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Original Research

Role of Graph Theory to Facilitate Landscape Connectivity: Subdivision of a Harary Graph

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Abstract

This work focuses on mapping landscape connectivity by making use of a subdivision of a Harary graph through super edge antimagic total labeling. This study employs a Harary graph by inserting h vertices in each edge, where $h = 2n, n \geq 1$ using the super (a, 2) edge antimagic total labeling and labeling the vertices and edges by taking the difference of arithmetic progression as 2 i.e. $d = 2$. We divided this paper into two parts. In first part, when the order of the subdivided harary graphs p varies then the distance t will remain the same, while in the other part, when the order p varies then distance t will also vary.

Keywords: Harary graph, super (a,d)-EAT, subdivision of Harary graph, landscape connectivity

Introduction

For a long time the surface and subsurface places have mirrored the social, traditional, and economic features considered as a marker of heritage. This should be passed on to upcoming generations for its protection, and for such an approach landscape connectivity activities should be given priority. The protection and sustainability of an area is a prime objective during modeling of the landscape connectivity design. Mapping landscape connectivity works best at landscape scale, where the given population or species are highly diversified with different forms of connectivity based on geographic information system (GIS) to quantify connectivity [1-8]. A growing number of quantitative approaches facilitate measuring and mapping connectivity, which can integrate large amounts of information needed to evaluate connectivity for a given population or species. Th hour is needed to identify an effective approach for maintaining and restoring connectivity [9-12], and GIS is a valuable technique in this regard [13-17]. A graph represents the landscape as a set of nodes and edges. The nodes are the distinct entities in the landscape where edges represent connectivity between nodes as shown in Fig.1. Edges may or may not be interconnected and deliver information about connectivity [18]. Landscape connectivity is characterized by graph-making with to base on GIS. Some of studies show that a GIS-based approach is used to quantify landscape connectivity [1-8]. Landscapes or networks connect the people in many ways and can be viewed as a network of environmental territory connected by scattering individuals in [19]. The arrangement of a network along with its nodes and connecting lines is worth noticing, as it is one of the growing properties that affects humanity in various ways [20].

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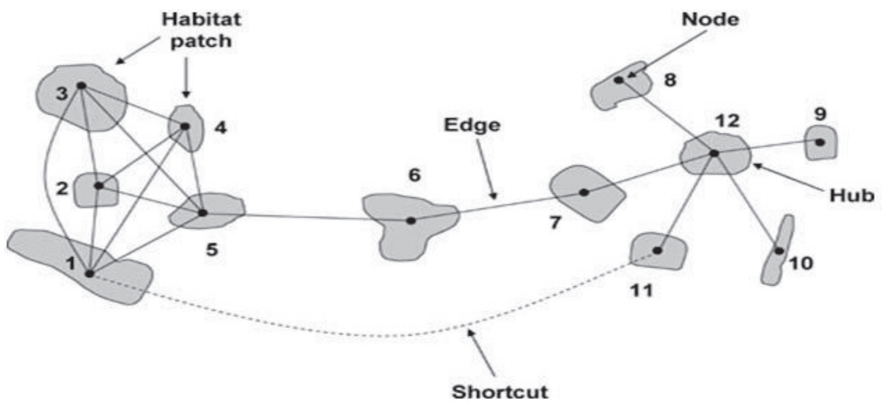

Fig. 1. Landscape connectivity.

In this paper we investigated finite, simple, and undirected graphs. Graph G consists of the vertex-set $V(G)$ and the edge-set $E(G)$.

A graph labeling is a mapping that assigns numbers to graph elements. The domain will be the set of all vertices, the set of all edges, or the set of all vertices and edges in this paper. Total labeling is a labeling in which the domain is set of vertices and edges. If the sum of edge weights constitutes an arithmetic progression with initial term and difference d, then the total labeling is said to be (a, d) edge antimagic total labeling. If the least labels are assigned to vertices, then the total labeling is known as super (a, d)-edge antimagic total labeling. Harmonius, cordial, graceful, and antimagic are the types of graph labelings. This paper deals with the super (a, d)-edge antimagic total labeling.

The super magic labeling was introduced in various classes [21-22]. For (V, E) graph G, a bijective mapping $f : V(G) \cup E(G) \rightarrow \{1,2,3,...|V| + |E|\}$ is an edge-magic total labeling of G if $f(x) + f(xy) + f(y) = k(constant)$, where k is a constant, independent of the choice of edge $xy \in E(G)$. The concept of edge magic total labeling and super edge-magic total labeling of graph G was appplied by various authors for landscape connectivity [23-26] as a bijective function $f : V(G) \cup E(G) \rightarrow \{1,2,3,...|V| + |E|\}$, such that in addition to being an edge-magic total labeling of G , if it satisfies the extra property that is $f(V(G)) = \{1,2,3,...,v\}$. Wallis named this labeling strongly edge-magic.

The concept of antimagic labeling was introduced by Kim and Keown in [27-28]. In their terminology, graph G is called antimagic if its edges are labeled with labels $\{1,2,3,...,e\}$ in such a way that all vertex-weights are pairwise distinct, where a vertex-weight of a vertex v is the sum of labels of all the edges incident with v.

An (a, d)-EAT labeling of graph G is defined as a one-to-one mapping f from $V(G) \cup E(G)$ to the set $\{1,2,3,..., v + e\}$ so that the set of edge-weights $\{f(x) + f(xy) + f(y) : xy \in E(G)\}$ equals $\{a, a + d, a + a2,...a + (e - 1) d\}$ for two integers $a>0$ and $d\geq 0$. Notice that the same labeling would be an edge-magic total labeling when $d = 0$. In other words an $(a,0)$ -EAT labeling is an EMT labeling of G.

An (a,d)-EAT labeling is called super if the smallest labels appear on the vertices of G, i.e., $f(V(G)) = \{1,2,3,...,v\}$. The (a,d)-edge antimagic total labeling and super (a,d)-edge antimagic total labelings are natural extensions of the notion of an edge-magic total labeling [29-31].

Ngurah et al. [9] proved that mC_n $(n\geq 3)$ has an (a,d)-edge antimagic total labeling in the following cases: $(a,d) = (\frac{5mn}{2}+2,1)$, where m is even, $(a,d) = (2mn + 2,2)$, $(a,d) = (\frac{3mn+5}{2},3)$ for m and n odd, $(a,d) = ((mn + 3),4)$ for m and n odd, and mC_n has a super $(2mn + 2, 1)$-edge antimagic total labeling. They also proved that the following C_n has a super (a,d)-edge antimagic total labeling if either d is 0 or 2 and n is odd, or $d = 1$; for odd $(n\geq 3)$ and $m = 1$ or 2, the generalized Peterson graph $P(n,m)$ has a super $(\frac{11n+3}{2},0)$-edge antimagic total labeling and a super $(\frac{5n+5}{2},2)$-edge antimagic total labeling; and for odd $n\geq 3$, $P(n,\frac{n-1}{2})$ has a super $(\frac{11n+3}{2},0)$-edge antimagic total labeling and a super $(\frac{5n+5}{2},2)$-edge antimagic total labeling.

Super edge-antimagic total labeling for Harary graphs C^t_p was constructed by Hussain et al. [28]. They worked on super (a,d)-edge antimagic total labeling and super (a, d)-vertex antimagic total labeling. They also constructed the super edge-antimagic and super vertex-antimagic total labelings for a disjointed union of k identical copies of the Harary graph.

Super edge-antimagic total labeling and super edge magic labeling for subdivided stars were made in [33-35]. Javaid et al. [36] proved (a,d)-EAT labeling of extended w-trees and super edge-magic total labeling on w-trees was defined [37]. All trees are super edge magic with at most 17 vertices proven [38-39].

Harary Graph

For $t \geq 2$ and $p \geq 4$, a Harary graph C^t_p is a graph constructed from a cycle C_p by joining any two vertices at distance t in C_p.

Subdivided Harary Graph

For $t \geq 6$ and $p \geq 6$, a subdivided Harary graph $C^{t,h}_p$ is a graph constructed from Harary graph C^t_p after the subdivision (for even $h \geq 2$) of each edge of graph C^t_p.

Super (a,2)-edge Antimagic Total Labeling of Subdivided Harary Graphs

Main Results

We prove that subdivided Harary graph is a super (a,2)-edge antimagic total labeling for even $h \geq 2$.

Theorem 2.1.1. For any $p \geq 25$ with $h = 2$ and for $t = 6$, $G \cong C^{6,2}_p$ admits a super ($4p + 1$,2) edge-antimagic total labeling.

Proof. Consider the vertex and edge set of G as $|V(G)| = p$ and $|E(G)| = q$. Let $V_1(G)$, $E_1(G)$ and $V_2(G)$, $E_2(G)$ denote the vertices on the outer and inner cycles, respectively. The vertex $[V(G) = V_1(G) \cup V_2(G)]$ and edge $[E(G) = E_1(G) \cup E_2(G)]$ sets of G are defined as follows:

$$V(G) = \{v_i : 1 \leq i \leq \frac{3p}{5}\} \cup \{v^j_\alpha : 1 \leq i \leq \frac{p}{5}, 1 \leq j \leq h\},$$

$$E(G) = \{v_i v_{i+1} : 1 \leq i \leq \frac{3p}{5}\} \cup \{v_{3i-2} v^1_\xi : 1 \leq i \leq \frac{p}{5}\} \cup$$
$$\{v^r_\xi v^{r+1}_\xi : 1 \leq i \leq \frac{p}{5}, 1 \leq r \leq h-1\} \cup \{v^h_\xi v_\eta : 1 \leq i \leq \frac{p}{5}\},$$

where $\zeta = 3i - 2, 3i - 2 + t$, $\eta = 3i - 2 + t$ and all indices are taken in mod $\frac{3p}{5}$. Now we define labeling $\lambda : V(G) \cup E(G) \rightarrow \{1,2,3,..., p + q\}$. We label the vertices on outer and inner cycle as follows:

$$\lambda(v_i) = \begin{cases} 2i - \frac{4p}{5} + 1, & for\ i = \frac{3p}{5}; \\ \frac{2p}{5} + 1 + 2i, & for\ 1 \leq i \leq \frac{3p}{5} - 1. \end{cases}$$

$$\lambda(v^r_\xi) = \begin{cases} 2p + (4 - 2s) - 2i, & for\ 1 \leq i \leq \frac{p}{5}, r = 2i - 1, s = 1; \\ \frac{2p}{5} - 3 - 2i, & for\ 1 \leq i \leq \frac{p}{5} - 2, r = 2; \\ \frac{4p}{5} - 4 - 2i, & for\ \frac{p}{5} - 1 \leq i \leq \frac{3p}{5}, r = 2. \end{cases}$$

We label the edges on outer and inner cycle as follows:

$$\lambda(v_i v_{i+1}) = \begin{cases} \frac{16p}{5} + 2q - 3 - 2i, & for\ \frac{3p}{5} - 1 \leq i \leq \frac{3p}{5}; \\ 2p + 2q - 3 - 2i, & for\ 1 \leq i \leq \frac{3p}{5} - 2. \end{cases}$$

$$\lambda((v_{3i-2} v^1_\xi) = \frac{12q}{6} + 1 - 2i, 1 \leq i \leq \frac{p}{5}.$$

$$\lambda(v^r_\xi v^{r+1}_\xi) = \frac{12q}{6} - (3 - 2s) + 2i, 1 \leq i \leq \frac{p}{5}, r = 2s - 1, s = 1.$$

$$\lambda(v^h_\xi v_\eta) = \begin{cases} \frac{8q}{3} - 3 - 2i, & for\ 1 \leq i \leq \frac{p}{5} - 2; \\ \frac{8q}{3} + \frac{2p}{5} - 3 - 2i, & for\ \frac{p}{5} - 1 \leq i \leq \frac{p}{5}. \end{cases}$$

Edge weights of all edges in $E_1(G)$ will form consecutive integers $4p+1, 4p+2, ..., \frac{26p}{5} - 1$, where the weight $4p + 1$ is obtained by the edge $v^h_{\frac{3p}{5}-5,1} v_1$ if $\frac{3p}{8} \neq t$. Edge weights of all edges in $E_2(G)$ will form consecutive integers $\frac{26p}{5}, \frac{26p}{5}+1, \frac{26p}{5}+2, ..., \frac{32p}{5} - 1$. Therefore, all the edge weights form consecutive integers $4p+1, 4p+2, ..., \frac{32p}{5} - 1$. Since all vertices receive smallest labels so λ is a super ($4p + 1$,2) edge antimagic total labeling. Fig. 2 shows super (141,2)–*EA*T labeling of $C^{6,2}_{35}$.

Theorem 2.1.2. For any $p \geq 20n + 5$, with $h = 2n, n \geq 1$ and for $t = 4n + 2, n \geq 1$, $G \cong C^{4n+2,2n}_p$ admits a super ($4p + 1$,2) edge-antimagic total labeling.

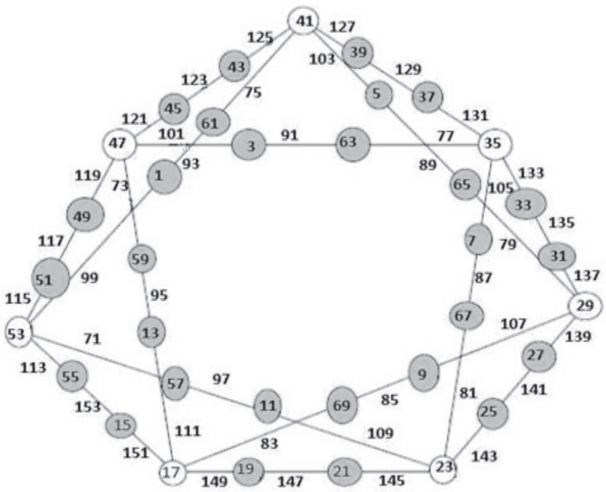

Fig. 2. Super (141,2)-EAT labeling of $C^{6,2}_{35}$.

Proof. Let us denote the vertex and edge set of G as $|V(G)| = p$ and $|E(G)| = q$. Let $V_1(G)$, $E_1(G)$ and $V_2(G)$, $E_2(G)$ denote the vertices on the outer and inner cycles, respectively. The vertex $[V(G) = V_1(G) \cup V_2(G)]$ and edge $[E(G) = E_1(G) \cup E_2(G)]$ sets of G are defined as follows:

$$V(G)=\{v_i : 1\le i\le \frac{(2n+1)p}{4n+1}, n\ge 1\}\cup\{v_\alpha^j : 1\le i\le \frac{p}{4n+1}, 1\le j\le h, n\ge 1\},$$

$$E(G)=\{v_iv_{i+1} : 1\le i\le \frac{(2n+1)p}{4n+1}, n\ge 1\}\cup \{v_{(2n+1)i-2n}v_\alpha^1 : 1\le i\le \frac{p}{4n+1}, n\ge 1\}$$

$$\cup\{v_\alpha^j v_\alpha^{j+1} : 1\le i\le \frac{p}{4n+1}, 1\le j\le h-1, n\ge 1\}\cup \{v_\alpha^h v_\beta : 1\le i\le \frac{p}{4n+1}, n\ge 1\},$$

where $\alpha = (2n + 1)i - 2n$, $(2n + 1)i - 2n + t$, $\beta = (2n + 1)i - 2n + t$ and all indices are taken in mod $\frac{(2n+1)p}{4n+1}, n\ge 1$.

Now we define labeling $\lambda : V(G) \cup E(G) \rightarrow \{1,2,3,..., p + q\}$. We label the vertices on outer and inner cycle as follows:

$$\lambda(v_i)=\begin{cases}2i-\frac{(2n+2)p}{4n+1}+2n-1, & \frac{(2n+1)p}{4n+1}-(n-1)\le i\le \frac{(2n+1)p}{4n+1}, n\ge 1;\\ \frac{2np}{4n+1}+2n-1+2i, & 1\le i\le \frac{(2n+1)p}{4n+1}-n, n\ge 1.\end{cases}$$

$$\lambda(v_\alpha^j)=\begin{cases}2p+\gamma, & 1\le i\le \frac{p}{4n+1}, j=2k-1, 1\le k\le n, n\ge 1;\\ \frac{(2n+2)p}{4n+1}+\gamma, & 1\le i\le \frac{p}{4n+1}, j=2k, 1\le k\le n-1, n\ge 2;\\ \frac{2np}{4n+1}-\delta, & 1\le i\le \frac{p}{4n+1}-2, j=h, n\ge 1;\\ \frac{4np}{4n+1}-\delta, & \frac{p}{4n+1}-1\le i\le \frac{p}{4n+1}, j=h, n\ge 1,\end{cases}$$

where $\gamma = (2n - 2k + 1) - 2ni$ and $\delta = (4n - 1) + 2ni$. The edges on the outer and inner cycles are labelled as:

$$\lambda(v_kv_{k+1})=\begin{cases}\frac{(12n+4)p}{4n+1}+2q-(2n+1)-2i, & \frac{(2n+1)p}{4n+1}-n\le i\le \frac{(2n+1)p}{4n+1}, n\ge 1;\\ 2p+2q-(2n+1)-2i, & 1\le i\le \frac{(2n+1)p}{4n+1}-(n+1), n\ge 1.\end{cases}$$

$$\lambda(v_{(2n+1)i-2n}v_\alpha^j)=\frac{(10n+2)q}{4n+2}+1-2ni, 1\le i\le \frac{p}{4n+1}, n\ge 1.$$

$$\lambda(v_\alpha^j v_\alpha^{j+1})=\begin{cases}\frac{(10n+2)q}{4n+2}-\zeta, & 1\le i\le \eta, j=2k-1, 1\le k\le n, n\ge 1;\\ \frac{(8n+2)q}{4n+2}-\rho, & 1\le i\le \eta, j=2k, 1\le k\le n-1, n\ge 2,\end{cases}$$

where $\zeta = (2n - 2k + 1) - 2ni$, $\rho = (2n - 1 - 2k) - 2ni$ and $\eta=\frac{\xi}{4n+1}$.

$$\lambda(v_\alpha^h v_\beta)=\begin{cases}\frac{(6n+2)q}{2n+1}-3-2i, & 1\le i\le \frac{p}{4n+1}-2, n\ge 1;\\ \frac{(6n+2)q}{2n+1}+\frac{2p}{4n+1}-3-2i, & \frac{p}{4n+1}-1\le i\le \frac{p}{4n+1}, n\ge 1.\end{cases}$$

Edge weights of all edges in $E_1(G)$ will form consecutive integers $4p+1, 4p+2, ..., \frac{(20n+6)p}{4n+1}-1, n\ge 1$, where the weight $4p + 1$ is obtained by the edge $v^h_{\frac{(2n+1)p}{4n+1}-(4n+1),1}v_1$, if $\frac{(2n+1)p}{6n+2}\ne t, n\ge 1$. Edge weights of all edges in $E_2(G)$ will form consecutive integers $\frac{(20n+6)p}{4n+1}, \frac{(20n+6)p}{4n+1}+1, ..., \frac{(24n+8)p}{4n+1}-1, n\ge 1$.

Therefore, all the edge weights form consecutive integers $4p+1, 4p+2, ..., \frac{(24n+8)p}{4n+1}-1, n\ge 1$. Since all vertices receive the smallest labels, λ is a super $(4p + 1,2)$-EAT labeling.

Theorem 2.1.3. For any even p, $p\ge 16$ with $h = 2$ and for any t (which is multiple of 3), $t\ge 6$, $G\cong C_p^{t2}$ admits a super $(4p + 1,2)$ edge-antimagic total labeling.

Proof. Let us denote the vertex and edge set of G as $|V(G)| = p$ and $|E(G)| = q$. Let $V_1(G)$, $E_1(G)$ and $V_2(G)$, $E_2(G)$ denote the vertices on the outer and inner cycles, respectively. The vertex $[V(G) = V_1(G) \cup V_2(G)]$ and edge $[E(G) = E_1(G) \cup E_2(G)]$ sets of G are defined as follows:

$$V(G)=\{v_i : 1\le i\le \frac{3p}{4}\}\cup\{v_\alpha^j : 1\le i\le \frac{p}{8}, 1\le j\le h\},$$

$$E(G)=\{v_iv_{i+1} : 1\le i\le \frac{3p}{4}\}\cup\{v_{3i-2}v_\alpha^1 : 1\le i\le \frac{p}{8}\}$$

$$\cup\{v_\alpha^j v_\alpha^{j+1} : 1\le i\le \frac{p}{8}, 1\le j\le h-1\}$$

$$\cup\{v_\alpha^h v_\beta : 1\le i\le \frac{p}{8}\},$$

where $\alpha = 3i - 2, 3i - 2 + t$, $\beta = 3i - 2 + t$ and all indices are taken in mod $\frac{3p}{4}$. Now we define labeling $\lambda : V(G) \cup E(G) \rightarrow \{1,2,3,..., p + q\}$. We label the vertices on outer and inner cycle as follows:

$$\lambda(v_i) = \begin{cases} 2i - \dfrac{10p}{8} + 1, & for\ i = \dfrac{3p}{4}; \\ \dfrac{2p}{8} + 1 + 2i, & for\ 1 \le i \le \dfrac{3p}{4} - 1. \end{cases}$$

$$\lambda(v_\alpha^j) = \begin{cases} 2p + (3-2k) - 2i, & for\ 1 \le i \le \dfrac{p}{8},\ j = 2k-1,\ k = 1; \\ \dfrac{4p}{8} - 3 - 2i, & for\ 1 \le i \le \dfrac{p}{8},\ j = 2k,\ k = 1. \end{cases}$$

The edges on outer cycle and inner cycle are labelled as:

$$\lambda(v_i v_{i+1}) = \begin{cases} \dfrac{14p}{4} + 2q - 3 - 2i, & for\ \dfrac{3p}{4} - 1 \le i \le \dfrac{3p}{4}; \\ 2p + 2q - 3 - 2i, & for\ 1 \le i \le \dfrac{3p}{4} - 2. \end{cases}$$

$$\lambda(v_{3i-2} v_\alpha^j) = \frac{18q}{9} + 1 - 2i, 1 \le i \le \frac{p}{8}.$$

$$\lambda(v_\alpha^j v_\alpha^{j+1}) = \frac{18q}{9} - (3-2k) + 2i, 1 \le i \le \frac{p}{8},\ j = 2k-1, k = 1.$$

$$\lambda(v_\alpha^h v_\beta) = \frac{22q}{9} + \frac{2p}{8} - 3 - 2i, 1 \le i \le \frac{p}{8}.$$

$E_1(G)$ has edge weights of all edges form consecutive integers as: $4p+1, 4p+2, ..., \frac{38p}{8} - 1$, where the weight $4p + 1$ is obtained by the edge $v^h_{1,\frac{3p}{8}+1} v_{\frac{3p}{8}+1}$ if $\frac{3p}{8} = t$.

$E_2(G)$ has edge weights of all edges form consecutive integers as: $\frac{38p}{8}, \frac{38p}{8}+1, \frac{38p}{8}+2, ..., \frac{50p}{8}-1$.

Combining both the sequences $E_1(G)$ and $E_2(G)$ as: $4p+1, 4p+2, ..., \frac{50p}{8} - 1$.

As all vertices obtain the smallest labels, so λ is a super $(4p + 1,2)$-EAT labeling (Fig. 3).

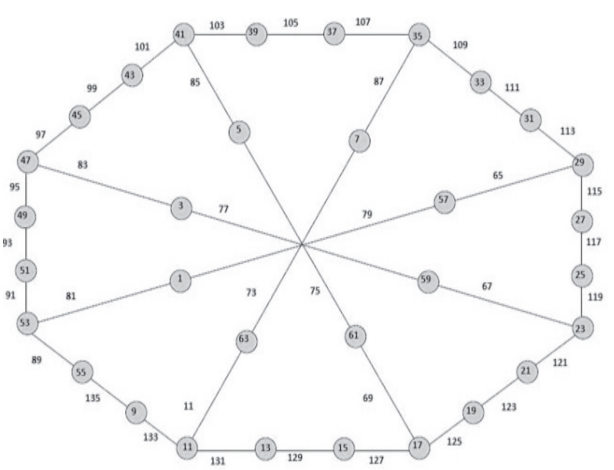

Fig. 3. Super (161, 2)-EAT labeling of $C_{40}^{12,2}$.

Theorem 2.1.4. For any even p, $p \ge 12n + 4$ with $h = 2n$, $n \ge 1$ and for any t (which is multiple of $2n + 1$), $t \ge 4n + 2$, $n \ge 1$, $G \cong C_p^{t,2n}$ admits a super $(4p + 1,2)$ edge-antimagic total labeling.

Proof. Let us denote the vertex and edge set of G as $|V(G)| = p$ and $|V(G)| = q$. Let $V_1(G)$, $E_1(G)$ and $V_2(G)$, $E_2(G)$ denote the vertices on the outer and inner cycles, respectively. The vertex $[V(G) = V_1(G) \cup V_2(G)]$ and edge $[E(G) = E_1(G) \cup E_2(G)]$ sets of G are defined as follows:

$$V(G) = \{v_i : 1 \le i \le \frac{(2n+1)p}{3n+1}, n \ge 1\}$$
$$\cup \{v_\alpha^j : 1 \le i \le \frac{p}{6n+2}, 1 \le j \le h, n \ge 1\},$$

$$E(G) = \{v_i v_{i+1} : 1 \le i \le \frac{(2n+1)p}{3n+1}, n \ge 1\}$$
$$\cup \{v_{(2n+1)i-2n} v_\alpha^1 : 1 \le i \le \frac{p}{6n+2}, n \ge 1\}$$
$$\cup \{v_\alpha^j v_\alpha^{j+1} : 1 \le i \le \frac{p}{6n+2}, 1 \le j \le h-1, n \ge 1\}$$
$$\cup \{v_\alpha^h v_\beta : 1 \le i \le \frac{p}{6n+2}, n \ge 1\},$$

where $\alpha = (2n + 1)i - 2n$, $(2n + 1)i - 2n + t$, $\beta = (2n + 1)i - 2n + t$ and all indices are taken in mod $\frac{(2n+1)p}{3n+1}$, $n \ge 1$. Now we define labeling $\lambda : V(G) \cup E(G) \rightarrow \{1,2,3,..., p + q\}$. We label the vertices on outer and inner cycle as follows:

$$\lambda(v_i) = \begin{cases} 2i - \dfrac{(6n+4)p}{6n+2} + 2n - 1, & \dfrac{(2n+1)p}{3n+1} - (n-1) \le i \le \dfrac{(2n+1)p}{3n+1}, \forall\ n \ge 1; \\ \dfrac{2np}{6n+2} + 2n - 1 + 2i, & 1 \le i \le \dfrac{(2n+1)p}{3n+1} - n, \forall\ n \ge 1. \end{cases}$$

$$\lambda(v_\alpha^j) = \begin{cases} 2i + \gamma, & 1 \le i \le \dfrac{p}{6n+2},\ j = 2k-1, 1 \le k \le n, n \ge 1; \\ \dfrac{(2n+2)p}{6n+2} + \gamma, & 1 \le i \le \dfrac{p}{6n+2},\ j = 2k, 1 \le k \le n, n \ge 1, \end{cases}$$

where $\gamma = (2n - 2m + 1) - 2ni$. We label the edges on outer and inner cycles as follows:

$$\lambda(v_i v_{i+1}) = \begin{cases} \dfrac{(10n+4)p}{3n+1} + 2q - (2n+1) - 2i, & \dfrac{(2n+1)p}{3n+1} - n \le i \le \dfrac{(2n+1)p}{3n+1}, n \ge 1; \\ 2p + 2q - (2n+1) - 2i, & 1 \le i \le \dfrac{(2n+1)p}{3n+1} - (n+1), n \ge 1. \end{cases}$$

$$\lambda(v_{(2n+1)i-2n} v_\alpha^j) = \frac{(14n+4)q}{6n+3} + 1 - 2ni, 1 \le i \le \frac{p}{6n+2}, n \ge 1.$$

$$\lambda(v_\alpha^j v_\alpha^{j+1}) = \begin{cases} \dfrac{(14n+4)q}{6n+3} - \delta, & 1 \le i \le \dfrac{p}{6n+2},\ j = 2k-1, 1 \le k \le n, n \ge 1; \\ \dfrac{(12n+4)q}{6n+3} - \zeta, & 1 \le i \le \dfrac{p}{6n+2},\ j = 2k, 1 \le k \le n-1, n \ge 2, \end{cases}$$

where $\delta = (2n + 1 - 2k) - 2ni$ and $\zeta = (2n - 2k + 1) - 2ni$.

$$\lambda(v_\alpha^h v_\beta) = \frac{(16n+6)q}{6n+3} + \frac{2p}{6n+2} - 3 - 2i, 1 \le i \le \frac{p}{6n+2}, n \ge 1.$$

Edge weights of all edges in $E_1(G)$ will form consecutive integers $4p+1, 4p+2, ..., \frac{(28n+10)p}{6n+2} - 1$, $n \ge 1$, where the weight $4p + 1$ is obtained by the edge

$v^{h}_{1,\frac{(2n+1)p}{6n+2}+1} v_{\frac{(2n+1)p}{6n+2}+1}$ if $\frac{(2n+1)p}{6n+2}=t$, $n \geq 1$. Edge weights of all edges in $E_2(G)$ will form consecutive integers $\frac{(28n+10)p}{6n+2}, \frac{(28n+10)p}{6n+2}+1, \ldots, \frac{(36n+14)p}{6n+2}-1$, $n \geq 1$.

Therefore, all the edge weights form consecutive integers $4p+1, 4p+2, \ldots, \frac{(36n+14)p}{6n+2}-1$, $n \geq 1$. Since all vertices receive smallest labels, λ is a super $(4p + 1,2)$-EAT labeling.

This work focuses on landscape connectivity as it is an important part of biodiversity preservation efforts. It subsidizes the assurance of approach to the genetic variability and persistence of extinct biota, which helps stabilize the adverse effects of habitat crumbling. It is also affected regarding species range in response to climatic diversification.

The application of graph-theory, particularly Harary graph, has been endorsed in the last decade for this purpose [41-45]. This can work as an effective analytical tool for the study of the landscape fragmentation effects on the fauna and to augment the selection of reserve networks. Particularly, the graph structures have been revealed to be an influential way of modeling landscape networks and performing complex analysis regarding [46].

The advantage of a graph-theory application over the other techniques is the special framework that is made applicable to very small sets of data. Graph theory can be applied for huge populations and also provide leverage on applications concerned with landscape connectivity [42-48].

Conclusions

This model is an effective approach for landscape connectivity in an authentic and reliable way. Graph theory and its implementation through subdivision of a Harary graph by antimagic total labeling helps to develop connectivity responses and to reduce the uncertainty associated with previous models. The hour is to preserve and protect the surface and subsurface environment without disturbing the original character of the region. Keeping this in view, landscape connectivity by using this graphic approach and design is worth applicable.

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