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Original Research

The Eutrophication Assessment Based on the Fuzzy C-means (FCM) Algorithm and Improved Grey Target Theory

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Abstract

Lake eutrophication is an important aspect of water quality assessment. The traditional grey target model is a valid method for evaluating lake eutrophication, but because of the correlation among assessment indicators, its predictive accuracy is questionable. To eliminate this influence, the FCM algorithm and improved grey target theory are introduced to evaluate the lake eutrophication. First, the FCM algorithm is applied to determine the weight coefficients; second, for the suggested model, the correlation coefficient matrix is used to replace the covariance matrix in the Mahalanobis distance; third, the positive and negative ideal solutions in the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) model are used to define the positive and negative target centers. Finally, the engineering example is verified using the improved model. Conclusions are drawn that the method is feasible for estimating the lake eutrophication, that its predictive accuracy is higher than that of traditional models, and that it can mine and develop data to the greatest extent using information from a few samples and limited information. These conclusions can provide a new approach and perspective for assessing lake eutrophication in the future.

Keywords: FCM algorithm, improved grey target theory, lake eutrophication, assessment

Introduction

The water resources of the lake constitute a vital component of the global water cycle system, providing humanity with a multitude of functions including flood control and irrigation, aquaculture, navigation, power generation, tourism and recreation, as well as water supply for industrial, agricultural, and domestic use [1, 2]. However, with rapid population growth, productivity gains, and booming economic development, the negative impacts of human activities on the lake have become increasingly significant [3]. For example, these activities include reclaiming land from the lake for farming, overfishing, indiscriminate clearing and logging of vegetation in the watershed, as well as the indiscriminate discharge of industrial and agricultural wastewater, along with domestic sewage, etc. [4, 5]. These adverse factors have not only caused tremendous losses to the production and livelihoods of people in the lake area, but also significantly accelerated the lake's

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disappearance. Therefore, the quality assessment and prediction of lake environments have emerged as an important research topic [6].

Lake eutrophication is an important aspect of water quality assessment. Statistics from surveys of over 130 lakes across the country reveal that eutrophic lakes account for 43.5% of the total, while mesotrophic lakes account for 45.1%, with the combined proportion reaching 88.6% [7]. The eutrophication problem caused by pollution of lake water resources not only affects industrial and agricultural production and the domestic water supply for urban residents in the lake basin, but also severely impacts the lake's ecosystem [8]. So, the accurate assessment of lake eutrophication has great practical significance [9].

In recent years, scholars have successively proposed many methods to assess lake eutrophication. Such as the comprehensive index method [10], fuzzy mathematics method [11], grey clustering method [12], etc. Although these methods provide certain scientific assessments of lake eutrophication, it cannot be denied that they have some shortcomings. For example, these traditional methods fail to adequately address the complex nonlinear relationships between evaluation factors and water quality grades. The utility functions and weights used in the evaluation process are artificially designed, limiting the model's generality and also affecting the reliability of the results [13]. Artificial neural network technology [14] has also been applied to assess lake eutrophication, overcoming the shortcomings of the aforementioned methods and providing an effective approach for comprehensive evaluation. However, there are still three difficult challenges that are hard to overcome in its application: (1) The training speed is slow, and it is difficult to meet the requirements of many scenarios that demand online learning; (2) The learning process is highly prone to converging to local minima, and it is hard to ensure learning accuracy; (3) The network has poor generalization ability, and it fails to guarantee good application performance on samples outside the training set after training, i.e., it has poor scalability [15].

To overcome the above shortcomings, the FCM algorithm and improved grey target theory are introduced to assess the lake eutrophication. It is a combination method of the FCM algorithm and improved grey target theory. It can effectively handle complex, numerous, and highly correlated research objects. In order to effectively combine the FCM algorithm with the improved grey target model [16, 17], this paper revises the traditional grey target evaluation model from two aspects: (1) the correlation coefficient matrix is adopted to replace the covariance matrix in the Mahalanobis distance; (2) the positive and negative ideal solutions in the TOPSIS model are introduced to define the positive and negative target centers.

The paper is organized as follows: in Materials and Methods Section, the methodology based on the FCM algorithm and grey target theory is presented and the engineering background is introduced; in Results and Discussion Section, an assessment model is constructed, and the assessment results are analyzed; in Conclusions Section, conclusions are drawn.

Materials and Methods

Study Area

Wuliangsuhai Lake is located in Urad Front Banner, Bayannao'er City, Inner Mongolia Autonomous Region (in Fig. 1). It is situated between 40°36′ and 41°03′ north

Fig. 1. Map showing the geographical location of the study area.

latitude and 108º43′ and 108º57′ east longitude, representing the largest lake within this latitude range globally and ranking as the eighth-largest freshwater lake in China. The existing water area of Wuliangsuhai Lake is 285.38 km^2, of which the reed-covered area accounts for 118.97 km^2, and the open water area covers 111.13 km^2. The lake has a long, narrow shape, stretching farther from north to south than from east to west. Specifically, it extends 35 to 40 km north-south and 5 to 10 km east-west. The lake's shoreline is 130 km long, and it has a storage capacity of 250 to 300 million m^3. The region where the lake is located experiences distinct seasonal changes with significant temperature variations. The multi-year average temperature is 7.3ºC, and the annual sunshine duration amounts to 3185.5 hours. Within the lake's basin, precipitation is scarce while evaporation is high, with a multi-year average rainfall of 224 mm and an evaporation rate of 1502 mm.

The Determination of Weights

Rough set theory is a new analysis theory of discrete data proposed by the Polish mathematician Pawlak in 1982 to handle fuzzy and uncertain knowledge [18, 19]. It is supposed that a quadruple $S=(U,A,V,f)$ is a knowledge representation system, where the universe of discourse U and the attribute set A are both non-empty finite sets, $A=C\cup D, C\cap D=\varnothing$, C and D are conditional attribute set and decision attribute set, respectively. V is the value domain set of attributes; f is the information function. This kind of knowledge representation system with conditional attributes and decision attributes is called a decision table. In rough set theory, a decision table is applied to describe objects in the universe of discourse. The decision table is a two-dimensional table that has been discretized based on the FCM (Fuzzy C-Means) algorithm. Each row describes an object, and each column describes an attribute of the object.

Let $U/C=\{x_1,x_2,...,x_n\}$, $U/D=\{Y_1,Y_2,...,Y_m\}$, then the support degree of the decision attribute D with respect to (or C's support over D with regard to) the conditional attribute C is defined as [20]

$$spt_c(D)=\frac{1}{|U|}\sum_{i=1}^{m}|pos_C(Y_i)|, Y_i\in U/D \tag{1}$$

Where, $spt_c(D)$ is called the degree to which decision attribute D depends on conditional attribute C. Its range of value is [0, 1]. When $spt_c(D)=1$, D is fully dependent on C; when $spt_c(D)=0$ D is fully independent of C; when $0<spt_c(D)<1$, D is roughly dependent on C.

$pos_C(D)$ denotes the positive region of Q under C; # is the cardinality or size of the set #, it represents the number of elements in the set for finite sets. The importance of a conditional attribute subset $C_i(\varnothing\subset C_i\subseteq C)$ with respect to the decision attribute D is defined as:

$$sig^{D}_{C-\{C_i\}}=spt_C(D)-spt_{C-\{C_i\}}(D) \tag{2}$$

Based on the magnitude of the importance of conditional attribute subsets, one can discern their strength of influence on the decision attribute. The weights of various conditional attribute subsets can be expressed as [21, 22]:

$$\omega_i=\frac{spt_C(D)-spt_{C-\{C_i\}}(D)}{\sum_{i=1}^{m}\left(spt_C(D)-spt_{C-\{C_i\}}(D)\right)} \tag{3}$$

The Calculation Method of Mahalanobis Distance

To accurately capture the relationship between the assessment indicators, a corresponding matrix is used to replace the covariance matrix in the Mahalanobis distance, thereby departing from traditional theory. Simultaneously, the Mahalanobis distance is applied to determine the positive and negative target center distances. Their calculative process is listed as follows:

1) The building of sample matrix

Suppose that there exist m samples in the set $X=(x_1, x_2, x_3, ..., x_m)$. n assessment indicators are constructed as the index set $V=(v_1, v_2, ..., v_4)$. The magnitude of sample x_k is $b_{kl}(k=1, 2, 3, ..., m)$ $(l=1, 2, 3, ..., n)$, then the corresponding matrix of X to V is

$$B=\begin{bmatrix} b_{11} & b_{12} & ... & b_{1n} \\ b_{21} & b_{22} & ... & b_{2n} \\ ... & ... & ... & ... \\ b_{m1} & b_{m2} & ... & b_{mn} \end{bmatrix} \tag{4}$$

2) The normalization of sample matrix

To reduce the dimensions of assessment index, it can be expressed as [23]:

$$D_i=\frac{1}{m}\sum_{k=1}^{m}b_{kl} \tag{5}$$

For the cost-type indicator:

$$r_{kl}=\frac{d_l-b_{kl}}{\max\left(\max_{1\le k\le m}(b_{kl})-d_l, d_l-\min_{1\le k\le m}(b_{kl})\right)} \tag{6}$$

where, $d_i=\frac{1}{n}\sum_{i=1}^{n}D_j$.

For the benefit-type indicator:

$$r_{kl} = \frac{d_{kl} - b_l}{\max\left(\max\limits_{1\le k\le m}\left(b_{kl}\right) - d_l, d_l - \min\limits_{1\le k\le m}\left(b_{kl}\right)\right)} \tag{7}$$

Based on the features of the normalization matrix, a standardized matrix R can be expressed as:

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ r_{n1} & r_{n2} & \cdots & r_{nn} \end{bmatrix} \tag{8}$$

3) Determining negative and positive target centers

Suppose that $r_l^+ = \max\{r_{kl}|1\le k\le n\}((l = 1, 2, 3, ..., n)$, then:

$$r^+ = \left(r_1^+, \quad r_2^+, \quad \ldots \quad r_n^+\right) \tag{9}$$

where, r^+ is the positive ideal solution.

Assume that $r_l^+ = \min\{r_{kl}|1\le k\le n\}((l = 1, 2, 3, ..., n)$, then:

$$r^- = \left(r_1^-, \quad r_2^-, \quad \ldots \quad r_n^-\right) \tag{10}$$

where, r^- is the positive ideal solution.

4) Determining positive and negative target center distances

$$i_l = \omega_l\left(r_{kl} - r_l^+\right), I = \left(i_1 \quad i_2 \quad \cdots \quad i_n\right)^T \tag{11}$$

$$j_l = \omega_l\left(r_{kl} - r_l^-\right), J = \left(j_1 \quad j_2 \quad \ldots \quad j_n\right)^T \tag{12}$$

where, I and J are the coefficient difference between sample x_k and the positive and negative target centers, respectively.

The positive target center distance is calculated as:

$$\varepsilon_k^+ = \left(I^T I^{-1} I\right)^{1/2} \tag{13}$$

The negative target center distance is:

$$\varepsilon_k^- = \left(J^T I^{-1} J\right)^{1/2} \tag{14}$$

where, II denotes the correlation coefficient matrix.

5) Determining the relative target center distance

The relative target distance ξ_k of k-th indicator is calculated as:

$$\xi_k^* = \frac{\xi_k^-}{\xi_k^+ + \xi_k^-} \tag{15}$$

6) The classification of quality grade

The set D is defined as the ordered intervals obtained by partitioning set ε based on t quality levels, namely, $C = (c_1, c_2, ..., c_t)$, let $1\le\phi<t$, and ϕ is the positive integer. ϕ-th quality grade $\gamma_\phi = \max\{c_\phi\}$, $\eta_\phi = \min\{c_\phi\}$, then the level classification Y is [24]

$$l_\phi = \mu\gamma_\phi + \left(1-\mu\right)\eta_{\phi+1}, \mu \in \left(0,1\right) \tag{16}$$

$$Y = \left\{c \middle| 0 \le c_1 < z_1, f_1 \le c_2 < z_2, \ldots, z_{i-1} \le c_t < +\infty\right\} \tag{17}$$

The Determination of the Evaluation Index

Taking into account many factors related to lake eutrophication evaluation, the evaluation indicators Cha, Total phosphorus (TP), Total nitrogen (TN), oxygen consumption (COD), and Transparency (SD) are selected as the set of indicators for lake eutrophication. By referring to existing research on criterion indicator systems [25], the classification standards for eutrophication grades corresponding to each indicator are presented in Table 1.

Twenty-two datasets were selected as the model test set to verify and evaluate the model's accuracy, as shown in Table 2.

In Table 2, the first 16 samples are used as training data, and the last 6 as test data.

Table 1. The classification standard of eutrophication level.

The assessment criteria	Cha (mg/m³)	TP (mg/m³)	TN (mg/m³)	COD (mg/L)	SD (m)
Oligotrophic (I)	0~1	0~2.5	0~30	0~0.3	5~10
Oligotrophic to mesotrophic (II)	1~2	2.5~5	30~50	0.3~0.4	1.5~5
Mesotrophic (III)	2~4	5~25	50~300	0.4~2	1~1.5
Mesotrophic to eutrophic (IV)	4~10	25~50	300~500	2~4	0.4~1
Eutrophic (V)	10~65	50~200	500~2000	4~10	0.3~0.4
Hypereutrophic (VI)	65~160	200~600	2000~6000	10~25	0~0.3

Table 2. The monitoring samples.

Serial number	Name of monitoring point	Cha (mg/m^3)	TP (mg/m^3)	TN (mg/m^3)	COD (mg/L)	SD (m)	Risk grade
1	I12	65.49	181	18380	110.2	0.37	V
2	J11	47.5	179	13750	108.4	0.52	V
3	J13	150.65	366	21570	113.3	0.25	VI
4	K12	106.43	453	24610	117.6	0.23	VI
5	L11	158.99	333	17660	96.9	0.19	VI
6	L13	188.62	325	20990	108.6	0.26	VI
7	L15	97.26	189	10857	102.6	0.43	V
8	M12	84.65	303	20990	98.2	0.33	V
9	M14	170.52	338	27940	116.1	0.28	VI
10	M16	7.857	85.5	1810	76	0.83	IV
11	N13	69.39	250	26060	93.5	0.32	V
12	O10	13.381	84.5	723	85.5	1.46	IV
13	P9	17.87	54.6	1737	83.2	0.93	IV
14	P11	1.413	80.3	2895	83.1	0.76	IV
15	Q8	6.142	79.6	1737	84.9	0.9	IV
16	Q10	5.876	74.8	869	80.8	1	IV
17	R7	59.6	49.7	1810	88.8	1.45	V
18	S6	5.845	64.6	1448	81.7	1.76	IV
19	S8	3.338	83.2	2171	73.4	0.91	IV
20	T5	1.75	75.3	1810	91.3	1.57	IV
21	U4	6.075	72.6	2171	89.2	1.07	IV
22	V3	6.551	84.1	1810	95.8	1.75	IV

Results and Discussion

The Determination of Weight Coefficients

Data discretization was performed on Table 2 based on the FCM (Fuzzy C-Means) algorithm, where *U* represents the universe of discourse and *A* denotes the attribute set. In this paper, continuous attribute values are discretized into the classification levels for lake eutrophication categories based on various influencing factors, as shown in Table 3.

In Table 3, the condition attribute is $C = \{c_1\ c_2\ c_3\ c_4\ c_5\}$, its number of clusters is 5; the decision attribute is $D = \{d\}$, its number of clusters is 6. For example, the indicator σ_c is selected as the instance, then $spt_c(d) = 0.5$, $spt^d_{c-c_1} = \frac{7}{22}$, $spt^d_{c-c_2} = \frac{1}{11}$, $spt^d_{c-c_3} = \frac{2}{11}, spt^d_{c-c_4} = 0,\ spt^d_{c-c_5} = \frac{5}{22}$.

Then based on Equations (1)-(3), the weight coefficients can be obtained as:

$$\omega = [0.1081 \quad 0.2432 \quad 0.1892 \quad 0.2973 \quad 0.1622]$$

The Standardization of Sample Matrix

Standardizing the sample data is performed. Cha, Total phosphorus (TP), Total nitrogen (TN), and oxygen consumption (COD) belong to the cost-type indicators. Transparency (SD) is a benefit-type indicator. They are standardized based on Equations (4) and (8), respectively. The matrix *R* is shown in Table 4.

Determining Relative Target Center Distance

Based on Table 4, the positive target center of samples is determined as $r^+ = (1\ 1\ 1\ 1\ 0.941\ 0.421)$, the negative target center is $r^- = (-0.6405 - 0.6464 - 0.8573 - 1\ -1)$; based on the standardized sample indicator matrix R, the correlation coefficient matrix among evaluation indicators can be obtained; it is shown in Table 5.

Table 3. Decision table of measured sample data.

Serial Number	Evaluation indicators					d
	c_1	c_2	c_3	c_4	c_5	
1	4	5	6	6	5	5
2	5	5	6	6	4	5
3	6	6	6	6	6	6
4	6	6	6	6	6	6
5	6	6	6	6	6	6
6	6	6	6	6	6	6
7	6	5	6	6	4	5
8	6	6	6	6	5	5
9	6	6	6	6	6	6
10	4	5	5	6	4	4
11	6	6	6	6	5	5
12	5	5	5	6	3	4
13	5	5	5	6	4	4
14	2	5	6	6	4	4
15	4	5	5	6	4	4
16	4	5	5	6	3	4
17	5	4	5	6	3	5
18	4	5	5	6	2	4
19	3	5	6	6	4	4
20	2	5	5	6	2	4
21	4	5	6	6	3	4
22	4	5	5	6	2	4

It can be found in Table 5 that evaluation indicator Cha has strong correlations with TP, TN, COD, and SD. To eliminate the impact of correlation among evaluation indicators, the correlation coefficient matrix is used to calculate the relative target center distance.

According to Equations (11)-(15), the target center distance is shown in Table 6.

The Determination of Classification Standard

Based on Tables 2 and 6, in combination with Equations (16) and (17), the standard for the relative target center distance is shown in Table 7.

Its classification target is plotted in Fig. 2.

The Prediction of the Model

The latter 6 groups of data in Table 1 were selected as the training sample; based on the quality grade classification shown in Table 7 and the relative target center distances, the quality grade of the training samples can be determined. The grade level obtained by the improved grey target model is consistent with the actual quality level. The actual grade level of this testing sample was determined based on exploration information. The comparative results are shown in Table 8.

According to the comparative results of the assessment model in Table 8, conclusions can be drawn that the results obtained by the suggested method are entirely consistent with the actual quality level for 6 samples. The proposed method achieves 100% accuracy. So, the conclusion demonstrates that it is feasible to estimate the lake eutrophication. In comparison with the traditional grey target theory [26], its accuracy is higher (the accuracy in the traditional theory is 67%), mainly because the traditional grey target model neglects the correlation between evaluation indicators during the evaluation process, which results in the repeated computation of information and a reduction in the accuracy of evaluation results. For example, the positive correlation coefficient between evaluation indicator Cha

Table 4. Standardized sample matrix R.

Sample	Cha (mg/m^3)	TP (mg/m^3)	TN (mg/m^3)	COD (mg/L)	SD (m)
1	0.079	-0.1241	0.3476	0.5958	0.2196
2	-0.2366	-0.1323	0.0317	0.5118	0.0517
3	0.6673	0.6405	0.5653	0.7404	0.3538
4	0.2798	1	0.7728	0.9411	0.3762
5	0.7404	0.5041	0.2985	-0.0248	0.421
6	1	0.471	0.5257	0.5211	0.3427
7	0.1994	-0.091	-0.1658	0.2412	0.1524
8	0.0889	0.3801	0.5257	0.0359	0.2643
9	0.8414	0.5248	1	0.8711	0.3203
10	-0.584	-0.5187	-0.7831	-1	-0.2951
11	-0.0448	0.1611	0.8717	-0.1834	0.2756
12	-0.5356	-0.5228	-0.8573	-0.5567	-1
13	-0.4963	-0.6464	-0.7881	-0.664	-0.407
14	-0.6405	-0.5402	-0.7091	-0.6687	-0.2168
15	-0.599	-0.5431	-0.7881	-0.5847	-0.3734
16	-0.6014	-0.5629	-0.8474	0	0

Table 5. The correlation coefficient matrix.

Correlation	Cha (mg/m^3)	TP (mg/m^3)	TN (mg/m^3)	COD (mg/L)	SD (m)
Cha (mg/m^3)	1	0.864	0.824	0.751	0.787
TP (mg/m^3)	0.864	1	0.904	0.784	0.799
TN (mg/m^3)	0.824	0.904	1	0.785	0.831
COD (mg/L)	0.751	0.784	0.785	1	0.74
SD (m)	0.787	0.799	0.831	0.74	1

Table 6. The synthetic target center distance.

Sample number	The relative target center distance ξ_k^*	Sample number	The relative target center distance ξ_k^*	Sample number	The relative target center distance ξ_k^*	Sample number	The relative target center distance ξ_k^*
1	0.6485	5	0.4826	9	0.9134	13	0.0798
2	0.6685	6	0.8414	10	0.0847	14	0.1317
3	0.9456	7	0.5515	11	0.6164	15	0.1059
4	0.9355	8	0.8502	12	0.0921	16	0.3186

Table 7. The classification of quality grade.

Risk level	Relative target center distance ξ_k^*	Description
I	(0 0.1622)	Oligotrophic
II	(0.1622 0.315)	Oligotrophic to mesotrophic
III	(0.315 0.435)	Mesotrophic
IV	(0.435 0.6664)	Mesotrophic to eutrophic
V	(0.6664 0.9456)	Eutrophic
VI	(0.9456 + ∞)	Hypereutrophic

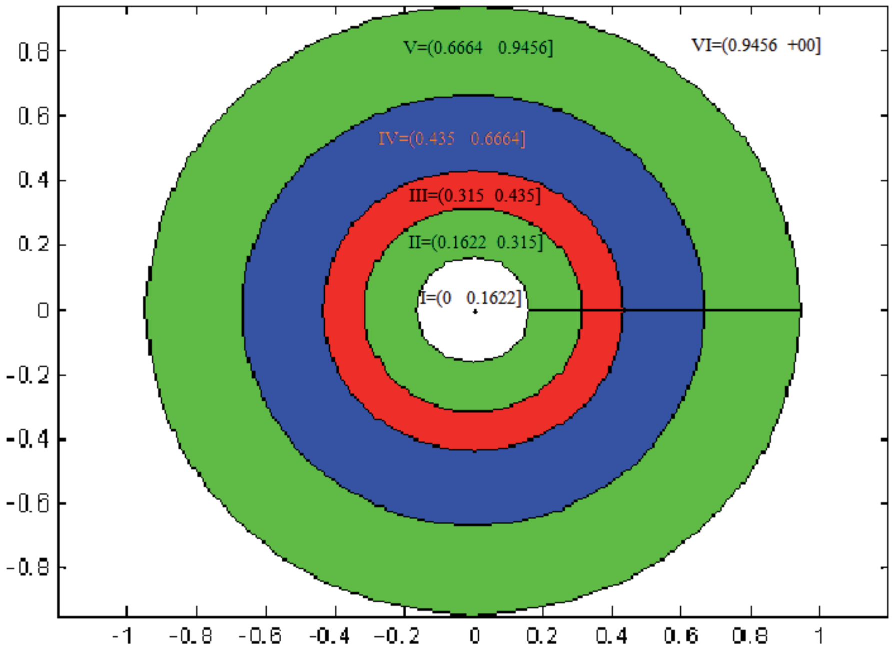

Fig. 2. The classification target figure of lake eutrophication grade.

Table 8. The quality grade prediction of lake eutrophication.

Sample number	Relative target center distance	The quality level in traditional grey target model	The quality level in the suggested method	Actual risk level
17	0.8632	V	V	V
18	0.4724	IV	IV	IV
19	0.5021	IV	IV	IV
20	0.5693	V	IV	IV
21	0.6229	III	IV	IV
22	0.6526	IV	IV	IV

and TP reaches 0.864. It can also be found in Table 8 that the quality levels of the lake eutrophication from 17 to 22# samples are different; the quality level from 18# to 22# samples is IV, which means 18#, 19#, 20#, 21#, and 22# samples are mesotrophic to eutrophic; one at 17# sample is eutrophic. So, the corresponding treatment measures should be performed.

Conclusions

Based on the characteristics of lake eutrophication assessment, a weighting method based on the FCM algorithm was constructed. The correlation coefficient matrix was adopted to replace the covariance matrix in the Mahalanobis distance and was introduced into the traditional grey target model. Based on the root causes of lake eutrophication, a quality evaluation index system for prevention and management was established. The positive and negative target center distances were calculated using Mahalanobis distances based on correlation coefficients; the relative target center distance was defined, and quality levels were classified.

Considering the influence of correlation among evaluation indicators, the correlation coefficient matrix is adopted to replace the covariance matrix in the Mahalanobis distance and is introduced into the traditional grey target model, thereby characterizing the correlation among evaluation indicators. Consequently, the improved method enhances the accuracy of evaluation results. Meanwhile, the positive and negative ideal solutions in the TOPSIS model are used to define the positive and negative target centers; thus, this drawback of the original grey target model, which uses only a single ideal optimal value as the target core, is overcome.

When the improved grey target model is applied to evaluate the practical lake eutrophication level, the quality levels it produces are consistent with the actual quality levels. From the perspective of indicator correlations, the validity and rationality of the improved grey target model in lake eutrophication evaluation have been verified, which indicates the feasibility of the improved grey target model to evaluate lake eutrophication quality.

However, the suggested model still has some limitations, such as complex calculations and the requirement for multiple variable parameters, which limit its application. Therefore, the suggested model has great potential for improvement in the future.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflict of interest.

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