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Original Research

Modeling the Profit from Hydropower Plant Energy Generation Using Dimensional Analysis

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Abstract

Our work presents the original mathematical model, which can be determined on the basis of the actual profits from electricity production in pumped storage hydropower plants (PSHP). The derived relationship was obtained by the dimensional analysis of the relevant physical parameters describing the production of energy. The main purpose of this paper is to prove whether a dimensional analysis can be a useful tool to describe the economic aspect of the phenomenon, as proved by its worth when examining technical phenomena. The mathematical model has been derived for PSHP Ružín in Slovakia, but its validity, based on the similarity theory, can be extended to any pumping power plant that works with the influx of water into the upper storage reservoir, and/or without the feed, respectively. The article discusses the effects of various parameters on the operating profit for the turbine operations and the costs associated with the pumping operation.

Keywords: dimensional analysis, operating profit, pumped storage hydroelectric power plant.

Introduction

Energy generation plays an important role in the economy of the country. In recent decades electricity production has more than doubled and electricity demand is rising rapidly. Energy consumption optimization is considered one of the pillars of principled management based on sustainability [1]. Hydropower plays a key role not only as a renewable and sustainable energy source, whose potential is estimated to be still very large, but also as a large-scale energy storage technology, commonly recognized as being one of most cost-efficient among those technologies currently available. Hydroelectric efficiency and profitability are important factors, but the environmental benefits and social responsibility are also significant for sustainable use of hydropower [2].

Nowadays renewed interest in pumped hydro energy storage plants and the huge demand for the rehabilitation and repowering of old mini and micro-hydro plants are

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emerging globally [3]. Pump storage plants are an important electricity storage technology at present [4]. Investments and innovations in this technology are expected to increase [5, 6]. The great interest nowadays generally is in the field of the profit from renewable energy sources. In order to evaluate these investments correctly the peculiarities of pump storage plants and the characteristics of liberalized power markets have to be considered. The main characteristics of power markets are the strong power price volatility and the occurrence of price spikes. The common approach for the appraisal of capital budgeting, and thus of power plants, represents the net present value method [5, 7]. Muche [7] developed a valuation approach that considers the real options in the valuation of a pump storage plant. These real options particularly arise from a price-based unit commitment planning that has been necessary or possible since the liberalization of the power markets. Unit commitment policy of the pump storage plants results in realizable cash flows, which is the base for the investment in pump storage plants valuation. The use of forecasted market prices is common because it offers an objective yardstick for investment comparison [5]. Day-ahead markets are mostly used for this investment valuation [8-10].

Catalão et al. [11] proposed a novel mixed-integer nonlinear approach to solve the short-term hydro scheduling problem in the day-ahead electricity market, considering not only head-dependency, but also start/stop of units, discontinuous operating regions, and discharge ramping constraints. Unit commitment based on forecasted market prices has to consider market price uncertainty [12]. Uncertainties within the energy cost savings are modelled in Deng et al. [13] stochastically using Monte-Carlo simulation, taking both the energy price fluctuation and the facility performance variability into account. Uncertainties may exist in various impact factors of electricity generation and also in CO_2 mitigation systems such as greenhouse gas emissions inventory, greenhouse gas reduction costs, electricity prices, and emission reduction credits [14]. Many studies have examined the economics of renewable energy project installations, including wave energy [15-17].

The forecasting of electricity demand and price has emerged as a major research task, not only in electrical engineering. A lot of researchers and academicians are engaged in the activity of developing tools and algorithms for load and electricity price forecasting.

The main methodologies used in electricity price forecasting have been reviewed [18, 19]. This study includes price-forecasting techniques and also techniques based on input and output variables. Most of the researchers have utilized past experience in selecting the input variables for their respective model, and choice of best input variables for a particular model is still an open area of research. Kanamura and Ōhahi [20] present a structural model for electricity prices based on demand and supply. They show that the structural model can generate price spikes that fit the observed data better than those generated by other preceding models such as the jump diffusion model [21]. The result shows that the structural model can provide more realistic optimal operational policies in a much simpler way than jump diffusion models. In this paper, the presented mathematical model of the profit from the energy generation in hydropower plants using dimensional analysis is also based on input parameter.

Mathematical models are trying to present studied phenomenon in space and time. The basis for development of models based on dimensional analysis for their practical application in technical disciplines was completed by Buckingham [22].

Dimensional analysis, described by this author, is a technique that allows the transformation of dimensional variables, describing the phenomenon, in the set of dimensionless numbers, whose number is always less than the number of used physical quantities [22-24]. This method uses the rule of equations' dimensional homogeneity, meaning that each complete physical equation represents the sum of the members, which altogether always have the same measurement. According to Buckingham, for every equation containing o dimensional variables it is possible to represent it by k dimensionless numbers — so-called π variables, and thus to describe the solution by a simpler criterion equation. This theory has been widely developed and confirmed [25-28].

Dimensional analysis is used by scientists for the description of diverse phenomena. For example, Lin et al. [29] uses the Buckingham theory for the description of transferred heat output from the channel plate heat exchanger. Demir et al. [30] presents developed models for prediction of pressure losses in irrigation facilities. Yurdem et al. [31] developed a model for prediction of pressure losses in the hydrocyclone and disk filters by employing a dimensional analysis approach. Fries and Dreyer [32] use the dimensionless method for description of the capillary rise of liquids in the porous media. Vilčeková and Šenitková [33] developed a mathematical model for the prediction of nitrogen oxides produced in an indoor environment of buildings, depending on the intensity of the combustion in gas stoves. Employing this method, Zeleňáková and Čarnogurská [34] describe the prediction of concentrated pollutants in the water flow. Zeleňáková and Čarnogurská [35] and Zeleňáková et al. [36] have used dimensional analysis for the prediction of nitrogen and phosphorus concentrations in the Laborec water flow in Slovakia.

The mathematical model described in this paper is authentic and on its basis as the actual profit from the production of electricity in the pumped storage hydropower plant Ružín in Slovakia has been determined, and then compared with the figures reported by the plant operator. The scientific contribution of this paper is based on a new approach to the formation of complex energy processes. The derived mathematical model showing the gain from energy production has universal validity and can be used for any pumped-storage hydropower plant, which satisfies the condition of approximate similarity.

Study Area

Until now, for all types of hydropower plants the needs of the power system grid are preferred over the interests of the optimized plant operation itself. Especially at PSHP, as the regulators of the power electricity supply system, their optimized operating mode is completely subordinated to the interests of the power grid. In spite of this situation there have always been attempts to find ways to improve production efficiency, while fully respecting the requirements of the power electricity supply system.

For the operation of the PSHP itself, the alternation between own production and pumping, which is the preparation for such production, is technologically essential. During turbine operation (and pumping as well), there is a fluctuation in water levels in the upper accumulating and lower retention reservoirs.

In terms of economic evaluation of the power plant the composition of total daily energy production is very important, i.e., what proportions of the daily production are contributed by:

a) Production from pumped storage
b) Production from the water feed into the storage reservoir

For pumped storage hydropower plants it is desirable if both kinds of production are involved in energy production. If the PSHP does not have a natural feed of water into the upper reservoir, the production referred to in "b" shall not apply.

The mathematical model for the description of profits gained from the production of electricity has been developed for the Ružín pumped storage hydro power plant (Fig. 1). The model represents the average value of daily profits from production, as the difference between the value of electricity produced and the energy taken from the grid during the pumping of water into the upper storage reservoir, i.e.:

$$Z = \sum_{i=1}^{n} Z_{t\,i} - \sum_{j=1}^{m} Z_{p\,j} \tag{1}$$

… where $Z_{t,i}$ is the profit from the i-th turbine operation during one day (€), $Z_{p,j}$ are the energy costs for the j-th pump operation in one day (€), n is the number of turbine installations during the day (1), and m is the number of pump operations during the day (1).

Model Development

For the development of the mathematical model, dimensional analysis requires that different physical variables affecting the given phenomenon are stated in SI units. For the representation of partial profits from electricity production and costs of its consumption the following relevant parameters have been identified, the dimensions of which will in the first step reflecting

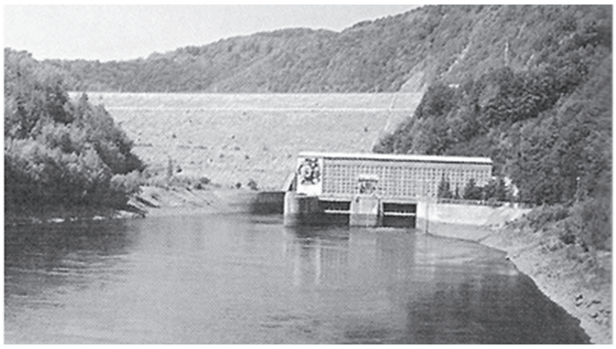

Fig. 1. View of the Ružín pumped storage hydropower plant.

a customary representation used in energy production operations (eventually they will be transformed into the SI system units using a conversion constant). Definition of the profit from production for turbine operation ($Z_{t,i}$) for a specific period is based on the following relevant parameters:

– *Cps:* price of peak electricity (€/MWh) = $1/(3.6\cdot10^9)$ €/Ws = $1/(3.6\cdot10^9)$ €/J = $1/(3.6\cdot10^9)$ €$\cdot$s^2/(kg$\cdot$m^2),
– $(H_{1,i} + H_{2,i})/2 = H_{t,i}$: average useful head of water in the turbine operation; where $H_{1,i}$ (m) is the water level in the upper reservoir before starting the production of electricity in turbine operation and $H_{2,i}$ (m) is the water level after stopping production
– ρ: water density (kg/m^3)
– $Q_{V,t,i}$: turbine flow capacity in a given period (m^3/s)
– $\tau_{t,i}$: duration of the turbine operation (s)

The relationship between relevant variables with different dimensions is expressed by the following complete physical equation:

$$f(Z_{t,i}, C_p, H_{t\,i}, \rho, Q_{V,t\,i}, \tau_{t\,i}) = 0 \tag{2}$$

Hence, the formulation of dimensionless arguments (parameters) is based on the relationship:

$$\pi = Z_{t\,i}^{x_1} \cdot C_p^{x_2} \cdot H_{t\,i}^{x_3} \cdot \rho^{x_4} \cdot Q_{V,t\,i}^{x_5} \cdot \tau_{t\,i}^{x_6} \tag{3}$$

The dimensional matrix for basic units, therefore, will have $o = 6$ columns and $p = 4$ rows, and has the following form:

$$\begin{array}{c} \\ € \\ \text{m} \\ \text{s} \\ \text{kg} \end{array} \begin{array}{c} \begin{array}{cccccc} Z_{t\,i} & C_p & H_{t\,i} & \rho & Q_{V,t\,i} & \tau_{t\,i} \end{array} \\ \left\| \begin{array}{cccccc} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & -3 & 3 & 0 \\ 0 & 2 & 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & 1 & 0 & 0 \end{array} \right\| \end{array} \tag{4}$$

Because the dimensional matrix (4) has a rank of $h = 4$ (number of rows), with the total number of variables

$o = 6$, $k = o - p$, i.e., two dimensionless arguments π can be created in total.

For the number of unknown $x_i > h$, the unknown variables cannot be unambiguously determined. Therefore the rectangular dimensional matrix must be split into two parts, so a square matrix with h columns and h rows is created. Columns from the original matrix must be chosen in a way so that the determinant of a square matrix has a nonzero result ($\Delta \neq 0$).

The expression of square matrix **A** and the vector of unknown variables $\boldsymbol{x_i}$ of equation (4) will be modified and written in a simplified form:

$$\mathbf{A} \cdot \mathbf{B} = (-1) \cdot \mathbf{D} \cdot \mathbf{E} \tag{5}$$

By modification of equation (4) to the square matrix and matrix of unknowns, in accordance with equation (5), subject to the rules for selection of the unknowns, we get the expression:

$$\left\| \begin{matrix} 1 & 0 & 0 & 0 \\ -2 & 1 & -3 & 3 \\ 2 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 \end{matrix} \right\| \cdot \left\| \begin{matrix} x_2 \\ x_3 \\ x_4 \\ x_5 \end{matrix} \right\| = (-1) \cdot \left\| \begin{matrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{matrix} \right\| \cdot \left\| \begin{matrix} x_1 \\ x_6 \end{matrix} \right\| \tag{6}$$

The determinant of matrix **A** has the value $\Delta_A = 1$.
The selection of redundant unknowns x_1 and x_6 will be made twice, where both selections must be linearly independent (see expression (7)):

$$\begin{array}{ccc} & x_1 & x_6 \\ \text{1.voľ.} & 1 & 0 \\ \text{2.voľ.} & 0 & 1 \end{array} \tag{7}$$

The selection matrix is expressed as:

$$\left\| \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \right\|$$

… and its determinant has the value of $\Delta = 1$. This meets the condition for further solution.

By multiplying matrices according to equation (5) we obtain the expression:

$$\|\mathbf{A}\|_{4\,\mathrm{x}\,4} \cdot \|\mathbf{B}\|_{4\,\mathrm{x}\,1} = \|\mathbf{C}\|_{4\,\mathrm{x}\,1}$$

$$\|\mathbf{D}\|_{4\,\mathrm{x}\,2} \cdot \|\mathbf{E}\|_{2\,\mathrm{x}\,1} = \|\mathbf{F}\|_{4\,\mathrm{x}\,1} \tag{8}$$

Then, equations (5) and (8) result in the expression:

$$\|\mathbf{C}\|_{4\,\mathrm{x}\,1} = (-1) \cdot \|\mathbf{F}\|_{4\,\mathrm{x}\,1} \tag{9}$$

As in equation (9), this is the same type of matrix, for the matrix elements must apply:

$$\begin{array}{rrrrcr} & x_2 & & & = & -x_1 \\ -2 \cdot x_2 & +x_3 & -3 \cdot x_4 & +3 \cdot x_5 & = & 0 \\ & 2 \cdot x_2 & & -x_5 & = & -x_6 \\ & -x_2 & & +x_4 & = & 0 \end{array} \tag{10}$$

Solving the set of linear equations (10) gives two independent criteria π_1 a π_2:

	x_1	x_2	x_3	x_4	x_5	x_6
π_1	1	-1	1	-1	-2	0
π_2	0	0	-3	0	1	1

Two dimensionless complex arguments correspond to this solution:

$$\pi_1 = \frac{Z_{t,i} \cdot H_{t,i}}{C_{pe} \cdot \rho \cdot Q_{V,t,i}^2} \quad (1) \tag{11}$$

$$\pi_2 = \frac{Q_{V,t,i} \cdot \tau_{t,i}}{H_{t,i}^3} \quad (1) \tag{12}$$

The dimensionless form of the homogeneous function, expressing the profit from turbine production in the morning peak hours, can be expressed by functional dependence:

$$\psi\,(\pi_1, \pi_2) = 0 \tag{13}$$

The dependence of dimensionless argument π_1, as a function of argument π_2, can be described by relation (14):

$$\pi_1 = \varphi\,(\pi_2) \tag{14}$$

Dependency in terms of relation (14) is assumed, e.g., in the form of a power function:

$$\pi_1 = A_1 \cdot \pi_2^{B_1} \tag{15}$$

The representation of this function in logarithmic coordinates is a linear functional relationship in the following form:

$$\log \pi_1 = \log A_1 + B_1 \cdot \log \pi_2 \tag{16}$$

… where A_1 is the intercept constant (1) and B_1 is regression coefficient (1).

The extended form of equation (15) for the expression of profits from electricity production in the turbine operation has the following relationship form for the relevant dimensional variables:

$$\frac{Z_{\mathrm{t},i} \cdot H_{\mathrm{t},i}}{C_{\mathrm{pe}} \cdot \rho \cdot Q^{2}_{V,\mathrm{t},i}} = A_1 \left(\frac{Q_{V,\mathrm{t},i} \quad {}_{\mathrm{t},i}}{H^{3}_{\mathrm{t},i}} \right)^{B_1} \text{(1)} \qquad (17)$$

… or, after the adjustment:

$$Z_{\mathrm{t},i} = A_1 \cdot C_{\mathrm{pe}} \cdot \rho \cdot Q^{2+B_1}_{V,\mathrm{t},i} \cdot H^{-3 \cdot B_1 - 1}_{\mathrm{t},i} \cdot \tau^{\,B_1}_{\mathrm{t},i} \text{ (€)} \qquad (18)$$

In determining the profits from production according to this relationship the individual variables must be entered in the following dimensions: cost of energy C_{pe} (€/J), volume of flow $Q_{V,t}$ (m^3/s), head H_t (m), density ρ (kg/m^3), and duration τ_t (s). The unit price of energy usually expressed in practice in €/MWh is necessary when entering C_{pe} in relation (18), then divide by $3.6 \cdot 10^9$.

By the same procedure the relationship for expression of costs ($Z_{c,j}$) for water pumping into the storage reservoir for a specific period has been obtained. Except for the density of water ρ, the model is based on the following relevant variables:

– C_{be}: base load electricity price (€/MWh) = $1/(3.6 \cdot 10^9)$ €·s^2/(kg·m^2)
– $(H_{1,j} + H_{2,j})/2 = H_{p,j}$: average pressure head at the time of water pumping; where $H_{1,j}$ (m) is the difference between the levels of the upper and lower reservoir before starting the pumping, operation and $H_{2,j}$ (m) is the difference after stopping the pumping
– $Q_{V,p,j}$: volume of turbine flow during the relevant pumping mode (m^3/s)
– $\tau_{p,j}$: duration of the relevant pumping operation (s)

The costs of pumping during the pumping mode of operation can be expressed by:

$$Z_{\mathrm{c},j} = A_2 \cdot C_{\mathrm{be}} \cdot \rho \cdot Q^{2+B_2}_{V,\mathrm{p},j} \cdot H^{-3 \cdot B_2 - 1}_{\mathrm{p},j} \cdot \tau^{\,B_2}_{\mathrm{p},j} \text{ (€)} \qquad (19)$$

Calibration of the Model

The developed model has been calibrated with the use of experimental data provided by the operator of the Ružín power plant. A total of 10 measurements were carried out with the turbine operation during the morning peak hours. The average efficiency of the turbine operation at the time of measurements was $\eta_t = 0.896$.

The dimensionless arguments π_1 a π_2 can be calculated from the experimentally obtained values of the relevant variables (Table 1). Intercept A_1 and regression coefficient B_1 can be obtained from the shown functional dependence of π_1 on π_2 in logarithmic coordinates (Fig. 2). Their values are $A_1 = 720925$ and $B_1 = 0.96130$.

A total of 12 measurements are available from the morning pumping for the calibration of the pumping operation. The average efficiency of the pumping operation was $\eta_p = 0{,}890$.

The intercept and regression coefficient data for the pumping operation in the morning mode of operation reached the following values: $A_2 = 834967$ and $B_2 = 0.73160$.

Discussion

The mean deviation between the profit from production during the turbine operation according to relationship (18) and the real value obtained from the PSHP operator is 13.1%. For the pumping operation this deviation is

Table 1. Measuring data of the relevant parameters and the calculated values of profits from production during the turbine operation in accordance with relationship (18).

W_t	H_1	H_2	τ_t	$Q_{V,t}$	Z_t	$Z_{t,sol}$	Deviation
(MWh)	(m)	(m)	(min)	(m^3·s^{-1})	(€)	(€)	(%)
126.11	52.18	49.83	270	62.51	6028	5173	14.18
133.33	49.80	48.31	300	61.88	6377	6464	-1.36
93.22	50.16	48.36	210	61.51	4456	4435	0.47
143.48	47.98	44.76	360	58.67	6858	8185	-19.35
70.37	45.82	44.13	185	57.74	3364	4635	-37.77
14.61	48.35	47.97	35	59.22	699	773	-10.55
290.14	52.15	47.68	600	66.13	13868	14321	-3.26
40.19	52.43	49.43	85	63.71	1922	1804	6.15
240.30	52.64	47.89	550	59.35	11486	9305	18.99
53.93	52.20	48.82	125	58.32	2578	2086	19.07

Table 2. Constants in relationship (21).

Turbine operation		Pumping operation	
Constant	Value	Constant	Value
$K_t = A_1 \cdot \rho$	$720.925 \cdot 10^6$	$K_c = A_2 \cdot \rho$	$834.967 \cdot 10^6$
$m_1 = 2 + B_1$	2.9613	$m_2 = 2 + B_2$	2.7316
$n_1 = -3 \cdot B_1 - 1$	–3.8839	$n_2 = -3 \cdot B_2 - 1$	–3.1948
$o_1 = B_1$	0.9613	$o_2 = B_2$	0.7316

12.1%. The difference between the calculated values of profits or costs from the developed mathematical model and the values determined from the measuring data of the relevant individual variables cannot be caused by owned model error, because it is built with particular emphasis on relevant constants influencing the given process. Deviation is likely due to low data precision provided by the plant operator (large rounding of provided data). This applies in particular to data on the duration of the production, water head values, and flow volumes. Another reason may be the fact that the scope of data from the experiment is relatively small. If they were available (results of at least several hundred measurements), it would be possible to statistically evaluate them. They would exclude so-called "outliers" and derive the constants in the model, and only this modified data would be applied and used.

In order to verify the model, 11 experimental measurements from the evening turbine operation and 11 measurements from the night pumping operation were available. Solutions using these data showed that values of intercept and regression coefficients reached nearly the same values as the data from morning turbine and morning pumping operations. Based on this, it can be concluded that for the specific power plant for all periods of the turbine or pumping operation the invariable pair of the intercept and regression coefficient values can be used:

a) For *turbine* operation A_1, B_1
b) For *pumping* operation A_2, B_2

The general form of the relationship for the capturing of the mean daily profits will be:

$$Z = \sum_{i=1}^{n} C_{\mathrm{pe}} \cdot \rho \cdot A_1 \; Q_{V,\mathrm{t},i}^{2+B_1} \cdot H_{\mathrm{t},i}^{-3.B_1-1} \cdot \tau_{\mathrm{t},i}^{B_1} - \\ - \sum_{j=1}^{m} C_{\mathrm{be}} \cdot \rho \cdot A_2 \; Q_{V,\mathrm{p},j}^{2+B_2} \cdot H_{\mathrm{p},j}^{-3.B_2-1} \cdot \tau_{\mathrm{p},j}^{B_2} \quad (€) \quad 20)$$

The values of intercepts and regression coefficients for the individual operations in the Ružín PSHP are as follows:

a) For turbine operation: **A_1 = 720925**
B_1 = 0.96130
b) For pumping operation: **A_2 = 834967**
B_2 = 0.73160

After substituting the values of regression coefficients and the intercepts in expression (20), separately for the turbine and for the pumping operation, the following equation is obtained

$$Z = \sum_{i=1}^{n} K_{\mathrm{t}} \cdot C_{\mathrm{pe}} \cdot Q_{V,\mathrm{t},i}^{m_1} \cdot H_{\mathrm{t},i}^{n_1} \cdot \tau_{\mathrm{t},i}^{o_1} - \sum_{j=1}^{m} K_{\mathrm{c}} \cdot C_{\mathrm{be}} \cdot Q_{V,\mathrm{p},j}^{m_2} \cdot H_{\mathrm{p},j}^{n_2} \cdot \tau_{\mathrm{p},j}^{o_2} \quad (€) \quad (21)$$

The individual constants (21) are presented in Table 2.

The calculated coefficients for expressing profits from electricity production with the use of relationship (21) are presented in Table 2, which takes into account not only the nature of the operation and the number of working machines, but also all the losses during electricity production and/or consumption that occurred.

The advantage of expressing profits from electricity production according to equation (21) is that in dimensionless argument π_1 (relationship 11), profit Z is in proportion to the unit price *C*. As a result of this proportion the amount of energy produced — which means that the dimensionless number π_1 remains, with the change of the current unit price for the energy produced — always remains the same. This statement applies to the pumping operation as well. Any change in price will not affect the calculated values of intercepts and regression coefficients, which is very important for the versatility of the obtained mathematical model.

Based on the described methodology, each new pumped-storage hydro power plant (further marked with "′"), which is not yet physically available, can have the profit from its power generation predicted based on the model laws (indicators of similarity). From Fig. 2 it is clear that each point which lies on the regression line corresponds, in terms of features $\pi_1 = \varphi\,(\pi_2)$, to one possible state in which dimensionless arguments π_1 and π_2 have constant values. In each such point of function $\pi_1 = \varphi\,(\pi_2)$ an infinite number of physically similar cases (′) can be plotted, which we can express as:

$$\pi_1 = \pi_1' \quad (1) \quad (22)$$

$$\pi_2 = \pi_2' \quad (1) \quad (23)$$

Based on the similarity constants (*c*) of various physical quantities between the model (PSHP Ružín) and future (new) plant, the expression for turbine operation is as follows:

$$\frac{Z'_1}{Z_1} = c_{Z_1},\ \frac{Q'_{V,t}}{Q_{V,t}} = c_{Q_{V,t}},\ \frac{H'_t}{H_t} = c_{H_t},\ \frac{\tau'_t}{\tau_t} = c_t,\ \frac{C'_1}{C_1} = c_{C_1} \tag{24}$$

… and model laws are in the form:

$$1 = \frac{c_{Z_1} \cdot c_{H_t}}{c_{C_1} \cdot c^2_{Q_{V,t}}} \quad (1) \tag{25}$$

$$1 = \frac{c_{Q_{V,t}} \cdot c_{\tau_t}}{c^3_{H_t}} \quad (1) \tag{26}$$

Using these laws the profit for the new plant can be predicted, which in comparison with PSHP Ružín has different operating conditions. Assuming that the efficiency during the turbine and the pump operation of the model and the plant itself will be the same (using the same turbines and pumps), there may be model laws (25) and (26) affecting the profit from the power generation at the future plant by changing the three selected similarity constants, and the other two coefficients are calculated from the model laws. Assuming that the price of the power generated in the plant and the model will be the same (c_{C_1} = 1), the flow in the future plant will be, for example, two times higher than in the model ($c_{Q_{V,t}}$ = 2); the duration of turbine operation at the future plant will be two times longer (c_{τ_t} = 2), from the other two constants of proportionality (24); the necessary head will be determined; and then the profit from power generation.

For the head is valid $c^3_{H_t} = c_{\tau_t} \cdot c_{Q_{V,t}} = 2 \cdot 2 = 4$, i.e., $c_{H_t} = \sqrt[3]{4} = 1{,}5874$. The future plant will need to be secured at its design volume of flow and the planned duration of turbine operation, and with head about 1.6 times larger than was determined in the model (PSHP Ružín).

Profit from power generation following the designed hydropower plant results from equation (25). For those selected similarity constants of relevant variables the profit at the future plant will be calculated as being about 2.5 times higher than in the model, as it applies:

$$c_{Z_1} = \frac{c_{C_1} \cdot c^2_{Q_{V,t}}}{c_{H_t}} = \frac{1 \cdot 2^2}{1{,}5874} = 2{,}5198$$

For any future PSHP the profit from power generation can be predicted using this procedure.

Conclusions

The issue of optimal operation of pumped storage power stations in the current market conditions involves deep consideration in the present research. Today we can profit from the production of the pumped storage determined as the product of the electricity produced in the specified time (e.g., a month) and the price of energy produced. The generated power must be so-called "pure energy" that is channeled into the power system. The resulting gain is reduced by the cost of energy, which in due time

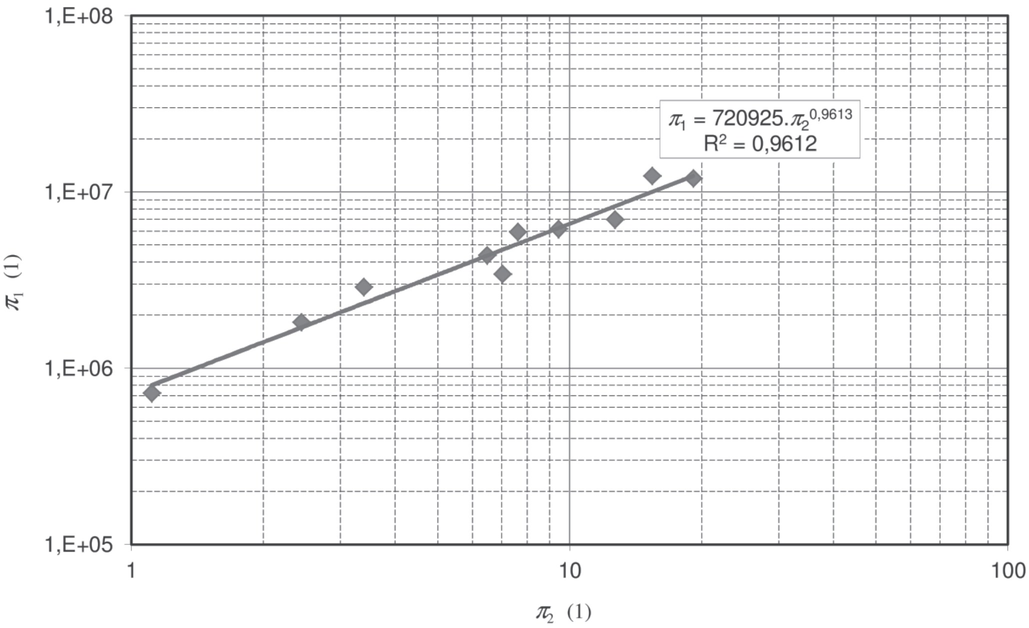

Fig. 2. Dependence of dimensionless arguments presented in logarithmic coordinates for turbine operation in the peak morning hours.

is consumed to operate a pump or the actual operation of a hydropower plant. Following the procedure described in the paper, a qualified definition of the effects of individual parameters on the resulting profit from production in the other way has been presented so far. The scientific contribution of the present contribution is based on a new approach to a comprehensive solution of complex energy processes. The derived mathematical model showing the profit from energy production has universal validity and can be used for any pumped-storage hydro power plant, which satisfies the condition of approximate similarity.

If the new plant satisfies the condition of similarity, i.e., if the indicators of similarity derived from similarity constants are equal to one, then the profit from power generation can be clearly expressed using equation (21). If this condition cannot be met, it is necessary to determine the intercepts and regression coefficients for such a facility on the basis of the experiment.

The present procedure statements of profit from production cannot be compared with other studies in the research area, because this information is missing from the area. The reason is that what has been described by mathematical interpretation has not yet been used for expressing economic indicators. This is a new approach that is currently being tested by the authors in several areas.

Based on the submitted results it can be concluded that dimensional analysis can be a suitable tool not only for expressing the technical characteristics, but also the economic characteristics. The results are gathered for the pump storage power plant case study as an important part of future energy system investments. Furthermore, the results can be adapted to investments other than pump storage plants.

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