{75} = S_{75}/T$$

Weights Determination for the Case Study

In this subsection, the LBWA and MEREC methods are used to assign both subjective and objective weights to the twelve criteria, which are then integrated through the weight combination programming.

Subjective weights for the case study. The criteria's subjective weights were assigned using the LBWA method, based on the judgments of the same five experts (Table 7). In Step 2.1, the most important criterion, C_1 , is selected, while the remaining criteria are grouped into four levels based on their relative significance. In Step 2.2, the defined levels (with the maximum size of each subset specified) are used to determine the value of $\lambda = \max(L_1, L_2, L_3, L_4) = 4$ for the comparisons. Within this range of $\lambda = 4$, the relationships between the most important criterion and the others are established according to their respective levels. Additionally, the elasticity coefficient, $\Delta\lambda$, is set to 5 based on practical considerations [30]. In Step 2.3, the influence function is calculated using Eq. (2); based on this, the subjective weight of each criterion is derived through Eq. (3), with the corresponding results presented in Eqs. (12) and (13), respectively.

$$L_1: f(C_1) = \frac{5}{1 \cdot 5 + 0} = 5, \quad f(C_4) = \frac{5}{1 \cdot 5 + 1} = \frac{5}{6}, \quad f(C_5) = \frac{5}{1 \cdot 5 + 3} = \frac{5}{8};$$

$$L_2: f(C_6) = \frac{5}{2 \cdot 5 + 1} = \frac{5}{11}, \quad f(C_9) = \frac{5}{2 \cdot 5 + 1} = \frac{5}{11}, \quad f(C_8) = \frac{5}{2 \cdot 5 + 3} = \frac{5}{13};$$

$$L_3: f(C_2) = \frac{5}{3 \cdot 5 + 1} = \frac{5}{16}, \quad f(C_{11}) = \frac{5}{3 \cdot 5 + 2} = \frac{5}{17};$$

$$L_4: f(C_7) = \frac{5}{4 \cdot 5 + 1} = \frac{5}{21}, \quad f(C_{10}) = \frac{5}{4 \cdot 5 + 2} = \frac{5}{22},$$

$$f(C_3) = \frac{5}{4 \cdot 5 + 3} = \frac{5}{23}, \quad f(C_{12}) = \frac{5}{4 \cdot 5 + 4} = \frac{5}{24}$$

(12)

$$ws_1 = f(C_1) / \sum_{j=1}^{12} f(C_j) = 0.190, \quad ws_2 = f(C_2) \cdot ws_1 = 0.060,$$

$$ws_3 = f(C_3) \cdot ws_1 = 0.041, \quad ws_4 = f(C_4) \cdot ws_1 = 0.159,$$

$$ws_5 = f(C_5) \cdot ws_1 = 0.119, \quad ws_6 = f(C_6) \cdot ws_1 = 0.087,$$

$$ws_7 = f(C_7) \cdot ws_1 = 0.045, \quad ws_8 = f(C_8) \cdot ws_1 = 0.073,$$

$$ws_9 = f(C_9) \cdot ws_1 = 0.087, \quad ws_{10} = f(C_{10}) \cdot ws_1 = 0.043,$$

$$ws_{11} = f(C_{11}) \cdot ws_1 = 0.056, \quad ws_{12} = f(C_{12}) \cdot ws_1 = 0.040$$

(13)

Table 6. Collected performance data with missing values in the case study [16, 20, 35-38, 42, 43].

x_{ij}	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
Unit	kg CO ₂ -eq	m ³	kg SO ₂ -eq	MJ	US dollar	US dollar	US dollar	Year	Score	Score	Score	m ²
A ₁	38977	63.49	76.41	390144	543.7	73.6	218.3	11	9	1.50	19	7.56
A ₂	555	1.35	4.12	2502	129.6	111.5	234.9	3	6	2.50	22	3.25
A ₃	11764	80	M.S	M.S	M.S	55.7	M.S	M.S	6	2.75	8	M.S
A ₄	2129	38.21	9.38	M.S	510.4	225.5	148.0	M.S	6	2.63	8	M.S
A ₅	2250	15.35	17.40	30302	445.2	275.1	231.9	30	4	3.13	26	9.26
A ₆	646	7.38	0.22	6871	M.S	M.S	M.S	6	8	1.25	14	10.73
A ₇	34	0.38	0.24	2365	217.3	157.1	96.2	9	8	2.38	15	2.86

Table 7. Levels and comparison values of criteria based on the level-based weight assessment (LBWA) for subjective weighting.

Most important-C1	L ₁	L ₂	L ₃	L ₄
Groups	{C ₁ , C ₄ , C ₅ }	{C ₆ , C ₇ , C ₈ }	{C ₂ , C ₁₁ }	{C ₇ , C ₁₀ , C ₃ , C ₁₂ }
Comparison values	{I ₁ =0, I ₄ =1, I ₅ =3}	{I ₆ =1, I ₉ =1, I ₈ =3}	{I ₂ =1, I ₉ =2}	{I ₇ =1, I ₁₀ =2, I ₃ =3, I ₁₂ =4}

Table 8. Information related to the objective weights derived from the method based on the removal effects of criteria (MERECE).

	Q _i	Q _{i1}	Q _{i2}	Q _{i3}	Q _{i4}	Q _{i5}	Q _{i6}	Q _{i7}	Q _{i8}	Q _{i9}	Q _{i10}	Q _{i11}	Q _{i12}
A ₁	0.345	0.345	0.331	0.345	0.345	0.345	0.264	0.341	0.265	0.345	0.334	0.292	0.286
A ₂	1.008	0.869	0.875	0.915	0.841	0.963	0.980	1.008	1.008	0.996	0.987	0.977	1.004
A ₃	0.717	0.667	0.717	0.636	0.556	0.671	0.650	0.716	0.659	0.700	0.684	0.717	0.696
A ₄	0.803	0.688	0.775	0.722	0.583	0.801	0.796	0.786	0.741	0.788	0.775	0.803	0.793
A ₅	0.816	0.705	0.753	0.760	0.717	0.809	0.816	0.815	0.727	0.786	0.782	0.772	0.772
A ₆	1.054	0.928	0.983	0.868	0.930	1.017	1.030	1.016	1.034	1.051	1.054	1.038	1.015
A ₇	1.207	1.014	1.064	1.052	1.071	1.184	1.193	1.185	1.179	1.204	1.191	1.191	1.207
		C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉	C ₁₀	C ₁₁	C ₁₂
E _j		0.734	0.452	0.653	0.907	0.160	0.222	0.084	0.337	0.081	0.143	0.160	0.178
ow _j		0.179	0.110	0.159	0.221	0.039	0.054	0.020	0.082	0.020	0.035	0.039	0.043

Objective weights for the case study. The MERECE method is used to generate the objective weights. Specifically, in Step 2.4, the imputed dataset is standardized (see Table A3 in the Appendix), and Eq. (4) is applied to compute the overall performance of the alternatives. Table A3 provides the standardized values used as the basis for deriving the MERECE objective weights. In Step 2.5, the performances of the alternatives, with each criterion removed, are calculated using Eq. (5). Then, in Step 2.6, the removal effect of each criterion on the overall performance is assessed using the deviation-based formula in Eq. (6). Finally, the objective weights of the criteria are determined using Eq. (7). Table 8 summarizes the key intermediate parameters and the MERECE-derived weights.

The weight combination programming for the case study is established in Eq. (14) and can be solved using Lingo. The integrated weights for the criteria are shown in Fig. 3. For comparison, the LBWA-based subjective weights and the MERECE-derived objective weights are also presented. The results indicate that climate change (C₁) and primary energy consumption (C₄) are the two most important criteria in terms of environmental performance, while the criteria related to the technical aspect are generally less important.

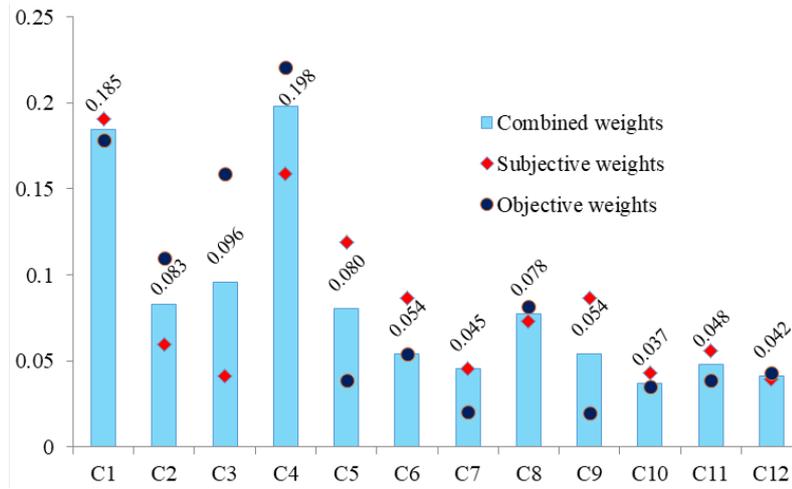


Fig. 3. Comparison of subjective, objective, and combined weights for fly ash reuse criteria in the case study.

$$\text{Minimize } D = \left(\frac{\sqrt{((0.190-w_1)^2 + (0.179-w_1)^2)/2}}{\sqrt{(0.190-0.179)^2}} + \frac{\sqrt{((0.060-w_2)^2 + (0.110-w_2)^2)/2}}{\sqrt{(0.060-0.110)^2}} + \frac{\sqrt{((0.041-w_3)^2 + (0.159-w_3)^2)/2}}{\sqrt{(0.041-0.159)^2}} \right. \\
 \left. + \frac{\sqrt{((0.159-w_4)^2 + (0.221-w_4)^2)/2}}{\sqrt{(0.159-0.221)^2}} + \frac{\sqrt{((0.119-w_5)^2 + (0.039-w_5)^2)/2}}{\sqrt{(0.119-0.039)^2}} + \frac{\sqrt{((0.087-w_6)^2 + (0.054-w_6)^2)/2}}{\sqrt{(0.087-0.054)^2}} \right. \\
 \left. + \frac{\sqrt{((0.045-w_7)^2 + (0.020-w_7)^2)/2}}{\sqrt{(0.045-0.020)^2}} + \frac{\sqrt{((0.073-w_8)^2 + (0.082-w_8)^2)/2}}{\sqrt{(0.073-0.082)^2}} + \frac{\sqrt{((0.087-w_9)^2 + (0.020-w_9)^2)/2}}{\sqrt{(0.087-0.020)^2}} \right. \\
 \left. + \frac{\sqrt{((0.043-w_{10})^2 + (0.035-w_{10})^2)/2}}{\sqrt{(0.043-0.035)^2}} + \frac{\sqrt{((0.056-w_{11})^2 + (0.039-w_{11})^2)/2}}{\sqrt{(0.056-0.039)^2}} + \frac{\sqrt{((0.040-w_{12})^2 + (0.043-w_{12})^2)/2}}{\sqrt{(0.040-0.043)^2}} \right)$$

st. $w_1 + w_2 + w_3 + w_4 + w_5 + w_6 + w_7 + w_8 + w_9 + w_{10} + w_{11} + w_{12} = 1;$
 $0.179 \leq w_1 \leq 0.190, 0.060 \leq w_2 \leq 0.110, 0.041 \leq w_3 \leq 0.159, 0.159 \leq w_4 \leq 0.221, 0.039 \leq w_5 \leq 0.119, 0.054 \leq w_6 \leq 0.087,$
 $0.020 \leq w_7 \leq 0.045, 0.073 \leq w_8 \leq 0.082, 0.020 \leq w_9 \leq 0.087, 0.035 \leq w_{10} \leq 0.043, 0.039 \leq w_{11} \leq 0.059, 0.040 \leq w_{12} \leq 0.043$

(14)

Alternatives Prioritization for the Case Study

In this subsection, the ranking of the seven alternatives is determined using the PROMETHEE II method. Specifically, the decision matrix required in Step 3.1 is presented in Eq. (15). By applying Eq. (10) in Step 3.2, the values of the preference function are calculated. In Step 3.3, the weighted sum of the preference functions is computed to determine each alternative’s leaving and entering flows. Finally, in Step 3.4, the net flow is calculated, enabling the prioritization of the seven alternatives. Table 9 presents the key intermediate parameters and the final ranking results, which indicate that the technologies for reusing fly ash as building materials are ranked from best to worst as follows: $A_2 > A_7 > A_6 > A_3 > A_4 > A_5 > A_1$.

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉	C ₁₀	C ₁₁	C ₁₂
A ₁	0.000	0.207	0.000	0.000	0.000	0.918	0.903	0.704	1.000	0.867	0.389	0.403
A ₂	0.987	0.988	0.949	0.999	1.000	0.746	1.000	1.000	0.400	0.335	0.222	0.950
A ₃	0.699	0.000	0.856	0.979	0.876	1.000	0.982	0.667	0.400	0.202	1.000	0.754
Matrix = A ₄	0.946	0.525	0.880	1.000	0.080	0.226	0.491	0.556	0.400	0.266	1.000	0.883
A ₅	0.943	0.812	0.775	0.927	0.238	0.000	0.982	0.000	0.000	0.000	0.000	0.187
A ₆	0.984	0.912	1.000	0.987	0.938	0.703	0.000	0.889	0.800	1.000	0.611	0.000
A ₇	1.000	1.000	1.000	0.999	0.788	0.538	0.188	0.778	0.800	0.399	0.667	1.000
Weight	0.185	0.083	0.096	0.198	0.080	0.054	0.045	0.078	0.054	0.037	0.048	0.042

(15)

Discussions

Based on the quantitative results of the case study, sensitivity analysis and comparative analysis are conducted to evaluate the effectiveness of the applied weighting and ranking methods.

Sensitivity Analysis

Considering that this study introduces a novel integrated weighting method, its robustness is examined through sensitivity analysis. Specifically, the weight of each criterion is individually increased or decreased by 30%, 60%, and 90% relative to its original value, while the remaining weights are proportionally adjusted to maintain a unit sum. This stepwise variation, commonly adopted in sensitivity analysis [44], enables a comprehensive examination of weight perturbations over a wide range. The proportional adjustment of the other criteria preserves their relative importance, ensuring an unbiased assessment of the effect of each attribute’s weight change on the overall ranking. Rankings are then recalculated under the adjusted weights to evaluate their impact on the overall prioritization. As shown in Fig. 4, among the 72 variations, only one-sixth of the cases display minor shifts, such as a reversal between ranks 4 and 5 or between ranks 2 and 3. These findings demonstrate the strong robustness of the proposed integrated weighting method, which ensures stable ranking results.

Comparative Study

Different types of methods can be employed to rank the alternatives. Accordingly, this study selects the weighted sum method (WSM) from the full aggregation approach, TOPSIS from the goal/aspiration/reference-level approach, and PROMETHEE II from the outranking family for comparative analysis. The ranking results of these three methods are presented

Table 9. PROMETHEE II results in the case study: outgoing/ingoing flows, net flow, and ranking.

Alternative	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇
Outgoing flow	0.108	0.264	0.198	0.163	0.128	0.253	0.247
Incoming flow	0.588	0.047	0.152	0.156	0.272	0.083	0.061
Net flow	-0.480	0.217	0.045	0.007	-0.145	0.170	0.186
Ranking	7	1	4	5	6	3	2

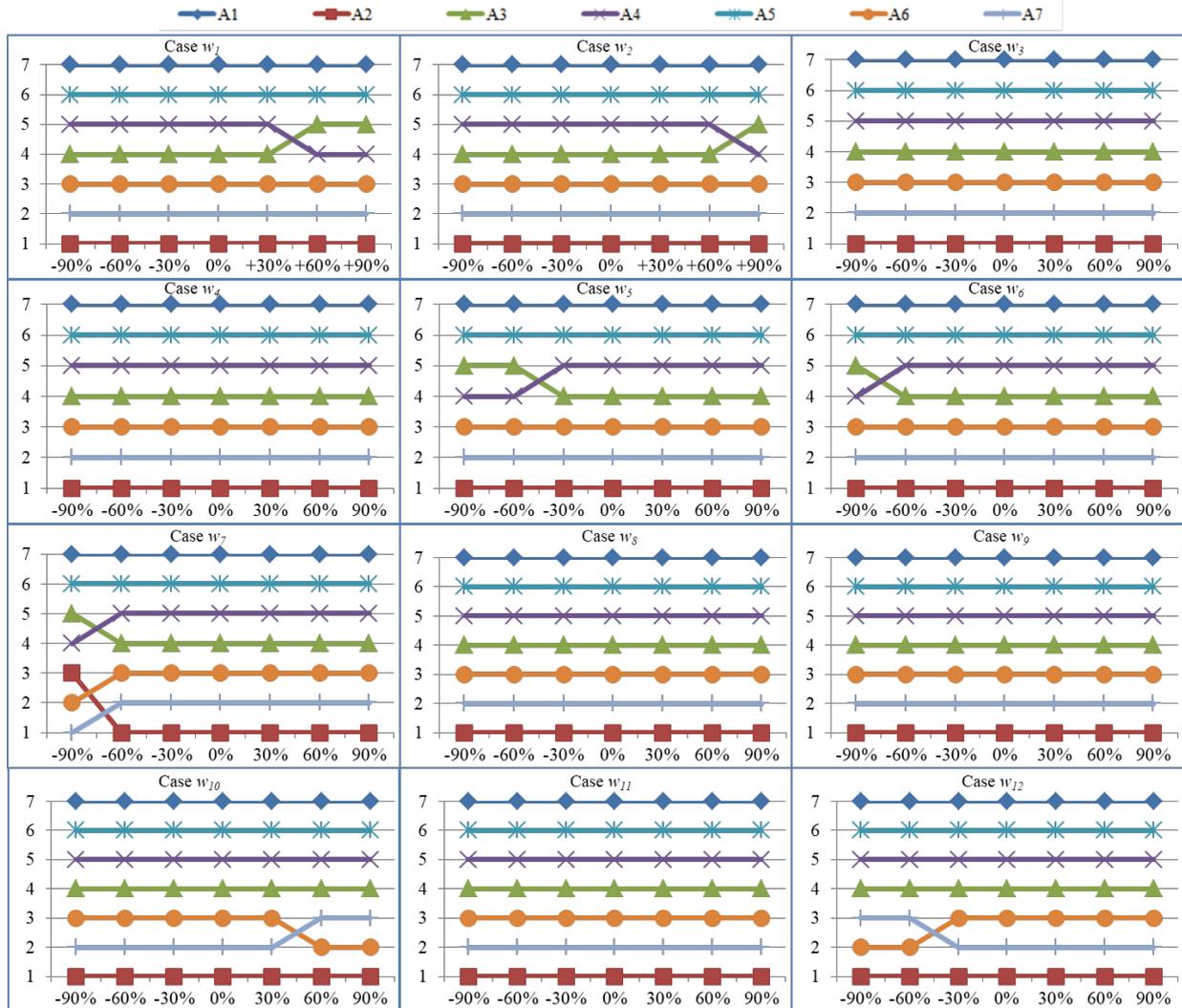


Fig. 4. Sensitivity analysis results: impact of ±90% variation in individual criteria on the alternatives' prioritization.

in Fig. 5. Both WSM and PROMETHEE II produce consistent rankings, while their results differ from TOPSIS in positions 4 to 6. On one hand, agreement among the three methods in identifying the best and worst alternatives indirectly validates PROMETHEE II's use for ranking. On the other, PROMETHEE II, by constructing preference functions that incorporate both precise data and uncertain information, offers clear advantages over WSM and TOPSIS in scenarios where the data is not entirely crisp or well-defined. This makes

PROMETHEE II more adaptable to real-world decision-making situations with missing data.

Advantages of the Proposed DSS

Beyond sensitivity and comparative analyses, the proposed DSS offers distinct advantages over existing multi-criteria frameworks in handling real-world decision-making under incomplete and heterogeneous data. Conventional MCDM approaches often rely on direct imputation (e.g., mean substitution) or expert-

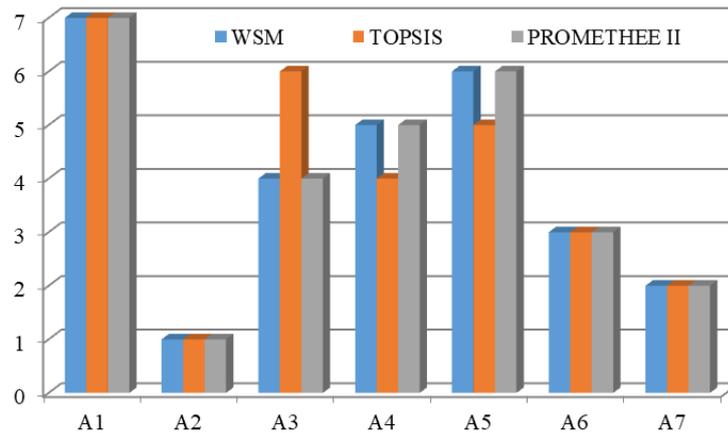


Fig. 5. Comparative study results of fly ash reuse alternatives using three MCDM methods.

dependent linguistic grading, which can introduce bias and reduce inclusiveness. In contrast, the DSS introduces a FUCOM-inspired imputation method that quantitatively estimates missing values by combining expert qualitative rankings with adjacent known data, preserving ordering consistency and proportional relationships, and enabling systematic inclusion of emerging, data-scarce technologies.

Traditional weighting methods are typically limited to purely subjective approaches (e.g., AHP or BWM) or objective ones (e.g., entropy), each with inherent drawbacks. The proposed hybrid strategy integrates LBWA-based subjective weights with MEREC-based objective weights, optimized via deviation-minimizing programming. Applied to MSWI fly ash reuse assessment, the proposed method yields enhanced robustness, transparency, and information adaptability.

Moreover, the unified integration of PROMETHEE II for ranking provides more stable and transparent prioritization than aggregation- or distance-based methods (e.g., WSM or TOPSIS), particularly when data are incomplete or imputed. Collectively, these methodological innovations improve objectivity, inclusiveness, and practical applicability, as demonstrated in the case study.

Implications

The implications of this study extend to both theoretical and practical domains, primarily through its methodological advancements in MCDM under data incompleteness.

Theoretically, the proposed DSS provides a unified framework that integrates mathematical innovations: a FUCOM-inspired imputation method that reconstructs missing data while preserving consistency and proportionality; a hybrid LBWA-MEREC weighting strategy balancing subjective expert priorities with objective criterion influence; and PROMETHEE II-based outranking for robust prioritization of imputed and heterogeneous data. This integration enables more

comprehensive, balanced, and inclusive sustainability assessments across environmental, economic, and technical dimensions, typically unattainable with fragmented or complete-data-dependent frameworks.

Practically, the DSS offers a structured and adaptable tool for decision-makers managing incomplete datasets in emerging waste-to-resource technologies. By systematically including data-scarce alternatives, it enables forward-looking evaluations that capture innovation potential and inform strategic industrial decisions. The case study indicates that environmental criteria carry the greatest weight, followed by economic criteria, with technical factors being less influential, and identifies FA-based ceramsite production and metal recovery as the most sustainable pathways. Based on these findings, industrial efforts and investments can be directed toward these environmentally and economically favorable options while ensuring technical feasibility. The framework's capacity to handle diverse criteria and incomplete data supports evidence-based planning and technology selection, facilitating sustainable and efficient utilization of MSWI fly ash.

Conclusions

Reusing MSWI fly ash in building materials enhances sustainable waste management and contributes to the circular economy within the construction industry. Given the wide range of fly ash-to-building material technologies, a comprehensive multidimensional assessment is essential for identifying the most sustainable solutions. This study proposes a novel decision support system (DSS) to address three key challenges: managing incomplete information, integrating subjective and objective criteria weighting, and ranking alternatives under data limitations.

Compared to existing studies, the contributions are threefold. First, the DSS provides a systematic framework for assessing and selecting MSWI fly ash reuse options, extending prior frameworks and pioneering applications

in fly ash recycling for construction materials. Second, a FUCOM-inspired imputation method combines expert pairwise comparisons with adjacent known data to estimate missing values, overcoming partial data limitations. Third, a hybrid LBWA-MEREC weighting strategy transforms subjective preferences and captures data characteristics to generate balanced criteria weights, avoiding bias. Additionally, PROMETHEE II, an outranking method using preference functions independent of data precision, is applied to rank alternatives under incomplete information.

The DSS was applied to seven fly ash-to-building material alternatives across twelve environmental, economic, and technical criteria. The imputation method effectively filled missing data, enabling quantitative comparison of all alternatives. Environmental factors, particularly climate change and primary energy consumption, had the greatest influence, followed by economic criteria, while technical factors were less significant, highlighting sustainability priorities in decision-making. Ranking results identified FA-based ceramsite production and metal recovery as the most sustainable pathways, balancing environmental and economic considerations with technical feasibility. Sensitivity analysis and comparative studies confirmed the robustness of the weighting and ranking outcomes. These results demonstrate that the proposed DSS effectively handles incomplete data, generates balanced criteria weights, and identifies the most sustainable MSWI fly ash reuse options, providing decision-makers with a transparent, inclusive, and evidence-based tool to support reliable and actionable sustainability assessments under heterogeneous datasets.

Despite its advantages, the DSS has some inherent limitations. First, the imputation method assumes that expert-provided qualitative rankings are reasonably reliable and internally consistent; substantial bias or disagreement among experts could affect the imputed values. Although this dependency can be mitigated through careful expert selection and consistency checks, it remains an inherent feature of the DSS. Second, future studies should expand the range of alternatives and criteria considered. Systematic indicator screening and structuring tools can be applied to establish a more comprehensive evaluation system for fly ash-to-building material pathways, while incorporating emerging reuse technologies and hybrid valorization schemes to better reflect technological innovation. Third, data quality improvement is essential, as many technologies are characterized under specific operating conditions, yielding single-point performance representations. Broader data coverage across operating ranges and process configurations would further enhance the robustness and applicability of the DSS.

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Conflict of Interest

The authors declare no conflict of interest.

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Appendix

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